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Result on Fixed Point for Convex Metris Space taking Random Operator for Kannan type Mapping

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***Abstract*--** **The aim of the present paper is to prove the existence of fixed points for Kannan mappings in convex metric spaces for random operator. The results are generalization forms of some known results.**

***Keywords*--** **Convex metric space, Common fixed point, Random operators, Kannan Type mappings.**

AMS Classifications: **47H10**

I. Introduction

Fixed-point theory plays an important role in solving the existence and uniqueness of solutions of differential equation, in solving Eigen value Problems and Boundary Value problems. Fixed-point theory also contributes in characterization of the completeness of matric spaces. Due to its applications in various disciplines of mathematical sciences, the Banach contraction and fixed-point theorems have been established. The ideas have a much wider scope than might be suspected and can be applied to establish many other existence theorem in the theory of differential and integral equations. There are numerous extension of Banach's fixed point theorem by generalization its hypothesis while retaining the convergence property of successive iterations the unique fixed point of mapping

Probabilistic functional analysis has emerged as one of the important mathematical disciplines in view of its role in analyzing probabilistic models in the applied sciences. The study of fixed points of random operators forms a central topic in this area. The Prague school of probabilistic initiated its study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [4]. Since then many interesting random fixed point results and several applications have appeared in the literature; for example the work of Beg and Shahazad [3], Lin [10], O'Regan [11], Papageorgiou [12] Xu [17].

In recent years, the study of random fixed points have attracted much attention some of the recent literatures in random fixed points may be noted in [1,3,5,12,15].In particular ,random iteration schemes leading to random fixed point of random operators have been discussed in [5,6,7].

Jungck introduced the concept of compatible mappings on metric spaces, as a generalization of weakly commuting mappings, which have been a useful tool for obtaining more comprehensive fixed point theorems .On the other hand, since Takahashi ([16]) introduced a notion of convex metric spaces, many authors have discussed the existence of fixed point and the convergence of iterative processes for nonexpansive mappings in this kind of spaces. Subsequently Guay et al., Talman[4] among others, have studied fixed point theorems in convex metric spaces. In this paper we prove existence of fixed points for Kannan mappings in convex metric spaces.

The purpose of this paper is to give existence of fixed points for Kannan mappings in convex metric spaces for random operator. The results are motivated by Beg and Azam[2]Choudhary[5],Guay et al [8]., Talman[15],

II. Preliminaries

Before starting the main results we give some preliminaries notes

Definition 2.1. The pair (A, B) of self-mappings of a metric space (X, d) is said to be compatible on X if whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to t \in X$, then $d(BAx_n, ABx_n) \to 0$.

Definition 2.2. Let (X, d) be a metric space and $J = [0,1]$. A mapping $W: X \times X \times J \to X$ is called a convex structure on X if for each $(x, y, \lambda) \in X \times X \times J$ and $u \in X$,

$$d\big(u, W(x, y, \lambda)\big) \leq \lambda d(u, x) + (1-\lambda) d(u, y).$$

A metric space X together with a convex structure W is called a convex metric space.

Definition 2.3. A nonempty subset K of a convex metric space (X, d) with a convex structure W is said to be convex if for all $(x, y, \lambda) \in K \times K \times J$, $W(x, y, \lambda) \in K$.

Definition 2.4. Let K be a nonempty subset of a convex metric space X. A mapping $T: K \to K$ is said to be Kannan mapping if

$$d(Tx, Ty) \leq \frac{d(x,Tx)+d(y,Ty)}{2} \quad \text{For all } x, y \in K.$$

Let T be a self mapping of a bounded subset K of a convex metric space X. Then T is said to have property (B) on K, if for every closed and convex subset F of K, which has non zero diameter and is invariant under T, there exists some $x \in F$ such that

$$d(x, Tx) < \sup_{y \in F} d(y, Ty)$$

Obviously, a Banach space, or any subset of a Banach space, is a convex metric space.

Note 2.4 : A convex metric space X is said to have property (C) if every decreasing net of nonempty closed and convex subsets of X has nonempty intersection.

Throughout this paper, (Ω, Σ) denotes a measurable space, C is non empty subset of K

Definition 2.5 Measurable function: A function $f: \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ foe every Borel subset B of X.

