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A Normalized Mathematical Model for Optimum Tilt Angles Based on the Desired Solar Fraction

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***Abstract*--This paper presents a normalized mathematical model that determines the basic factors influencing domestic solar water heating systems, and in specific, discusses the optimum design time as well as the optimum tilt angle of the solar collector, bearing in mind the contribution of the energy obtained to meet the targeted load. The paper considers the required thermal capacity that should be provided by the solar collector. The study is thoroughly based on normalized mathematical parameters. In the case of 100% solar contribution, for heated water temperature of 45oC and consumption of 100 liters/Person per day for a given house sited at Tripoli city, 32.68°N latitude, the optimum design time is the month of December, the optimum tilt angle is 75°, and the solar collector area, fulfilling the following specifications: glazed type, Fr. (τα) = 0.58, a (d Fr UL=4.0(W/m2.oC), is 2.83 m2. The results designate an optimum tilt angle of 50°, which is the same angle obtained by the traditional famous technique, and solar collector area of 1.6 m2 if up to 80% of demand is to be covered by solar energy. Results obtained by RETScreen commercial computer package are compared and validated.**

***Keywords*-- Capacity Of Storage Unit, Domestic Water Heating Systems, Optimum Tilt Angle, Solar Collector Area, Solar Fraction.**

I. INTRODUCTION

It is well known that the design capacities of different components of solar energy systems are linked with each other. These components depend on several factors including: the location, the orientation, the quality and intensity of solar radiation incident on the Earth's surface, weather conditions prevailing in the region, quality and time to load, the number of cloudy days, the minimum amount of energy required to operate the system, and the percentage contribution of solar energy in the coverage of the load [1].

Khasawneh, et al [2] made a review of the main scientific concepts related to solar radiation, the models used to describe the solar radiation properties, and for the methods of calculating the optimum tilt angle for PV applications.

The optimum tilt angle for northern Jordan is then calculated by applying the appropriate methods and models.

Haixiang Zang, et al, [3] suggest a simple and universal method to obtain the optimum tilt angles by estimating the monthly mean daily global solar radiation on tilted surfaces facing directly towards the equator, which is based on monthly average daily global solar radiation data produced from Typical Meteorological Year (TMY) data. The results indicate that changing the monthly, seasonal, and yearly optimum tilt angles causes a significant yearly gain in the solar radiation for the region. General correlations are generated to estimate the optimum tilt angle of solar collectors at six typical climatic stations of China.

The yearly optimum tilt angle is determined for eight different Indian cities. A mathematical correlation is proposed to estimate the optimum tilt angle for any Indian city based on its latitude [4]. An analytical model has been developed and used to estimate the solar irradiation passing through the transparent cover of a flat collector, both with and without a bottom reflector [5]. The analytical model is based on the anisotropic sky model and takes into account a finite length system with different angular configurations and reciprocal shading and reflections between reflector and collector. Optimal inclinations of the collector and reflector for each month at 39° N latitude have been identified.

There are many good methods to search for the optimum tilt angle, however, it is hard to realize it when solar collectors are placed under obstacle's shadows. Shaoning Wang, and Bo Hong [6] present algorithms used to achieve the best tilt angle and horizontal direction for solar collector's performance under the free-form surfaces 3D obstacle's shadow.

Several studies conducted in order to obtain the best way to design different components of solar energy systems. The most common method of design is the "f-chart" method that has been deduced by Beckman and Duffie, Wisconsin University [7].

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However, this method is not able to compare and to deal with systems with different storage capacities. As well as it has limited use, since it is only valid for heating systems, in addition to that it gives good results if the temperature ranging between 20°C to 70°C. In 1979 both Klein and Beckman developed the "Φ-f chart" method which is more usable and more accurate [8]. This method can only be applied for liquid heating systems, but with less restriction related to the temperature range compared with the "f-chart" method. This paper presents a mathematical model that determines and discusses the optimum design time and the optimum tilt angle of the flat plate solar collector, keeping in mind the desired contribution of the solar collector energy to cover the required water heating load. Here, both are different from other published studies; the study site and mathematical scheme used. The mathematical model is thoroughly based on normalized mathematical parameters, where the final work outcome is the evaluation of the size of the flat plate solar collector corresponding to the optimum design time and optimum tilt angle of the solar collector that are already determined. The details are presented in the following sections.

