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A Numerical Approach with Variable Temperature Boundary Conditions over a Continuous Moving Plate

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***Abstract*— A study of thermal boundary layer on a continuously moving semi-infinite flat plate, whose temperature varies as Ax^n, where A is a constant and x is measured from the leading edge of the plate has been presented. Similarity solutions have been derived and the resulting equations are integrated. The effect of various governing parameters, such as Prandtl number Pr, temperature exponent n and Schmidt number Sc which determine the concentration profiles and the local shear stress coefficient at the surface are studied. It has been observed that the value of the Sherwood number increases with increasing n.**

***Keywords*— Heat transfer, mass transfer, variable temperature, Prandtl number, Schmidt number**

I. Introduction

The boundary layer flow past a continuously moving semi-infinite plate was first studied by Sakiadis [1-3]. The axisymmetric case was considered by Koldenhof [4] whereas the heat transfer aspect of Refs.[1-4] was considered by Tsou, Sparrow and Goldstein [5] in case of flat plate, and in case of continuously moving cylinders, it was discussed by Pechoc [6], Bourne and Elliston [7], Rotte and Beck [8], Gampert [9, 10] and Karnis and Pechoc [11]. In Ref. [5], the viscous dissipation effects were not considered. These problems are important in technology, it is necessary to study the effects of variable plate temperature on the temperature field. Such a phenomenon has been studied in case of stationary plate by Tifford and Chu [12]. Heat transfer in flow past a continuous moving plate with variable temperature was studied by Soundalgekar and Ramana Murty [13]. It has been shown in Schlichting [14] that similarity solution to energy equation does not exist when viscous dissipation and variable plate temperature are considered simultaneously. Hence without considering viscous dissipation effects, the problem is considered here on taking into account the variable temperature of the plate.

Motivated by the above mentioned investigations and applications, we present in the paper the study of mass transfer in flow past a continuous moving plate with variable temperature. This study may be regarded as the extension of [13] by considering mass transfer, while [13] considered heat transfer. In Sec. 2, the mathematical analysis has been presented and in Sec. 3, the conclusions have been set out.

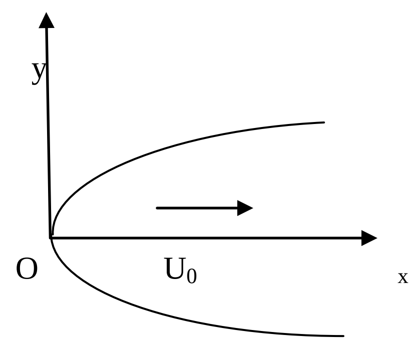

Fig.1. schematic representation of the boundary layer on a continuous moving surface.

II. Mathematical Analysis

The plate is assumed to be moving in a stationary fluid which is incompressible and viscous. The x- axis is taken along the moving plate and the y-axis is taken normal to the plate. If u and v are the velocity components along x and y – axis respectively, then under usual boundary layer approximation, the flow and the mass transfer are given by the system of boundary layer equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{.................................(1)}$$

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Here ν is the kinematic viscosity, α the thermal diffusivity, T is the temperature of the fluid, C is the concentration of the fluid and D is the coefficient of the mass diffusivity.