Definition 2.6 Random operator: A function $f: \Omega \times C \to C$ is said to be random operator, if $F(., X): \Omega \to C$ is measurable for every X $\in$ C

Definition 2.7 Continuous Random operator: A random operator $f: \Omega \times C \to c$ is said to be continuous if for fixed $t \in \Omega,\ f(t,.): C \to C$ *is continuous*

Definition 2.8. Random fixed point: A measurable function $g: \Omega \to C$ is said to be random fixed point of the random operator

$$f: \Omega \times C \to C, if\ f(t, g(t)) = g(t), \forall t \in \Omega$$

Definition 2.9 : Let (X, d) be a metric space and (Ω, Σ) is a measurable space, J= [0,1]. A mapping $W: X \times X \times J \to X$,is called a convex structure on X for random operator if for each

$(x(t), y(t), \delta) \in X \times X \times J$ and u(t) $\in X$

$$d(u(t), W(t, (x(t), y(t), \delta)) \leq \delta\, d(u(t), x(t)) + (1-\delta) d(u(t), y(t))$$

A metric space X together with a convex structure w and random operator is called a convex random metric space

Definition 2.10: A nonempty subset K of a convex random metric space (X,d) with a convex structure w is said to be convex if for all

$$(x(t), y(t), \delta) \in K \times K \times J,\ w[t, (x(t), y(t), \delta)] \in K$$

Definition 2.11. Let K be a nonempty subset of a convex metric space X. A mapping $T: K \to K$ is said to be Kannan mapping for random operator if

$$d(T(t, x(t)), T(t, y(t))) \leq \frac{d(x(t), T(t, x(t))) + d(y(t), T(t, y(t)))}{2}$$

For all $x(t), y(t) \in K$.

Let T be a self mapping of a bounded subset K of a convex metric space X. Then T is said to have property (B) on K, if for every closed and convex subset F of K, which has non zero diameter and is invariant under T, there exists some $x \in F$ for random operator such that

$$d(x(t), T(t, x(t))) < \sup_{y \in F} d(y(t), T(t, y(t)))$$

It is to be remembered that note2.4 is also true for random operator.

III. MAIN RESULTS

Throughout this section, $(\Omega, \sum)$ *denotes a* measurable spaces $t \in \Omega$ .We assume that (X, d) is a complete convex metric space with a convex structure W and K is a nonempty closed convex subset of X.

Theorem:-3.1 Let F be a Kannan mapping of a nonempty bounded closed and convex subset K of a convex metric space X having property (C) into itself. $(\Omega, \sum)$ *denotes a* measurable spaces $t \in \Omega$,If $\sup_{y(t) \in F} d(y(t), F(t, y(t)) < \delta(F), \{\delta(F)$ being the diameter of $F\}$ for every nonempty bounded closed and convex subset F of K which has non zero diameter and is mapped into itself by F, then F has a unique fixed point in K.

PROOF:- Let Γ be family of all bounded closed and convex subsets of K, mapped into itself by F. Obviously Γ is nonempty and has a minimal element S, S being minimal with respect to being nonempty bounded closed and convex and invariant under F. If $\delta(S) = 0$, then the point in S is a fixed point of F. Suppose $\delta(S) > 0$. For any $x(t), y(t) \in S$, we have

$$d(F(t, x(t)), F(t, y(t))) \leq \frac{d\left(x(t), F(t, x(t))\right)}{2} + \frac{d\left(y(t), F(t, y(t))\right)}{2} \leq \sup_{y \in S} d\left(y(t), F(t, y(t))\right).$$

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Hence $F(S)$ is contained in the closed sphere S_0 with $F(t,x(t))$ as centre and $\sup_{y\in S} d\left(y(t),F(t,y(t))\right)$ as radius. Also $S\cap S_0$ is invariant under F. Therefore by the minimality of S it follows that $S\subset S_0$.

Hence for any arbitrary but fixed $x(t)\in S$, we have

$$\sup_{y\in S} d\left(F(t,x(t)),y(t)\right)\le \sup_{y\in S} d\left(y(t),F(t,y(t))\right) \quad ....(3.1.1)$$

Let

$$S'=\left[z(t)\in S:\sup_{y\in S}\mathrm{d}(z(t),y(t))\le \sup_{y\in S}\mathrm{d}\left(y(t),F(t,y(t))\right)\right\}.$$

Obviously S' is nonempty.