II. Mathematical Model

1.1 Solar Irradiation And Ambient Temperature

This study is based on the monthly average daily data of both total solar irradiation data [9] and the ambient air temperature [10] for the location of Tripoli City-Libya; Latitude = 32.68° (N). The intensity of solar irradiation on a tilted surface was estimated by using the Liu and Jordan model [11].

1.2 Domestic Water Heating Load

It is known that the monthly domestic water heating load depends on several variables, including the number of days of the month, the number of people, the consumption of heated water, density and specific heat of water. The load investigated in this study was assumed to be a function of the ambient temperature (Ta) as in the following form;

$$LDWH \ \alpha \ (45° - Ta) \qquad (1)$$

Where, (Ta) is the ambient temperature, °C, (45°) is the required temperature for the hot water.

1.3 Normalization

For the general results as much as possible and to avoid the assumptions which may be far from reality, the estimated solar irradiation on a tilted surface and the load profile were normalized relative to the maximum value for each profile.

1.4 Optimized Parameters

1.4.1 Solar Collector Tilt Angle (β)

The optimum tilt angle considered in this work is the angle that provides the best matching of load curve with the solar irradiation profile. The determining criteria for the optimum tilt angle is based on an optimization factor (Fopt), defined as**;**

$$Fopt = Havg\ (\beta) \ / \ SDEV\ (\beta) \qquad (2)$$

Where, Havg (β) = Monthly average daily total solar irradiation (kJ/m^2.day) during the application period at certain collector tilt angle, and SDEV (β) is the standard deviation of the curve that represents the difference between the normalized load curve and normalized solar irradiation curve at the specified tilt angle. It can be clearly seen that increasing the optimization factor results in a better utilization of solar collection sub-system. Therefore, the optimum tilt angle occurs at the maximum value of Fopt.

1.4.2 Solar Energy Systems Design Time (TD)

The importance of the design time designation lies in the fact that solar energy is intermittent. Therefore, all solar energy systems must be designed according to a specified time. The design time, as used in this study, is the month of maximum deficit.

1.4.3 Collector Array Capacities (CA)

In the present work, two sizing parameters for the collection sub-system were considered. These parameters are; the solar collector array capacity, C_A, and the specific collector area, A_C.

The solar collector capacity, also called the solar-to-load ratio, can be defined as the ratio between the collector energy output and the load energy demand for the design month. This can be formulated as;

$$C_A = CL_{dm} / L_{dm} \qquad (3)$$

Where CL_{dm} is the monthly average of daily collector output energy for the design month and L_{dm} is the monthly average of daily load energy consumption for the design month. The specific collector area, A_C, can be defined, as the collector area required for each unit of energy demand at the design month. It can be estimated by dividing the collector array capacity, C_A, by the corresponding amount of the real radiation in the design month $I_{S,\,dm}$ (kJ/m^2.day). This can be written as;

$$A_C = C_A / I_{S,dm} \tag{4}$$

Now, it should be noted that the total solar collector area required could be calculated as follows;

$$\text{Collector area} = A_C * L_{dm} / \eta_C \tag{5}$$

Where η_C = the collection sub-system efficiency, and L_{dm} is the real value of the load demand for the design month (kJ/day), and could be calculated as follows;

$$L_{dm} = N_{person}\, V_{person}\, \rho\, c_p\, (45° – Ta) \tag{6}$$

Where L_{dm} = Daily domestic water heating load (kJ/day), N_{person} = Number of persons occupying the house, V_{person} = Volume of hot water required per person (100 liters/day), ρ = Density of water (1 kg/liter), c_p = The specific heat of water (4.18 kJ/kg.°C), and Ta = The ambient temperature, °C. More detailed expressions and discussions can be found elsewhere; Elwani [12].

III. Results And Discussions

The intermittent solar energy supply necessitates sizing the solar thermal systems to help covering a significant portion of the demand. In other words, the share of the solar energy in satisfying the energy demand should be predetermined to enable specifying accurately each of the different components of the system, and find out the factors that govern their functions. The following sections are the outcomes of the study regarding the basic factors needed for sizing the main components of solar domestic water heating systems under different meteorological and load conditions.

1.5 The Optimum Solar Collector Tilt Angle

Because solar energy is intermittent, it is necessary to have a supplementary energy source that partly covers the demand in case of insufficient solar supply, especially in the cloudy and/or low solar radiation days.

Accordingly, the estimation of the solar energy participation in satisfying the demand and computation of the flat plate collector tilt angle is carried out by the use of the aforementioned mathematical model. Figure (1) illustrates the relationship between the optimum collector tilt angle and the optimization factor (Fopt) at different shares of solar energy. However, Figure (2) shows the relation between the optimum tilt angle and solar fraction.

