

Simulation of Active Control of Chatter Vibrations

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Abstract—. This paper outlines a strategy for the active control of the self-excited vibration of machine tools which arises when metal is turned, ground or milled. The requisite sensor and actuators for successful operation of such a scheme are outlined. Examples of the results of typical sensors are illustrated in a brief review of previous work. A simple single degree of freedom model of chatter is simulated, although a higher order system could be included to model the chatter process for control purposes. The effect of using different sensors feedback is also considered. Several control strategies are demonstrated. These show that the vibration can be reduced to less than 10% of its' original value in less than 10 cycles. The implications regarding problems of implementation are discussed

Keywords— Chatter Vibration, Active control, Simulation, PDF control, PID control, LQR control, Production engineering, Turning.

I. INTRODUCTION

Chatter is a self-excited vibration caused by the interaction of the chip removal process and the structure of the machine tool and it is a major concern today when trying to achieve high product quality. The vibrations can be of quite large amplitude and result in the following:

- Poor surface finish
- Dimensional inaccuracy of the work
- Premature wear, damage and ultimately failure of the cutting tool. This is particularly important in the case of ceramic tipped tools.
- Damage to machine components from vibration.
- Loud objectionable noise.

Regenerative chatter is the most important type of selfexcited vibration. This is when the tool cuts a surface which has roughness or disturbances from the previous cuts. Chatter can exist in lathes, milling machines, grinding machines and in a drilling process. The theory of chatter in grinding machines is similar to that of regenerative chatter in lathes except that both work-piece and the grinding wheel will develop irregular surfaces and in consequence results in two separate finite time delays. In drilling machines the results will have a sinusoidal motion superimposed on its axis, and the depth of cut taken by one flute depends on the cut taken by one before it. The number of flutes therefore alters the time delay involved in the regenerative chatter process.

In milling machines the chatter is generally regenerative but has to allow for more than one tooth being in contact with the work, simultaneously serious vibration may also be present due to the cut applying force and a frequency of once per tooth and some higher harmonics may be present. Planers and shapers also exhibit chatter. The amplitude of the forces depends on the thickness of the work-piece. Chatter (Figure 1) is often initiated by a disturbance such as a lack of homogeneity in the material. Changes in the cutting friction due to insufficient cooling can also start the vibration. The dynamic cutting force element acts on the machine frame forcing the frame into vibration, this causes a change in the relative position of the work-piece and the tool cutting edge. This again alters the cutting force. The chip thickness will vary due to the amplitude of the vibration. In regenerative chatter this effect is amplified as each cut comes around again next to the cutting surface.

II. PREVIOUS WORK

Arnold [1] appears to be the first person to have systematically investigated the phenomena of chatter, while Hahn [2] discovered the principle of regenerative vibrations in grinding operations. Arnold reported on early American and German research on chatter spending some effort analysing the distortion of machine tool and relating this to the depth of cut in chatter, he also established the link between cutting speed and amplitude of vibration. He also clearly established nonlinear nature of chatter with limiting conditions on speed and frequency. Most of the early work was directed to obtain the stability borderlines for chatter prevention. Hahn for example used Nyquist criteria to predict conditional stability. Merritt [3] by devising a control system model obtained a simple one degree of freedom stability model with a very simple criterion for stability. Tlusty [4] solved the problem of stability in the case of an n degree of freedom system ignoring cutting dynamics. Tobias and Fishwick [5] made an exact solution to the three borderlines of stability while Gurney and Tobias [6] also developed boundaries for n degree of freedom systems involving chatter. Tobias [7] discusses the limitations of these approaches in his book. He also analyses vibration dampers such as the Lanchester damper and provides guidance on their use.



