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On generalized $\mathcal{T}$-Curvature Tensor

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***Abstract*-- The object of the present paper is to generalize $\mathcal{T}$-curvature tensor of para-Kenmotsu manifold with the help of a new generalized (0,2) symmetric tensor $\mathcal{Z}$ introduced by Mantica and Suh [7]. Various geometric properties of generalized $\mathcal{T}$-curvature tensor of para-Kenmotsu manifold have been studied.**

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I. INTRODUCTION

Several years ago, the notion of paracontact metric structures were introduced in [4].

Since the publication of [15], paracontact metric manifolds have been studied by many authors in recent years. The importance of para-Kenmotsu geometry, have been pointed out especially in the last years by several papers highlighting the exchanges with the theory of para-Kähler manifolds and its role in semi-Riemannian geometry and mathematical physics [3, 5, 6, 11, 8].

Tripathi and Gupta [14] had developed the notion of $\mathcal{T}$-curvature tensor in pseudo-Riemannian manifolds. They defined $\mathcal{T}$-curvature tensor as follows.

Definition 1.1 In a n-dimensional pseudo-Riemannian manifold (M, g), a $\mathcal{T}$-curvature tensor is a tensor of type (1,3) defined by

$$\begin{aligned}\mathcal{T}(X,Y,Z) = \ & c_0R(X,Y,Z) + c_1S(Y,Z)X + c_2S(X,Z)Y \\ & + c_3S(X,Y)Z + c_4g(Y,Z)QX + c_5g(X,Z)QY \\ & + c_6g(X,Y)QZ + rc_7[g(Y,Z)X - g(X,Z)Y],\end{aligned} \tag{1.1}$$

where $X,Y,Z \in \mathfrak{X}(M)$; $c_0,c_1,c_2,c_3,c_4,c_5,c_6,c_7$ are smooth functions on M, S, Q, R, r, g are respectively the Ricci tensor, Ricci operator, curvature tensor, scalar curvature and pseudo-Riemannian metric tensor.

Definition 1.2 The Riemannian curvature tensor R of type (0,4) on M is a quadri-linear mapping $R: \mathfrak{X}(M) \times \mathfrak{X}(M) \times \mathfrak{X}(M) \times \mathfrak{X}(M) \longrightarrow C^\infty(M)$ defined by $'R(X,Y,Z,W) = g(R(X,Y,Z),W)$ *for any* $X,Y,Z,W \in \mathfrak{X}(M)$

$\mathcal{T}$-curvature tensor reduces to many other curvature tensors for different values of $c_0,c_1,c_2,c_3,c_4,c_5,c_6,c_7$.

Definition 1.3 A $\mathcal{T}$-curvature tensor of type (0,4) is defined by

$$\begin{aligned}'\mathcal{T}(X,Y,Z,W) = \ & c_0\,'R(X,Y,Z,W) + c_1S(Y,Z)g(X,W) + c_2S(X,Z)g(Y,W) \\ & + c_3S(X,Y)g(Z,W) + c_4g(Y,Z)S(X,W) + c_5g(X,Z)S(Y,W) \\ & + c_6g(X,Y)S(Z,W) + rc_7[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)],\end{aligned} \tag{1.2}$$

where $X,Y,Z \in \mathfrak{X}(M)$, R is the Riemannian curvature tensor, S is the Ricci tensor, g is the pseudo-Riemannian metric tensor and $'T(X,Y,Z,W) = g(T(X,Y,Z),W)$.

In this paper, we consider the generalized $\mathcal{T}$ curvature tensor of para-Kenmotsu manifolds and study some properties of generalized $\mathcal{T}$ curvature tensor. The organisation of the paper is as follows:

After preliminaries on para-Kenmotsu manifold in section 2, we describe briefly the generalized $\mathcal{T}$ curvature tensor on para-Kenmotsu manifold in section 3 and also we study some properties of generalized $\mathcal{T}$ curvature tensor in para-Kenmotsu manifold. In section 4, we study a generalized $\mathcal{T}$ semi-symmetric para-Kenmotsu manifold is an η-Einstein manifold. Further in the section 5, we show that a generalized $\mathcal{T}$ Ricci semi-symmetric para-Kenmotsu manifold is η-Einstein manifold.