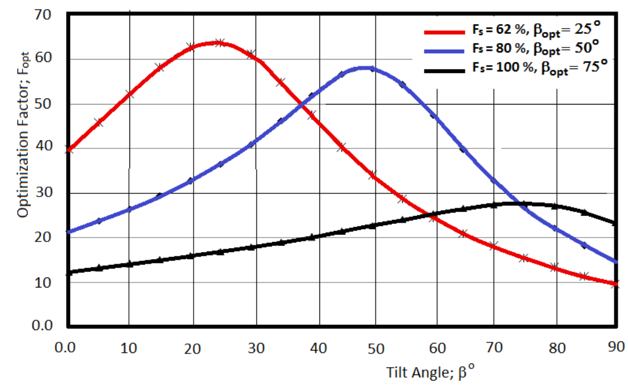

Figure 1 Optimization factor with collector tilt angle.

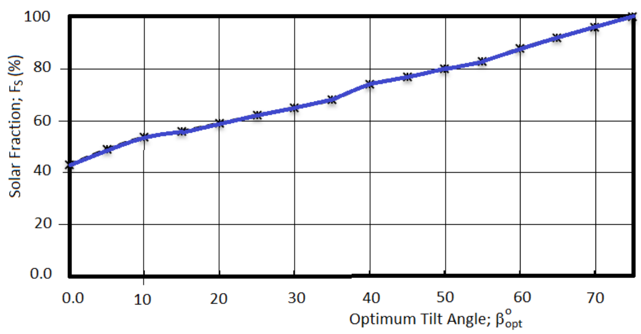

Figure 2 Solar fraction with optimum collector tilt angle.

Figure (2) shows that the lowest level of solar fraction in accordance with the climatic data for Tripoli city would be within 43% when the installation of solar collectors is on the horizontal level. This share increases with the increase of the collector tilt angle. It should be noted here that the optimum collector tilt angle is the angle that gives the best matching between the solar radiation curve and the load curve required to be covered. This principle has been formulated in the mathematical model; equation (2) described above.

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If solar fraction within 80% of the load, then the findings suggest that the optimum tilt angle within 50 degrees. This result is fully compatible with the traditional method for determining the tilt angle represented by; β = Latitude ± 15°.

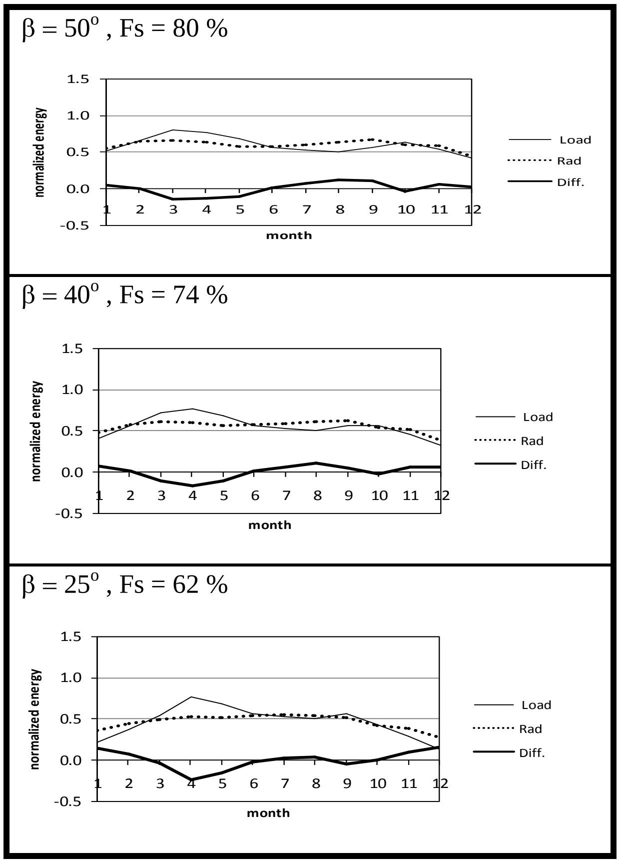

Figure 3 Normalized collector output with load demand for relatively low collector tilt angles.

1.6 The Optimum Design Time

By the determination of the optimum tilt angle, which based on the desired contribution required from the collector solar energy, the designer has the full knowledge of the change time of the required water heating load. Thus, the knowledge of the times of deficit and surplus energy can be drawn from the solar system. According to this concept, the appropriate time of designing and sizing of different incomplete components of the system could be obtained for the maximum deficit in solar energy.