Lemon and Long [8] worked at the Cincinnati milling machine company, and analysed the driving point impedances of their milling machines and compared these to rigid machines, matching this to Merritt's' stability criteria obtaining precise transfer functions. They postulated a frequency dependency of the cutting force. They were able to infer a non constant cutting force distribution with time. Henke [9] produce state models, using identification theory, and machine transfer functions of experimental systems using hydraulic force recording and chip size indicators, producing results of cutting force versus chip thickness compared to these two criteria. Welbourn and Smith [10] give an analysis of machine tool dynamics with a detailed dynamic vector description of the relative forces involved, including an analysis of vibration dampers and their ability to reduce chatter. Developments by Tlusty and Ismail [11] have included a non-linear model of chatter. Shiraishi and Kume [12] devised a control system to eliminate chatter using a micro drive and a state-space control system with an estimator for the missing states. They obtained good experimental results, showing it was possible to control chatter with this method. Tsai et. al. [13] provides a simulation system to predict chatter in end milling. They derive a model using tool deflection with a two degree of freedom system model to obtain a stability criterion. Mei et.al. [14] devised a plan for an active control system for chatter suppression by online variation of the rake and clearance angles of the tool in turning. The experiment results are very impressive, with good agreement between the simulated and experimental results. Hively et.al [15] investigated the concepts of chaos control while Håkansson et al. [16] proposed to control chatter in turning by vibrating the tool itself. This work is similar to that of Okrongli et.al. [17]. Pratt and Nayfeh [18] examined the problem of turning with a boring bar using a two mode model. They identified jump type instability, caused by non-linear dynamics, as well as sub critical vibrations. Semercigil and Chen [19] have proposed the use of a passive vibration damper to reduce the vibrations of an end mill. Andrén and Hårkansson [20] used an active control system on a MAZAK lathe. This mechanism reduces the bending of the tool using piezoelectric actuators. Sims and Zhang [21] used piezoelectric actuation of the work-piece to control chatter in milling processes with excellent results. Control techniques investigated include feedforward for more robust control. Ganguli [22] in his PhD thesis gives an excellent review of the current explanations of chatter while proposing active vibration damping and later Ganguli et.al. [23] used a two degree of freedom model for a milling process to create a mechatronic simulator.

They used displacement sensors and voice coil actuators to achieve active damping. The stability analysis has been brought up-to-date by Eynian [24] who has used a piezoelectric actuator and laser measurements to obtain very good experimental data on chatter and then analysing the results with Nyquist criterion to obtain the stability bounds. Siddhpura and Paurobally [25] give the best review of chatter research extant comparing stability prediction, detection and control techniques following this with an analysis of turning operations and chatter [26]. The latest efforts are illustrated by the work of Kim et al. [27] who used real time compensation via the machine tool controller.

To sum up; three basic control techniques have been used with some success in controlling chatter:

- Motion of the cutting tool in the direction of the cut
- Control of the rake and clearance angle of the cutting tool
- and controlling tool stiffness with additional actuators causing opposite deflection of the tool to that induced by the vibration

These can be grouped as vibration suppression and two techniques to prevent it from occurring, one by design and the other by active damping:

- Chuck damping in lathes and tool damping in mills
- Design for stability bounds.

Here we wish to prevent the chatter happening at all. The rest of this paper will describe possible solutions to implement control systems to reduce chatter.



Fig. 1: Chatter on Turned bar, showing a laser reflector mount



III. BASIS FOR ACTIVE CONTROL

To control any process we need an adequate theoretical model or physical measurements of a real system, suitable control strategies, appropriate sensors and actuators. The requirements of sensors and actuators are now examined

A. Sensors

Suitable sensors fall into three main classes [28], [29]. The first are position detectors. Three sub types are useful here, magnetic (typically eddy current devices), capacitive and optical. The relative merits of these are well established. Magnetic transducers are readily available, are relatively expensive. The main disadvantage of these devices is the location next to the work-piece which may interfere with the machining process. Capacitive transducers are of similar performance to magnetic devices but are prone to the stray electrical fields which may be present. A second approach is to use an accelerometer to measure the vibrations and is not so straightforward however, since the sensor cannot be placed at the location of the chatter the transmission of the vibrations to another location is important. This reduces the relative amplitude of the signal and may make the problem of singling out the chatter vibration difficult. However the danger of damage is substantially reduced. The variable gain for different work-piece sizes and locations is a major problem. The third is to use force sensors which are readily available from Kistler and others but are expensive and may interfere with the actuator. They are however in the correct location and are unlikely to be damaged. Our experiments have shown that these commercial force transducers are not rapid enough for control purposes. Contacting sensors are unlikely to be used because of the certainty of damage. Experiments by Delio [30] have shown the possibility of using microphones to detect chatter; these have the problem of variable gain as do accelerometers.