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II. PRELIMINARIES

The notion of an almost para-contact manifold was introduced by I. Sato [10].

An n-dimensional differentiable manifold M^n is said to have almost para-contact structure (ϕ, ξ, η), where ϕ is a tensor field of type (1,1), ξ is a vector field known as characteristic vector field and η is a 1-form satisfying the following relations

$$\phi^2(X) = X - \eta(X)\xi, \tag{2.1}$$

$$\eta(\phi X) = 0, \tag{2.2}$$

$$\phi(\xi) = 0, \tag{2.3}$$

and

$$\eta(\xi) = 1. \tag{2.4}$$

A differentiable manifold with almost para-contact structure (ϕ, ξ, η) is called an almost para-contact manifold. Further, if the manifold M^n has a semi-Riemannian metric g satisfying

$$\eta(X) = g(X, \xi) \tag{2.5}$$

and

$$g(\phi X, \phi Y) = -g(X, Y) + \eta(X)\eta(Y). \tag{2.6}$$

Then the structure (ϕ, ξ, η, g) satisfying conditions (2.1) to (2.6) is called an almost para-contact Riemannian structure and the manifold M^n with such a structure is called an almost para-contact Riemannian manifold [1, 10].

On a para-Kenmotsu manifold [2, 11, 9], the following relations hold:

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.7}$$

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.8}$$

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y), \tag{2.9}$$

$$\eta(R(X, Y, Z)) = g(X, Z)\eta(Y) - g(Y, Z)\eta(X), \tag{2.10}$$

$$R(X, Y, \xi) = \eta(X)Y - \eta(Y)X, \tag{2.11}$$

$$R(X, \xi, Y) = -R(\xi, X, Y) = g(X, Y)\xi - \eta(Y)X, \tag{2.12}$$

$$S(\phi X, \phi Y) = -(n-1)g(\phi X, \phi Y), \tag{2.13}$$

$$S(X, \xi) = -(n-1)\eta(X), \tag{2.14}$$

$$Q\xi = -(n-1)\xi, \tag{2.15}$$

$$r = -n(n-1), \tag{2.16}$$

For any vector fields X, Y, Z , where Q is the Ricci operator that is $g(QX, Y) = S(X, Y)$, S is the Ricci tensor and r is the scalar curvature.

In [2], Blaga has given an example on para-Kenmotsu manifold:

A para-Kenmotsu manifold is said to be η-Einstein if its Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y),$$

For arbitrary vector fields X and Y, where a and b are smooth functions on M^n.

III. GENERALIZED $\mathcal{T}$-CURVATURE TENSOR OF PARA-KENMOTSU MANIFOLD

In this section, we give a brief account of generalized $\mathcal{T}$-curvature tensor of para-Kenmotsu manifold and studied various geometric properties of it.

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The $\mathcal{T}$-curvature tensor is defined by Tripathi and Gupta

$$\begin{aligned} T(X,Y,Z) = \ & c_0 R(X,Y,Z) + c_1 S(Y,Z)X + c_2 S(X,Z)Y \\ & + c_3 S(X,Y)Z + c_4 g(Y,Z)QX + c_5 g(X,Z)QY \\ & + c_6 g(X,Y)QZ + rc_7[g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{3.1}$$

such a tensor field $\mathcal{T}$ is known as $\mathcal{T}$-curvature tensor.
Also, the type $(0,4)$ tensor field $'\mathcal{T}$ is given by

$$\begin{aligned} 'T(X,Y,Z,W) = \ & c_0\, 'R(X,Y,Z,W) + c_1 S(Y,Z)g(X,W) + c_2 S(X,Z)g(Y,W) \\ & + c_3 S(X,Y)g(Z,W) + c_4 g(Y,Z)S(X,W) + c_5 g(X,Z)S(Y,W) \\ & + c_6 g(X,Y)S(Z,W) + rc_7[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)], \end{aligned} \tag{3.2}$$

where

$$'\mathcal{T}(X,Y,Z,W) = g(\mathcal{T}(X,Y,Z),W)$$

and

$$'R(X,Y,Z,W) = g(R(X,Y,Z),W)$$

For the arbitrary vector fields X,Y,Z,W.