For any $u(t),v(t)\in S'\,and\ \lambda\in[0,1]$, we have

$$d\{w(t,u(t),v(t),\lambda),y(t)\}\le \lambda d(u(t),y(t))+(1-\lambda)d(v(t),y(t)),$$

$$\le \sup_{y\in S}\mathrm{d}(y(t),F(t,y(t)).$$

It follows that $w(t,u(t),v(t),\lambda)\in S'$ for all $u(t),v(t)\in S'$ and $\lambda\in[0,1]$. Therefore S' is convex.

Next, suppose that $z(t)\in Cl(S')$, Closure of S'. Then there exists a sequence $z_n(t)$ in S' such that $z_n(t)\ \to z(t)$, and

$$d(z_n(t),y(t))\le \sup_{y\in S}\mathrm{d}(z_n(t),y(t))\le \sup_{y\in S}\mathrm{d}\left(y(t),F(t,y(t))\right)$$

For all $y(t)\in S$. Letting n tend to infinity, we have

$$d(z(t),y(t))\le \sup_{y\in S}\mathrm{d}\left(y(t),F(t,t(y))\right).$$

It follows that $z(t)\in S'$, and therefore S' is closed.

For all $z(t)\in S'$, eqn. (3.1.1) implies that

$$\sup_{y\in S}\left(F(t,z(t),y(t))\right)\le \sup_{y\in S}\mathrm{d}\left(F(t,y(t),y(t))\right).$$

Using definition of S', we have $F(t,z(t))\in S'$ for all $z(t)\in S'$. Thus S' is invariant under F. Also $\delta(S')\le \sup_{y\in S}\mathrm{d}\left(y(t),F(t,y(t))\right)<\delta(S)$, by hypothesis. Hence S' is a proper closed and convex subset of S, which contradicts the minimality of S.

Uniqueness- Suppose that F have two fixed points $x(t)$ and $y(t)$. Then

$$d(x,(t),y(t))=d\left(F\left(t,x(t),F(t,y(t))\right)\right)$$
$$\le\frac{d\left(x(t),F(t,x(t))\right)+d\left(y(t),F(t,y(t))\right)}{2}=0.$$

It follows that $x(t)=y(t)$.Hence fixed point is unique.

Theorem 3. 2:- Let X be a convex metric space having property (C) and K be a nonempty bounded closed and convex subset of X. Let $F:K\to K$ be a continuous Kannan mapping. $(\Omega,\sum)$ *denotes a* measurable spaces $t\in\Omega$ .Suppose F has property (B) over K. Then F has a unique fixed point in K.

PROOF:- As in the previous theorem, let S be the minimalel ement in Γ with respect to being nonempty bounded closed and convex and invariant under F. If $\delta(S)=0$, the theorem is obvious. If $\delta(S)\neq 0$, by property (B), there exists $x(t)\in S$ such that

$$d\left(x(t),F(t,x(t))\right)=r<\sup_{y\in S}\mathrm{d}\left(y(t),F(t,y(t))\right). \quad (3.2.1)$$

Let $P=\left\{x(t)\in S:d\left(x(t),F(t,x(t))\right)\le r\right\}$. If $x(t)\in P$, then since

$$d\left(F(t,x(t)),F^2(t,x(t))\right)\le \frac{d\left(x(t),F(t,x(t))\right)+d\left(F\left(t,x(t),F^2(t,x(t))\right)\right)}{2}.$$

We have $d\left(F\left(t,x(t),F^2(t,x(t))\right)\right)\le r$ which implies $F(t,x(t))\in P$ for all $x(t)\in P$. Hence it follows that $F(P)\subset P$.

Let $P'=ClCo\ (FP)$, the closed and convex hull of FP. If $z(t)\in P'$, then any one of the following three cases may arise:

(i) $z(t)\in FP$ and since $FP\subset P$, hence $F(t,z(t))\in FP\subset P'$.