B. Actuators

Actuators are always a problem under control vibrations and so may not be fast enough for main types are available traditional electric motors suitable apart from the provocation the gears is loaded with the chosen device in those cases. A major problem is that of backlash and friction non-linearity's. Anti-backlash gearing has been developed for robots and could be used here. Four main types are available. The traditional electric motor is suitable and apart from the problem of backlash in the gears is likely to be the chosen device in most cases. DC servos and stepper motors are adequate for the job in hand. A hydraulic actuator of the type described by Kanai and Miyashita [31] would be suitable. Electromagnetic vibrators are conventionally used in vibration testing and clearly have sufficient performance to provide the force and frequency response. However their static performance is lacking in stiffness for our application and would have to be provided by a spring element. Lastly, piezo-electric stack actuators have now reached a level of performance to provide an alternative approach (Dahl and Wider [32]).Other actuators which are being researched such as the shape memory device (Hirose et al [33]) are simply insufficiently developed at this time.

C. Control Strategy

To control the chatter vibrations it is possible to divide the possible solutions into two major divisions: Active damping and counter movement.

Active Damping: In a number of machines with vibration problems vibration dampers have been installed. Usually these are passive dampers such as the Lanchester damper (described in Tobias' book). A different approach was used by De Ro [34] however who has applied magnetic damping to the chuck of a lathe with some success. It would be possible; to design a damper where the orifice that controls the rate of motion could be actively altered to give different damping rates.

Counter Movement: This can be sub-divided into two; movement of the tool separately from the main machine slide or alteration of the main machine drive speed. No applications of a modified normal slide drive have been found and it will be clear from later analysis why this is not a wise course of action to be adopted.

In principle the application here will use a micro drive mounted on the machine slide. If the motion in chatter can be adequately described by a single degree of freedom system then conventional control algorithms can be used but if two degrees of freedom are to be controlled then a state space controller must be designed.

IV. SAMPLE CONTROL SCHEME

A Cutting Process

The process described uses a single point tool to performing orthogonal cutting on a lathe. This is illustrated in figure 2. Instantaneous depth of cut u(t) is decreased as the work-piece moves away from the cutting tool causing an increase in chip thickness. As this occurs a raised portion is left on the work-piece this lump increases the uncut thickness in one revolution of the work. This was represented by figure 3. The equations can be written:



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TOOL RIGIDLY MOUNTED

Figure 2 Cutting Geometry (from Merritt 1965)

$$u(t) = u_0(t) - y(t) + \mu y(t - T) = 1$$

Where $T = \frac{1}{N}$

And μ is the overlap factor in successive cuts which impinge on each other. The most machining operations μ is equal to 1. The overlap factor is used to account for the geometric effects of rounding at the tool cutting edge and of the tool clearance angle and both of these effects tend to smear the machine surface and reduce the amplitude of the periodic variations. The Laplace transform of equation 1 is:

$$U(s) = U_0(s) - Y(s) + \mu e^{-Ts}Y(s)$$
 2

The resultant cutting force F(t) is related to instantaneous uncut chip thickness u(t) by the dynamics of the cutting process.



Figure 3 Chatter model (from Merritt)

In the single degree of freedom analysis which follows the stiffness and damping of the work-piece and the machine structure together are described by one lumped parameter single second order differential equation modelled as a second-order transfer function.

$$F(t) = K_{c}u(t)$$
 3

$$\frac{Y(s)}{F(s)} = \frac{1}{k_m \left[\frac{s^2}{\omega_1^2} + \frac{2\delta_1}{\omega_1}s + 1\right]}$$

$$4$$

The cutting force K_c is normally a function of the material, the shape of the tool, the speed of rotation and the width of the chip produced, which most authors describe as a constant value. There is some evidence that there is a phase lag between this force and the cutting process.