Differentiating covariantly equation (3.1) with respect to P, we get

$$\begin{aligned} (\nabla_P \mathcal{T})(X,Y)Z) = \ & c_0(\nabla_P R)(X,Y)Z) + c_1(\nabla_P S)(Y,Z)X + c_2(\nabla_P S)(X,Z)Y \\ & + c_3(\nabla_P S)(X,Y)Z + c_4 g(Y,Z)(\nabla_P Q)X \\ & + c_5 g(X,Z)(\nabla_P Q)Y + c_6 g(X,Y)(\nabla_P Q)Z \\ & + dr(P)c_7[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{3.3}$$

A new generalized $(0,2)$ symmetric tensor $\mathcal{Z}$ is defined by Mantica and Suh [7]

$$\mathcal{Z}(X,Y) = S(X,Y) + \psi g(X,Y), \tag{3.4}$$

where ψ is an arbitrary scalar function.

From equation (3.4), we have

$$\mathcal{Z}(\phi X,\phi Y) = S(\phi X,\phi Y) + \psi g(\phi X,\phi Y), \tag{3.5}$$

which on using equations (2.6) and (2.13), gives

$$\mathcal{Z}(\phi X,\phi Y) = [\psi - (n-1)][-g(X,Y) + \eta(X)\eta(Y)]. \tag{3.6}$$

From equation (3.4) in (3.2) equation reduces to

$$\begin{aligned} '\mathcal{T}(X,Y,Z,W) = \ & c_0\, 'R(X,Y,Z,W) + c_1 \mathcal{Z}(Y,Z)g(X,W) + c_2 \mathcal{Z}(X,Z)g(Y,W) \\ & + c_3 \mathcal{Z}(X,Y)g(Z,W) + c_4 g(Y,Z)\mathcal{Z}(X,W) + c_5 g(X,Z)\mathcal{Z}(Y,W) \\ & + c_6 g(X,Y)\mathcal{Z}(Z,W) + rc_7[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] \\ & - \psi[c_1 g(Y,Z)g(X,W) + c_2 g(X,Z)g(Y,W) + c_3 g(X,Y)g(Z,W) \\ & + c_4 g(Y,Z)g(X,W) + c_5 g(X,Z)g(Y,W) + c_6 g(X,Y)g(Z,W)]. \end{aligned} \tag{3.7}$$

Let

$$\begin{aligned} '\mathcal{T}^*(X,Y,Z,W) = \ & c_0\, 'R(X,Y,Z,W) + c_1 \mathcal{Z}(Y,Z)g(X,W) + c_2 \mathcal{Z}(X,Z)g(Y,W) \\ & + c_3 \mathcal{Z}(X,Y)g(Z,W) + c_4 g(Y,Z)\mathcal{Z}(X,W) + c_5 g(X,Z)\mathcal{Z}(Y,W) \\ & + c_6 g(X,Y)\mathcal{Z}(Z,W) + rc_7[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)], \end{aligned} \tag{3.8}$$

In the above equation, we get

$$'\mathcal{T}^*(X,Y,Z,W) = \; '\mathcal{T}(X,Y,Z,W) + \psi[c_1 g(Y,Z)g(X,W) + c_2 g(X,Z)g(Y,W) + c_3 g(X,Y)g(Z,W) + c_4 g(Y,Z)g(X,W) + c_5 g(X,Z)g(Y,W) + c_6 g(X,Y)g(Z,W)]. \tag{3.9}$$

Thus $'\mathcal{T}^*$ defined in equation (3.8) is called generalized $\mathcal{T}$- curvature tensor of para-Kenmotsu manifold.

If ψ=0, then from eqauation (3.9), we have

$$'\mathcal{T}^*(X,Y,Z,W) = \; '\mathcal{T}(X,Y,Z,W). \tag{3.10}$$

Lemma 1 If the scalar function ψ vanishes on para-Kenmotsu manifold, then the $\mathcal{T}$- curvature tensor and generalized $\mathcal{T}$-curvature tensor are identicle.