(ii) $z(t)\in Co(FP)=\bigcup_{i\in N}\mathrm{A_i}$,

Where

$A_1=W(FP\times FP\times[0,1])$,

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$A_2 = W(A_1 \times A_1 \times [0,1])$,

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It follows that there exists some $m \in N$ such that $z(t) \in A_m$, Applying principle of mathematical induction, we get

$$d\left(z(t), F\left(t, y(t)\right)\right) \le \frac{r}{2} + \frac{d\left(F\left(t, y(t), y(t)\right)\right)}{2}$$

For all $z(t) \in A_m$, and $y(t) \in K$. Thus, $d\left(z(t), F\left(t, z(t)\right)\right) \le r$, which implies $z(t) \in P$ and hence $F\left(t, z(t)\right) \in FP \subset P'$.

(iii) $z(t)$ is a limit point of $Co(FP)$, then there exists a sequence $z_n(t)$ in $Co\ (FP)$ such that $z_n(t) \to z(t)$. Since F is continuous, we have $Fz_n(t) \to F\left(t, z(t)\right)$ and

$$d\left(z(t), F\left(t, z(t)\right)\right) = \lim_{n\to\infty} d\left(z_n(t), F\left(t, z_n(t)\right)\right) \le r.$$

It follows that $z(t) \in P$ and $F\left(t, z(t)\right) \in FP \subset P'$,

Thus P' is closed and convex subset of S which is invariant under F and, for every element $z(t)$ of $P', d\left(z(t), F\left(t, z(t)\right)\right) \le r$, which implies by equation ((3.2.1)), that P' is a proper subset of S. This contradicts the minimality of S. Hence $\delta(S) = 0$, and the point in S is a fixed point o F.

Theorem 3.3- Let X be a convex metric space having property (C) and H be a closed and convex subset of X. Let K be a nonempty bounded closed and convex subset of H. $(\Omega,\sum)$ *denotes a* measurable spaces $t \in \Omega$ .Let $F: K \to H$ be a continuous Kannan map such that

(i) T maps $\partial_H K$ the boundary of K relative to H, into K.

(ii) If L is any closed and convex subset of K which has non zero diameter and if G is a subset of L such that $LG \subset F$, then there exists $x(t) \in G$ such that $d\left(x(t), F\left(t, x(t)\right)\right) \le \sup_{y\in F} d\left(y(t), F\left(t, y(t)\right)\right)$.

Then F has a unique fixed point in K.

PROOF:- Let Γ be the family of all closed and convex subsets E of H such that $E \cap K \neq \phi$ and $T: E \cap K \to E$. Obviously $H \in \Gamma$. Let $\{F_\alpha\}$ be a descending chain of subsets of Γ.

Property (C) implies that $F \cap K \neq \phi$ where $F = \cap\ F_\alpha$. Because $F: F_\alpha \cap K \to F_\alpha$ for each α thus $F: F \cap K \to F$. Hence by Zorn's lemma there exists a minimal element S in Γ, S being minimal with respect to being nonempty closed and convex and such that $S \cap K \neq \phi$ and $F: S \cap K \to S$.

If $\partial_s K = \phi$, then $S \subset K$ and $F: S \cap K \to S$ implies F maps S into S. Condition (ii) implies that F has property (B). Now theorem 2 further implies that F has unique fixed point in S.

If $\partial_s K \neq \phi$, then $\partial_s K \subset \partial_H K$ and condition (i) implies that F maps $\partial_s K$ into K. Also F maps $S \cap K$ into S. Hence F maps $\partial_s K$ into $S \cap K$. If $\partial(S \cap K) = 0$, then $S \cap K$ contains only one element z(t). Nonemptiness of $\partial_s K \subset S \cap K$ implies that $z(t) \in \partial_s K$ and $F: \partial_4 K \to S \cap K$ further implies that $F(t, z(t)) = z(t)$, which proves the theorem.