Low backlash geared drive



Figure 4 Schematic of micro-drive system

Lemon and Long [8] and Henke [9] describe this as a simple delay or lag of the order of 2 ms. Okrongli et al. al [17] gives a similar value for more modern steels. This is later modelled here as:

$$cutting \ force(s) = \frac{K_c}{(T_c s + 1)}$$
5

Where T_c is around 2 ms.

The lumped parameter description gives an adequate stability margin as shown by Merritt [3]. This stability criterion has shown good accuracy compared to the data measured by equipment used by Okrongli et al. [29] and others. This model is included in the SIMULINK model shown later.

To sum up:

- The cutting force is proportional to the tool position and depends on a time lag.
- The dynamics of the machine are given by: a second order transfer function (equation 4)
- and the delay is given by *Gd*(s) where μ is the factor which allows for overlap of different cuts, with a value between 0 and 1.
- The process block diagram is shown in figure.3.



The main problem for control is the regenerative time delay. Shiraishi and Kume [18] achieved suppression of chatter using a state-space controller with an estimator for the extra states inferred from the Pade' approximation to the time delay.

They used a stepper motor to actuate a micro drive, with an eddy-current sensor and a microcomputer to complete the control system. It is not clear whether their implementation is robust to changes in conditions for chatter.

The work described here (figures 4 & 5) uses a micro drive mounted on top of the lathe cross slide. An Electrocraft servomotor is proposed to be used and the displacement of the work-piece measured with the laser sensor similar to that described by Okrongli et. al.[29], as the system was designed for machining non-conduction materials. The movement was achieved by using a low backlash recirculating ball linear drive with a gear ratio of 0.01. As the analysis work progressed it became apparent that the ability to control the ability to control the chatter vibrations depended strongly on the value of the mass of the moving parts of the micro drive.

The output from the controller power amplifier is V and the motor current can be approximated by:

6

$$i = \frac{v}{R_{cr}}$$

The torque from the servomotor:

$$T_m = K_T i$$
 7

The acceleration of the tool tip is given by:

$$\ddot{u} = \frac{\left(\frac{T_m}{nr} - F - c\dot{u}\right)}{M_e}$$
8

In this case the electrical time constant is small enough to be neglected.

The control methodology used is the traditional servo techniques of Proportional Integral and Derivative (PID), Proportional and Integral (PI) or Pseudo Derivative Feedback (PDF) control. We have also tried Fuzzy and Neural Network controllers but they have proved to be not as robust. The other methods tried were a state-space techniques linear quadratic regulator (LQR) controller since we have sensor input. The techniques chosen to be investigated here were chosen for simplicity. The equations were simulated with SIMULINK[™] and are shown in figures 5 & 6.



Figure 5 Model with Backlash and time delay included



Figure 6 Micro-drive subsystem SIMULINK diagram

V. RESULTS

Several of the control schemes mentioned previously, were tested with a simulation program. One of these is shown here, the PDF controller (figure.5). The system described here uses a position sensor but with pseudo-differential feedback controller originated by Phelan [35]. The output of the controller power amplifier is applied to the servo motor sub-model and this delivers a certain movement u(t). This is applied to the chatter loop. That it can control chatter is clearly seen in figure.8.

This simulation is made in three parts:

- 1. Control without micro-drive dynamics
- 2. Control with dynamics of the micro-drive but without time dependence of the cutting force and without backlash.
- 3. Control with dynamics and with time dependence and with back lash.

The simulations (figure 7) of three types of controllers without micro-drive dynamics show clearly that all three can reduce the chatter vibrations to insignificant levels. The LQR controller user sensor feedback and is designed without the regenerative effects but works very well as expected. In the use of LQR the velocity and displacement were multiplied by constants and then fed back. In fact all types of controllers tested, were quite adequate. Different size motors were also investigated. In a commercial installation having excess installed power would be wasteful; hence the minimum size motor that can cope with the chatter forces is what would be chosen. The conditions for chatter were taken from Merritt [3].



When the dynamics are included (figure 8) the PID controller is worse than the PDF control but a PI controller produces results indistinguishable from the PDF system. In this case, the mass of the moving parts of the micro drive as initially proposed were 2.5 kg. The PDF gains were KC1=5 and KC2=10.