Lemma 2 Generalized $\mathcal{T}$-curvature tensor of para-Kenmotsu manifold satisfies Bianchi's first identity.

Remark 1 Generalized $\mathcal{T}$ -curvature tensor $'\mathcal{T}^*$ of para-Kenmotsu manifold is

- skew symmetric in first two slots.
- skew symmetric in last two slots.
- symmetric in pair of slots.

Proposition 1 Generalized $\mathcal{T}$ -curvature tensor of para-Kenmotsu manifold satisfies the following identities:

$$\begin{aligned}(a)\mathcal{T}^*(\xi,Y,Z) = -\mathcal{T}^*(Y,\xi,Z) = \; & c_0[\eta(Z)Y - g(Y,Z)\xi] + c_1[S(Y,Z) + \psi g(Y,Z)]\xi \\ & + c_2\eta(Z)Y[\psi - (n-1)] + c_3\eta(Y)Z[\psi - (n-1)] \\ & + c_4 g(Y,Z)\xi[\psi - (n-1)] + c_5\eta(Z)[QY + \psi Y] \\ & + c_6\eta(Y)[QZ + \psi Z] + rc_7[g(Y,Z)\xi - \eta(Z)Y],\end{aligned} \tag{3.11}$$

$$\begin{aligned}(b)\mathcal{T}^*(X,Y,\xi) = \; & c_0[\eta(X)Y - \eta(Y)X] + c_1\eta(Y)X[\psi - (n-1)] \\ & + c_2\eta(X)Y[\psi - (n-1)] + c_3[g(X,Y)\psi + S(X,Y)]\xi \\ & + c_4\eta(Y)[\psi X + QX] + c_5\eta(X)[\psi Y + QY] \\ & + c_6 g(X,Y)\xi[\psi - (n-1)] + rc_7[\eta(Y)X - \eta(X)Y],\end{aligned} \tag{3.12}$$

$$\begin{aligned}(c)\eta(\mathcal{T}^*(X,Y,Z)) & = c_0[g(X,Z)\eta(Y) - g(Z,Y)\eta(X)] + c_1\eta(X)[g(Z,Y)\psi + S(Z,Y)] \\ & + c_2\eta(Y)[g(Z,X)\psi + S(Z,X)] + c_3\eta(Z)[g(Y,X)\psi + S(Y,X)] \\ & + c_4\eta(X)g(Y,Z)[\psi - (n-1)] + c_5\eta(Y)g(X,Z)[\psi - (n-1)] \\ & + c_6\eta(Z)g(X,Y)[\psi - (n-1)] + rc_7[g(Y,Z)\eta(X) - g(Z,X)\eta(Y)].\end{aligned} \tag{3.13}$$

IV. GENERALIZED $\mathcal{T}$ SEMI-SYMMETRIC PARA-KENMOTSU MANIFOLD

Definition 4.1 Para-Kenmotsu manifold is said to be semi-symmetric if it satisfies the condition

$$R(X,Y) \cdot R = 0, \tag{4.1}$$

where $R(X,Y)$ is considered as the derivative of the tensor algebra at each point of the manifold.

Definition 4.2 Para-Kenmotsu manifold is said to be generalized $\mathcal{T}$ semi-symmetric if it satisfies the condition

$$R(X,Y) \cdot \mathcal{T}^* = 0, \tag{4.2}$$

where $\mathcal{T}^*$ is generalized $\mathcal{T}$-curvature tensor and $R(X,Y)$ is considered as the derivative of the tensor algebra at each point of the manifold.

Theorem 4.1 A generalized $\mathcal{T}$ semi-symmetric para-Kenmotsu manifold is an η-Einstein manifold.