Figures (3 and 4) compare the normalized collector energy output with the load demand at different desired solar fractions.

The curves at the bottom of each slide in the figures show the amount of surplus or deficit as compared to the load. Hence, the lowest value of this curve gives the required design month. Table (1) shows the designated design month with the desired solar fraction.

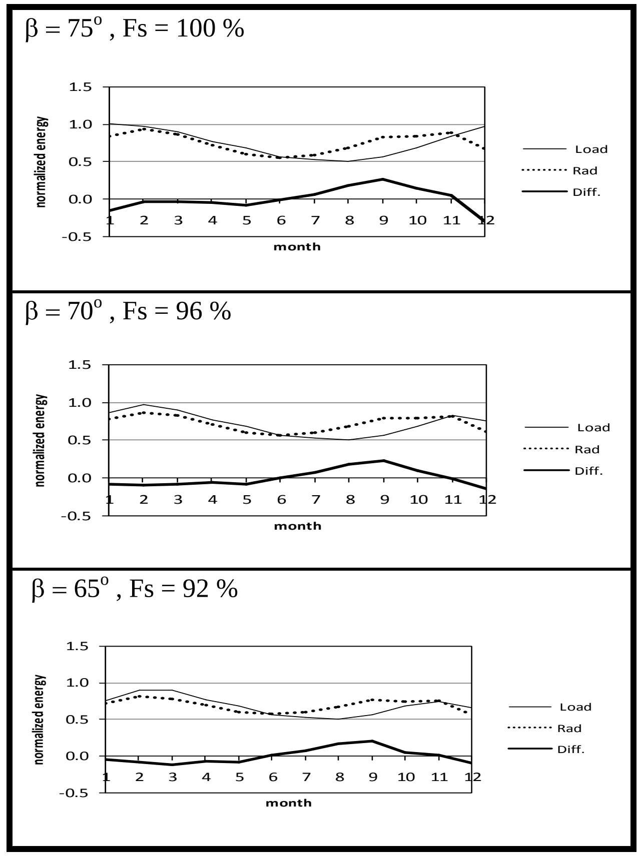

Figure 4 Normalized collector output with load demand for relatively high collector tilt angles.

Table 1
Design time with the solar fraction.

Fs(%)	Tilt angle (β)	Design Time (DT)
100	75	December
96	70	December
92	65	December
88	60	March
80	50	April
74	40	April
68	35	April
62	25	April

In the case of high desired solar fraction of 90% and above, the optimum design time would be December. This result is due to the fact that the low intensity of solar radiation and the high level of load occur in December. However, for solar fractions equal to or less than 80%, the optimum design time will be within April.

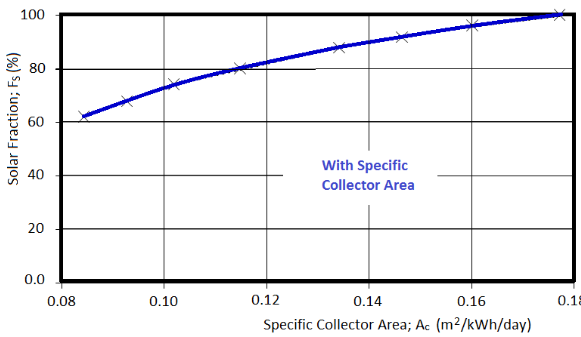

Figure 5 Solar fraction with specific collector area.

1.7 Solar Collector Capacity

One of the main components of solar energy systems is the solar collector unit. It is usually sized according to the energy that can be drawn from it at the design time. As mentioned above, this study is devoted to adopt a method for sizing the collector unit, which includes estimating the solar collector capacity. Table (2) presents the solar collector capacity and the specific collector area as a function of the tilt angle. Figure (5) shows the relationship between the desired solar fraction and specific collector area.

Table 2
Variations of solar collector capacity and specific collector area with the collector tilt angle.

Tilt angle β (Deg.)	C_A	A_C (m^2/kWh.day)
75	0.693	0.177
70	0.813	0.160
65	0.861	0.147
60	0.822	0.134
50	0.821	0.115
40	0.782	0.102
35	0.727	0.093
25	0.680	0.084

According to the daily domestic water heating load, the calculation of the solar collector area, was made by employing the mathematical model; equation (6). Figure (6) shows the recommended solar collector area in square meters for different desired solar fractions.