Since the PDF controller of those tested was slightly better than the others its' performance was investigated further, when the effects of frequency on the cutting force were added to the model. In figures 5&6 the model is modified by the addition of a time delay to the cutting force and a backlash term to the output stage of the micro-drive. To get an adequate response the notional mass of the moving parts of the micro-drive had to be reduced to 1.5 kg. The curves show the effect of introducing a step cut at 0.2 s. The transient caused by switching on the control at 0.5 s is large but dies away quickly. The chatter is reduced in less than 0.5s. Figure 8 now illustrates a response that is comparable with the earlier results. Figure 9a shows the results for PDF control without time delay on cutting force or backlash, fig 9b shows the effect of time delay, the chatter cannot be totally eliminated but it is reduced. It is important to note that the displacement of the surface is less than the value of backlash. Figure 9c illustrates the effect of backlash, making the chatter end result worse, whilst reducing the mass of the moving parts allows the chatter to be substantially reduced in fact to a better position than before (figure 9d). The size of the backlash introduced here is between 0.01 and 0.1 mm. This is as measured for a recirculating ball drive unit. This value of backlash is very critical to the performance of all the control systems in dealing with chatter. If it reaches values of 0.25mm then no controller investigated could effectively control the system for the conditions specified here.

However, as in the case of PI control which will also works in this case, the performance at this value of the gains is not sufficiently robust for all possible chatter speeds. From the diagram the reduction takes place in less than 10 cycles with only moderate control effort. It is therefore possible to design a PDF controller with moderate gains which will cope with the full range of chatter conditions. This is as good as the state-space method of Shiraishi mentioned earlier but with the added advantage that it requires much less programming and less computing power.

However the PDF controller is not very sensitive to disturbances in cutting force of around 10% variation in cutting force as shown in figure10a&b. It is quite probable that many different controllers can be designed to cope with these chatter vibrations.

VI. CONCLUSIONS

To control chatter previous workers have illustrated that two strategies are possible:-

- Active damping with reaction on the chuck, for example as used by De Ro.
- Motion compensation with several possible sensors and actuators being suitable choices.

Several control algorithms would appear to be suitable.

These include conventional servo practice such as Lead/lag, PID, PI, and PDF. Modern LQG state space control has been used, and LQR control with sensor input which has been investigated here.

If the drive system is very much faster than the response of the chatter mechanism then several controllers work effectively, including PI, PID, PDF and LQR with sensors. When the effects of an additional drive corrector are included then the best systems are PDF and PI controllers. With the addition of realistic backlash and the cutting force time delay included then all the controllers struggle to reduce the chatter to small enough values. This is critically dependent on the value of backlash in the drive system and on the mass of the moving parts. However the PDF controller does reduce the size of the chatter to a much smaller value than without control.

The effect of backlash and the low moving mass that need to be included in the micro-drive illustrate why it is unlikely that a chatter control mechanism could be incorporated into the normal drive system of the cross slide. The mass is far too high and the likely backlash would militate against it. The effects of wear would eventually render the scheme prone to failure, at critical moments. These considerations of mass and backlash would suggest the approach of controlling the stiffness of the tool or the rake/clearance angle would be better choices to provide a consistent method to eliminate chatter



Figure 7 Comparative Control Effectiveness with ideal drive





Figure 8 Control effectiveness with real design of micro drive



Figure 9 the effects of cutting force time delay and backlash on controllability



Figure 10 Effect of Robustness to change in cutting force of 10%.

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Nomenclature

- c Micro-drive friction damping N/m/s
- F cutting force N
- i Motor current amp
- *K_c* Cutting force per unit cut *N/m*
- K_T Motor torque constant Nm/amp
- N Rotational speed revs/s
- n gear ratio
- *M_e* Equivalent moving mass of micro-drive kg
- t time/s
- T period of rotation s
- *T_c Time constant of cutting force s*
- *T_m* Motor torque Nm
- R Radius of drive shaft m
- R_a Motor electrical resistance Ohm
- s Laplace variable
- U,u depth of cut m
- U_0 Initial depth of cut m
- V Controller output voltage volts
- y displacement of work-piece m
- μ overlap factor
- ω rotational frequency