III. Result And Discussion

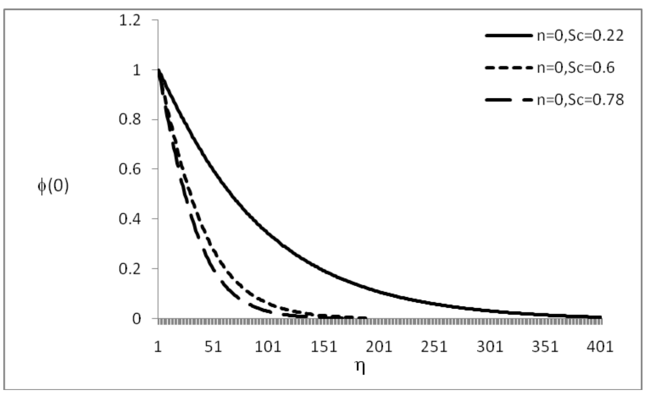

Fig.2. Concentration profiles with Pr = 0.7

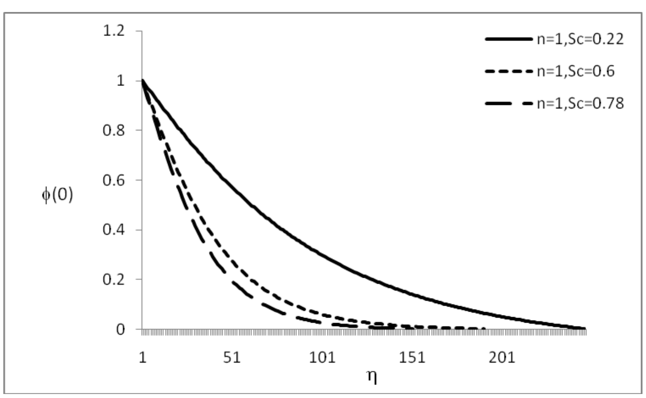

Fig.3. Concentration profiles with Pr = 0.7

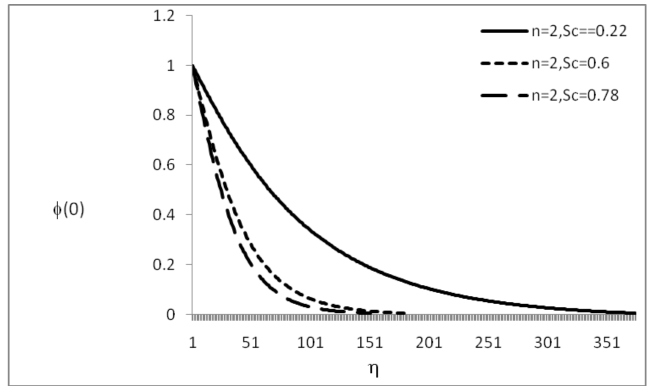

Fig.4. Concentration profile with Pr = 0.7

The boundary conditions are

$u = U_0$, $v = 0$, $T=T_w$, $C = C_w$ at $y = 0$

$u = 0$, $T\to T_\infty$, $C \to C_\infty$ at $y \to \infty$ …………….(5)

We assume that the plate temperature varies as

$T_w(x) - T_\infty = Ax^n$ ………………………….(6)

Introducing the usual similarity transformations

$$\eta = y\sqrt{\frac{U_0}{\nu x}}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

$$\psi=\sqrt{\nu x U_0}\ f(\eta)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The velocity components are given by

$$u=\frac{\partial\psi}{\partial y}\qquad and \qquad v=-\frac{\partial\psi}{\partial x}\ldots.(9)$$

Equations (1) – (4) reduce to

$$f'''+\frac{1}{2}f\,f''=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

$$\theta''-\Pr n\,f'\theta+\frac{1}{2}\Pr f\,\theta'=0\ \ldots\ldots\ldots\ldots\ldots(11)$$

$$\phi''+\frac{1}{2}Sc\,f\,\phi'=0\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

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Where

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}; \qquad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

$$\Pr=\frac{\mu c_p}{k}; \qquad Sc=\frac{\nu}{D}$$

and the primes denote differentiation with respect to η. The transformed boundary conditions are given by

At $\eta = 0$; $f= 0$; $f'=1$; $\theta=1$; $\varphi = 1$

As $\eta\to\infty$; $f'(\infty) = 0$; $\theta(\infty) = 0$; $\phi(\infty) = 0$ ………(13)

Numerical solutions of (10) and (12) for n = 0 have been given by Tsou et al an incompressible fluid with different Prandtl numbers. We have carried out solutions for n = 0, 1 and 2 and for sc = 0.22, 0.6, 0.78. The concentration profiles are shown in fig. 2. We observe from the figure that there is a fall in concentration of fluid with increasing Sc or n.