If $\partial(S \cap K) \neq 0$, we will show that we arrive at a contradiction. As $S \cap K$ is a closed and convex subset of K, containing more than one element and $F: \partial_s K \to S \cap K$. Thus condition (ii) implies that there exists $x(t) \in \partial_s K$ such that

$$d\left(x(t), F\left(t, x(t)\right)\right) = r < \sup_{y\in S\cap K} d\left(y(t), F\left(t, y(t)\right)\right).$$

(3.3.1)

Let $P = \left\{z(t) \in S \cap K: d\left(z(t), F\left(t, z(t)\right)\right) \le r\right\}$ and let $P' = ClCo\ (FP)$. Then $P' \cap K \neq \phi$. Indeed there exists $x(t) \in \partial_s k$, satisfying eqn. (3.3.1)which implies $x(t) \in P$. Therefore

$$d\left(F\left(t, x(t)\right), F^2\left(t, x(t)\right)\right) \le \frac{d\left(x(t), F\left(t, x(t)\right)\right) + d\left(F\left(t, x(t)\right), F^2\left(t, x(t)\right)\right)}{2}$$

from which is follows that $d\left(F\left(t, x(t)\right), F^2\left(t, x(t)\right)\right) \le r$, which implies $F\left(t, x(t)\right) \in P \subset P'$. Also $x(t) \in \partial_s K$ implies that $F\left(t, x(t)\right) \in S \cap K$ that is $F\left(t, x(t)\right) \in K$ which further implies that $P' \cap K \neq \phi$.

Next we show that F maps $P' \cap K$ into P'. If $z(t) \in P' \cap K = \{ClCo\ FP\} \cap K$, then we have following three possibilities.

(a) $z(t) \in FP$ and $z(t) \in K$. Then there exists $z_1(t) \in P \subset S \cap K$ such that $F\left(t, z_1(t)\right)$. Since $F: S \cap K \to S$, therefore $z(t) = F\left(t, z_1(t)\right) \in S$. Hence $z(t) \in S \cap K$.

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Therefore

$$d\left(z(t), F\left(t, z(t)\right)\right) \leq \frac{1}{2}\left\{d(F(t, z_1(t), z_1(t)) + d\left(F^2(t, z_1(t), F\left(t, z_1(t)\right)\right)\right\}.$$

Therefore $d\left(z(t), F\left(t, z(t)\right)\right) \leq r$ which implies $z(t) \in P$ and $F\left(t, z(t)\right) \in FP \subset P'$.

(b) $z(t) \in Co(FP)$ and $z(t) \in K$. Then there exists $m \in N$ such that $z(t) \in A_m$ and (as in theorem 2) $d\left(z(t), F\left(t, z(t)\right)\right) \leq r$. Hence $z \in A_m \cap K$. For $m = 1, z(t) = w\left(t, F\left(t, u(t)\right), F\left(t, v(t)\right), \lambda\right)$ for some $u(t), v(t) \in P \subset S \cap K$ and $\lambda \in [0,1]$. Since S is convex and $F: S \cap K \to S$, therefore $z(t) = w\left(F\left(t, u(t)\right), F\left(t, v(t)\right)\lambda\right) \in S$, which implies $z(t) \in S \cap K$. So by using principle of mathematical induction, it can be easily shown that $z(t) \in A_m \cap K$ implies $z(t) \in S \cap K$ for any m. Hence $z(t) \in P$ and $F\left(t, z(t)\right) \in FP \subset P'$.

(c) $z(t)$ is a limit point of $Co\ (FP)$ and $z(t) \in K$. Then there exists a sequence $z_n(t)$ in $Co\,(FP)$ such that $z_n(t) \to z(t)$. By case (b), $z_n(t) \in P$, therefore $z_n(t) \in S \cap K$ and $d\left(z_n(t), F(t, z(t)_n)\right) \leq r$. Since $S \cap K$ is closed, therefore $z(t) \in S \cap K$. Moreover, continuity of F implies that $Tz_n(t) \to Tz(t)$ and we get $d\left(z(t)\right), F\left(t, z(t)\right) = \lim_{n\to\infty} d\left(z_n(t)\right), F\left(t, z_n(t)\right) \leq r$

Thus, $z(t) \in P$ and $F(t, z(t) \in FP \subset P'$.

Hence we find that P' is a closed and convex subset of S such that

$P' \cap K \neq \emptyset$ and $F: P' \cap K \to P'$. Also

$d\left(z(t), F\left(t, z(t)\right)\right) \leq r < \sup_{y \in S \cap K} \mathrm{d}\left(\mathrm{y(t)}, \mathrm{F}\left(\mathrm{t}, \mathrm{y(t)}\right)\right)$ for any $z(t) \in P' \cap K$,

That is $P' \cap K$ is a proper subset of $S \cap K$. Hence P' is a proper subset of S which is contradiction.

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