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Proof. Consider

$$(R(\xi,X)\cdot\mathcal{T}^*)(U,V,Y)=0,$$

for any $X,Y,U,V\in K_LM$, where $\mathcal{T}^*$ is generalized $\mathcal{T}$-curvature tensor.Then we have

$$\begin{aligned}0 &= R(\xi,X,\mathcal{T}^*(U,V,Y))-\mathcal{T}^*(R(\xi,X,U),V,Y)\\ &-\mathcal{T}^*(U,R(\xi,X,V),Y)-\mathcal{T}^*(U,V,R(\xi,X,Y)).\end{aligned} \quad (4.3)$$

In view of the equation (2.12) above equation takes the form

$$\begin{aligned}0 &= \eta(\mathcal{T}^*(U,V,Y))X-{}'\mathcal{T}^*(U,V,Y,X)\xi-\eta(U)\mathcal{T}^*(X,V,Y)\\ &+g(X,U)\mathcal{T}^*(\xi,V,Y)-\eta(V)\mathcal{T}^*(U,X,Y)+g(X,V)\mathcal{T}^*(U,\xi,Y)\\ &-\eta(Y)\mathcal{T}^*(U,V,X)+g(X,Y)\mathcal{T}^*(U,V,\xi).\end{aligned}$$

Taking inner product of above equation with ξ and using equations (1.2), (2.4),(2.5),(2.10),(2.15), (3.9), (3.11), (3.12), (3.13), we get

$$\begin{aligned}-c_0\,{}'R(U,V,Y,X) &= c_0[g(X,V)g(Y,U)-g(X,U)g(Y,V)]+2c_1S(Y,V)g(X,U)\\
&-c_1\eta(U)\eta(V)S(X,Y)-c_1\eta(Y)\eta(U)S(X,V)-\psi c_1g(X,V)\eta(Y)\eta(U)\\
&+2\psi c_1g(Y,V)g(X,U)-c_1S(U,Y)g(X,V)-\psi c_1g(Y,U)g(X,V)\\
&-(n-1)c_1g(X,Y)\eta(V)\eta(U)-c_2S(X,Y)\eta(V)\eta(U)\\
&-c_2S(X,U)\eta(V)\eta(Y)-(n-1)c_2g(X,U)\eta(V)\eta(Y)\\
&-\psi c_2\eta(U)\eta(Y)g(X,V)+(n-1)c_2g(X,V)\eta(U)\eta(Y)\\
&-(n-1)c_2g(X,Y)\eta(V)\eta(U)+\psi c_2g(X,V)g(U,Y)\\
&-c_3S(X,V)\eta(Y)\eta(U)-2\psi c_3g(X,V)\eta(U)\eta(Y)\\
&-c_3S(X,U)\eta(Y)\eta(V)-(n-1)c_3g(X,U)\eta(V)\eta(Y)\\
&+(n-1)c_3g(X,V)\eta(U)\eta(Y)+2\psi c_3g(X,Y)g(U,V)\\
&+2c_3g(X,Y)S(U,V)-\psi c_4g(X,V)\eta(U)\eta(Y)\\
&+(n-1)c_4g(X,V)\eta(U)\eta(Y)+2\psi c_4g(X,U)g(Y,V)\\
&-(n-1)c_4g(Y,V)g(X,U)-\psi c_4g(X,V)g(U,Y)\\
&+(n-1)c_4g(Y,U)g(X,V)+(n-1)c_5g(X,V)\eta(U)\eta(Y)\\
&-\psi c_5g(X,V)\eta(U)\eta(Y)+\psi c_5g(X,V)g(U,Y)\\
&-2\psi c_6g(X,V)\eta(U)\eta(Y)+2(n-1)c_6g(X,V)\eta(U)\eta(Y)\\
&+2\psi c_6g(X,Y)g(U,V)-(n-1)c_6g(X,Y)g(U,V)\\
&+2rc_7g(X,U)g(Y,V)-2rc_7g(Y,U)g(X,V)\\
&+c_2S(Y,U)g(X,V)+c_4S(X,U)g(Y,V)+c_5S(X,V)g(Y,U)\\
&+c_6S(X,Y)g(V,U)\end{aligned}$$

Let $\{e_i: i=1,2,\ldots,n\}$ be an orthonormal basis vector putting $X=U=e_i$ in above equation and taking summation over i, we get

$$S(Y,V)=Ag(Y,V)+B\eta(V)\eta(Y)$$

where

$$A=\left[\frac{-nc_0+2n\psi c_1+2n\psi c_4-2n(n-1)c_4-(n-1)c_6+2nr\,c_7}{c_1-c_0-2nc_1-c_3-c_5-c_6}\right]$$

and

$$B=\left[\frac{c_0-c_1(2+2\psi+n-1)+(n-1)c_3+2c_4(n-1+\psi)+(n-1)c_5+2(n-1)c_6-2rc_7}{c_1-c_0-2nc_1-c_3-c_5-c_6}\right]$$

This shows that generalized $\mathcal{T}$ semi-symmetric para-Kenmotsu manifold is an η-Einstein manifold.