We now study the rate of heat and mass transfer, in terms of the Sherwood number. It is given by

$$Sh=\frac{xM_w}{D_m\Delta C}=-(\mathrm{Re})^{\frac{1}{2}}\phi'(0)$$

i.e., $Sh(\mathrm{Re})^{-\frac{1}{2}}=-\phi'(0)$ ………..(14)

Where M_w is the mass flux at the wall and are given by

$$M_w=-D_M\left(\frac{\partial C}{\partial y}\right)_{y=0}=-D_M\Delta C\sqrt{\frac{U_0}{\nu x}}\,\phi'(0) \quad ..(15)$$

here $\Delta C = C_w - C_\infty$.

The numerical values of - ϕ′(0) are given in Table 1.

Sc \ n	0	1	2
0.22	0.1456	0.1456	0.1456
2	0.68325	0.68325	0.68325
5	1.1538	1.1538	1.1538

We conclude from this table that an increase in Sc leads to an increase in the value of the Sherwood number.

IV. Conclusions

An increase in Sc or n leads to a fall in the concentration and a rise in the value of the Sherwood number.

REFERENCES

[1] Sakiadis, B.C.: Boundary Layer Behaviour on continuous solid surfaces: 1. Boundary-layer equations for two dimensional and axisymmetric flow. Amer. Inst. Chem. Eng. J. 7 (1961) p. 26

[2] Sakiadis, B.D.: Boundary Layer Behaviour on continuous solid surfaces: II. The boundary layer on a continuous flat surface. Amer. Inst. Chem. Eng. J. 7 (1961) p.221

[3] Sakiadis, B.C.: Boundary layer behavior on continuous solid surfaces: III. The Boundary layer continuous cylindrical surface. Amer. Inst. Chem. J. 7(1961) p. 467

[4] Koldenhof, E.A.: Laminar boundary layer on continuous flat and cylindrical surfaces. Amer. Inst. Chem.J. 9 (1965) p. 411

[5] Tsou, F.K.; Sparrow, E.M.; Goldstein, R.J.: Flow and heat transfer in the boundary layer on a continuous moving surface. Int. J. Heat Mass Transfer 10 (1967) p. 219

[6] Pechoc, V.: Cooling of Synthetic fibres. Thesis. Inst. Chem. Technol. Prague 1967

[7] Bourne, D.E.; Elliston, D.G.: Heat Transfer through the axially symmetric boundary layer on a moving fibre. Int. J. Heat Mass Transfer 13 (1970) p. 583

[8] Rotte, J.W.; Beck, W.J.: Some models for the calculation of heat transfer coefficients to a moving continuous cylinder. Chem. Engg. Sci. 24 (1969) p.705

[9] Gampert, B.: Grenzschichttheoretische Problems des aerodynamischen Schmelzspinnprozesses. Thesis. Berlin: Technical University 1973

[10] Gampert, B.: Berechnung des Warmeuberganges an einem in ruhenden fluid kontinuierlich bewegten sschlanken Kreiszylinder fur kleine Werte des Krummungsparameters auf der Basis von Reihenansatzen. Zeit. Angew. Math. Mech. 54 (1974) p. 118

[11] Karmis, J; Pechoc, V.: The thermal laminar boundary layer on a continuous cylinder. Int. J. Heat Mass Transfer 21 (1978) p. 43

[12] Tifford, A.N.; Chu, S.T.: Heat Transfer in Laminar Boundary Layers Subject to Surface Pressure and Temperature Distributions. Proc. Second Midwestern Conf. Fluid Mech. (1949) p.363

[13] Soundalgekar, V.M.; Ramana Murty T.V.: Heat Transfer in Flow Past a Continuous Moving Plate with Variable Temperature. Warme-und Stoffubertragung 14, (1980) p.91

[14] Schlichting, H. : Boundary Layer Theory. McGraw-Hill Book Company, 6 th Ed. (1968)