For result comparison purpose, the collector area was also calculated by employing the well known commercial "RETScreen" software, for the same daily domestic water heating load and for the same desired solar fraction [13]. This was done for a flat plate solar collector with the following specifications [11];

1. Glazed Collector Type
2. Fr(τα) coefficient = 0.58
3. Fr U_C coefficient =4.0 (W/ m^2 .°C)

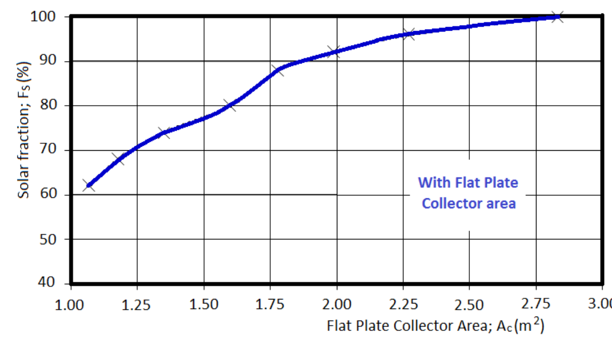

Figure 6 Solar fraction with flat plate collector area.

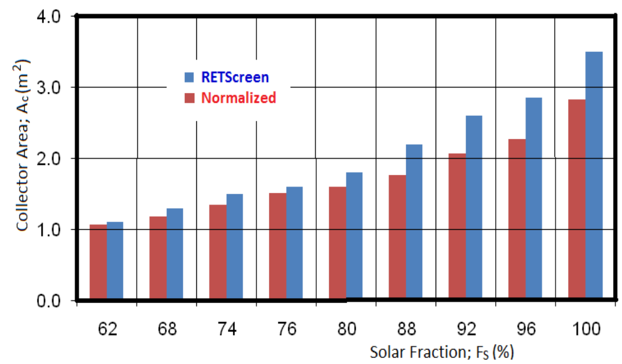

Figure 7 Flat plate collector area with solar fraction by the two methods.

Figure (7) shows the recommended flat plate solar collector area that is calculated by using both methods; RETScreen and the present normalized schema. Here, the variation between the collector areas obtained by both methods are closer as the desired solar fraction goes to low values. Referring to the normalized outcomes, to cover the total water heating energy demand fully by the solar energy system, the recommended solar collector capacity (C_A) is around 0.693, solar collector specific area (A_C) is a 0.177 (m^2/kWh), and the solar collector area is 2.83 (m^2).

However, for a solar fraction of up to 80%, the study suggests a solar collector capacity of around 0.821, solar collector specific area (A_C) of 0.115 (m^2/kWh), and solar collector area of 1.60 (m^2).

IV. Conclusions

1. The parameters; the time of design, the tilt angle, and the area of the solar collector, largely depend on the desired share of the solar energy system in meeting the load demand, since they simultaneously change as the share of the solar energy does.
2. The tilt angle shouldn't be calculated by the traditional technique if the total demand is to be covered by the solar supply.
3. The minimum of solar fraction in accordance with climatic data for Tripoli is within 43% for horizontally installed collectors.
4. The month of design changes according to the share of the solar energy; it would be in April if the share of the solar energy is equal or less than 80%. For solar share of equal or greater than 90%, the recommended design month is December.

V. Recommendations

1. This research work could be extended to consider the storage capacity related to the load profile, solar fraction, and the performance of the solar collector.
2. Also, this could be extended to cover different applications; Refrigeration, Air conditioning, Water desalination, and Power plant with different collector types.

NOMENCLATURE

Symbols	Description
A_C	Specific solar collector area
C_A	Solar collector array capacity
F_{opt}	Optimization factor
F_r	Solar collector heat-removal factor
F_s	Annual solar fraction
H_{avg}	Monthly average daily total solar irradiation (kJ/m^2.Day)
$I_{S,dm}$	Real radiation at design month (kJ/m^2.Day)
L_{dm}	Realistic value of the load demand at design month (kJ/day)
LDWH	Daily domestic water heating load (kJ/day)
N_{person}	Number of persons occupying the house
SDEV	Standard deviation of the curve
T_a	Ambient temperature (°C)
T_D	Design time
U_C	Collector overall heat-loss coefficient (W/m^2.K)
V_{person}	Volume of hot water required per person (Liters/day)
ρ	Density of the water (kg/liter).
c_p	Specific heat of the water (kJ/kg.°C).
β	Solar collector tilt angle (deg)
η_C	Solar collector efficiency
$(\tau\alpha)$	Transmissivity-absorptivity product ratio

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BIOGRAPHIES

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