V. Generalized $\mathcal{T}$ Ricci semi-symmetric para-Kenmotsu manifold

Definition 5.1 Para-Kenmotsu manifold M is said to be Ricci semi-symmetric if the condition

$$R(X,Y)\cdot S = 0, \tag{5.1}$$

holds for all $X, Y \in K_L M$.

Definition 5.2 Para-Kenmotsu manifold is said to be generalized $\mathcal{T}$ Ricci semi-symmetric if the condition

$$\mathcal{T}^*(X,Y)\cdot S = 0, \tag{5.2}$$

holds for all X, Y, where $\mathcal{T}^*$ is generalized $\mathcal{T}$ curvature tensor of para-Kenmotsu manifold.

Theorem 5.1 A generalized $\mathcal{T}$ Ricci semi-symmetric para-Kenmotsu manifold is an η-Einstein manifold.

Proof. Consider

$$(\mathcal{T}^*(\xi,X)\cdot S)(U,V) = 0,$$

which gives

$$S(\mathcal{T}^*(\xi,X,U),V) + S(U,\mathcal{T}^*(\xi,X,V)) = 0,$$

Using equations (2.14) and (3.11) in above equation, we get

$$\begin{aligned}
&c_0 S(X,V)\eta(U) + (n-1)g(X,U)\eta(V) - (n-1)c_1 S(X,U)\eta(V) \\
&-\psi(n-1)g(X,U)\eta(V) + c_2 S(X,V)\eta(U)[\psi-(n-1)] \\
&+c_3 S(U,V)\eta(X)[\psi-(n-1)] - c_4(n-1)g(X,U)\eta(V)[\psi-(n-1)] \\
&+c_5 S(QX,V)\eta(U) + \psi c_5 S(X,V)\eta(U) + c_6 S(QU,V)\eta(X) \\
&+\psi c_6 S(U,V)\eta(X) - rc_7(n-1)g(X,U)\eta(V) - rc_7 S(X,V)\eta(U) \\
&+c_0 S(X,U)\eta(V) + (n-1)g(X,V)\eta(U) - (n-1)c_1 S(X,V)\eta(U) \\
&-\psi c_1(n-1)g(X,V)\eta(U) + c_2 S(X,U)\eta(V)[\psi-(n-1)] \\
&+c_3 S(V,U)\eta(X)[\psi-(n-1)] - c_4(n-1)g(X,V)\eta(U)[\psi-(n-1)] \\
&+c_5 S(QX,U)\eta(V) + \psi c_5 S(X,U)\eta(V) + c_6 S(QV,U)\eta(X) \\
&+\psi c_6 S(V,U)\eta(X) - rc_7(n-1)g(X,V)\eta(U) - rc_7 S(X,U)\eta(V) = 0,
\end{aligned}$$

Putting $U = \xi$ in the above equation and using (2.4), (2.5) (2.14) and (2.15), we get

$$S(X,V) = Ag(X,V) + B\eta(V)\eta(X),$$

where

$$A = \left[\frac{(n-1)-\psi c_1(n-1)-c_4(n-1)[\psi-(n-1)]-rc_7(n-1)}{c_0+c_2[\psi-(n-1)]-(n-1)c_5+\psi c_5-rc_7-(n-1)c_1}\right]$$

and

$$B = (n-1)\left[\frac{[1+(n-1)c_1-\psi-(2c_3+c_4+c_2)(\psi-n+1)-c_6(n-1)-2\psi c_6-c_0-\psi c_5}{c_0+c_2[\psi-(n-1)]-(n-1)c_5+\psi c_5-rc_7-(n-1)c_1}\right].$$

This shows that generalized $\mathcal{T}$ Ricci semi-symmetric para-Kenmotsu manifold is an η-Einstein manifold.

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