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A Study on Influence of Altering The Tooth-Sum on Bending Stress In External Spur Gears Under Static Loading

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***Abstract* - Gears are used to provide a nonslip drive in power transmission. While transmitting power, if specific requirements like load carrying capacity, wear characteristics, noise level etc. are imposed, then the shape and size of the tooth become critical parameters for analysis, thus necessitating a detailed study and modification of tooth geometry. Of the above, the load carrying capacity depends on the bending strength of the gear tooth which is normally computed using Lewis Equation. Usually standard tooth geometry is modified by profile shift using S-gearing, may be S_o or $S_{\pm}$ type, with the former being more common. Tooth geometry can also be modified by way of altering the tooth-sum for a given center distance and module. This study is focused on introducing the profile shift by way of altering the tooth-sum and investigating its effect on tooth bending strength and contact ratio. Involute spur gears having tooth-sum 100, 2 mm module and 20 degree pressure angle are considered. When compared to standard gears, a reduction in bending stress by 12% for HPSTC loading and 35.28% for tip loading is observed with negative teeth alterations while 24.05% increase in contact ratio is observed with positive teeth alterations. Such betterment in performance aspects can be traded off to derive several other gearing benefits. Hence altered tooth-sum way is a unique and novel approach to profile shift in gear design. The unique advantage of this method is that it needs no structural modifications as there is no change in center distance.**

***Keywords* - Altered tooth-sum, spur gears, bending stress, profile shift, high contact ratio, operating pressure angle.**

Abbreviations and Nomenclature:

STS Standard tooth-Sum
ATS Altered tooth-sum
HPSTC Highest point of single tooth contact
BS Bending stress
BS-D2 Bending stress (HPSTC load)
BS-Tip Bending stress (Tip load)
CR Contact ratio
HCR High contact ratio
GR Gear ratio
Z_s,Z_s' STS, ATS
Z_e Number of teeth altered.
Z_1, Z_2 Teeth on STS pinion, gear.
Z_1', Z_2' Teeth on ATS pinion, gear.
α,α_e Standard, operating pressure angle.
X_e Total profile shift.
X_1,X_2 Profile shift coefficient on pinion, gear

I. Introduction

Gears are prominent elements used in a vast array of mechanical devices ranging from domestic applications to heavy engineering like ship building and aerospace industries. The tooth geometry rules the design and operating characteristics of gears. Most often design for strength is important. The power that gears can transmit depends on the maximum permissible tooth load along the path of contact during mesh which in turn depends on tooth bending stress. Lower teeth stresses enhance the durability of gears. The geometry of most spur gears is designed to provide the best compromise between tooth bending strength, durability and cost. Several alternatives, involving modification of tooth geometry and surface treatments are practiced to increase the load carrying capacity of gears. For instance, the bending strength can be improved by altering the pressure angle, introducing profile shift or addendum correction, fillet modification, tip relief, asymmetric tooth design etc. A stronger gear is one in which less bending stress is induced. The most common practice for modifying the tooth geometry is to introduce profile shift, also known as profile correction or addendum modification in gears. Profile correction is used when smaller number of teeth has to be generated in order to overcome interference or undercutting [4]. This is done by S-gearing, either So or S$\pm$. In So gearing, the mating gears receive equal amounts of profile shift, but in opposite directions, thereby ensuring that the sum of profile shifts is equal to zero with no change in center distance. In S$\pm$ gearing, the sum of profile shifts is not equal to zero, it is positive for S+ gearing and negative for S- gearing. The amount of profile shift among mating gears may not be equal as well. This gives rise to change in center distance and needs structural changes in order to house such profile shifted gears. The So gearing is normally preferred over S$\pm$ gearing. On the contrary, this study introduces the profile shift by way of altering the tooth-sum while preserving the center distance and module that needs no structural changes to house them. Here, negative teeth alteration leads to positive profile shift and vice versa. In addition, when teeth alteration is made, tooth topping takes place which is necessary to preserve the center distance and to ensure a proper backlash.

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The present study is indeed indebted, in many ways, to the efforts of earlier researchers. **Maag** [1] was the first person to use the principle of generating the involute tooth using rack type cutter and to generate profile shifted involute tooth. **Maitra** [2] has discussed the amount of profile shift required, its calculation and use. **Oda.S and Tsubokura.K** [3] have reported that the root stresses decrease with increase in positive profile shift. **Sanders** [4] has shown that an elliptical profile can be used to create larger root fillets that can yield lower stress than the circular root fillets, while also claiming that each gear size will have a unique, optimum elliptical shape. **Joseph Gonsalvis** and **GVN Rayudu** [5] have studied the effects of varying the number of teeth on a tooth-sum for a specified center distance in external gears.

II. OBJECTIVES

Sensing the needs of good gearing, this study intends to achieve the following objectives:

a. To understand the gear tooth geometry.
b. To alter the tooth-sum, compute the resulting total profile shift and its distribution among the mating gears.
c. To study the effects of altering the tooth-sum for specified center distance and module on performance of gears.
d. To identify the optimum profile shift for each tooth altered that result in equal bending strength or highest contact ratio as needed.
e. To indentify a domain of altered tooth-sum and profile shift taking care of undercutting, minimum top land of 0.4m, minimum contact ratio of 1.2 and location of pitch point within the length of contact.

III. PROFILE SHIFT

Profile shifting in involute gears is mainly employed to overcome the problems of undercutting while generating smaller number of teeth on the pinion. It is also used to impart better tooth-strength, to obtain the prescribed center distance or to have desired tooth action along the path of contact. Profile shifting means that while generating involute teeth on the gear reference line of the cutter, which is tangential to the pitch circle in case of the standard gear, is shifted either towards or away from the center of the gear being cut. This amount of shift is expressed as a coefficient X of the module m. The profile shift provided on pinion is represented by X_1 and that on gear by X_2. The shift towards the center of the gear is known as negative correction while shift away from the center of the gear is known as positive correction. Fig.1 shows the method of profile shifting using rack type cutter.

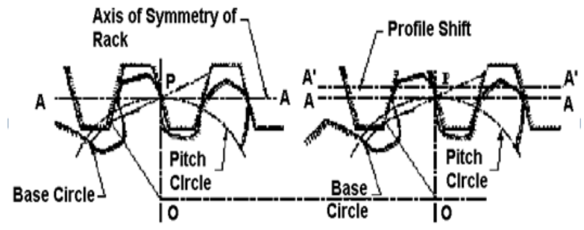

Fig 1. Method of profile shifting.

There are two types of profile-shifted gears, namely So gears and S± gears. In So type, the sum of profile shift coefficients is equal to zero (Xe=0, $X_1+X_2=0$, X_1-input, $X_2=-X_1$), hence the gears operate on standard center distance and pressure angle. Here, the profile shift on pinion is positive while on the gear it is negative. Since the amount of profile shift is same, but in opposite sense, there is a long addendum on pinion and short addendum on gear. Hence, it is also known as long and short addendum gearing. In the S± type, the sum of profile shift is not equal to zero (Xe≠0, $X_1 \neq X_2$, X_1=input, X_2=Xe- X_1) while gears operate on different center distance and pressure angle. **Maitra**[2] has successfully accommodated different tooth numbers for a given center distance and gear ratio.

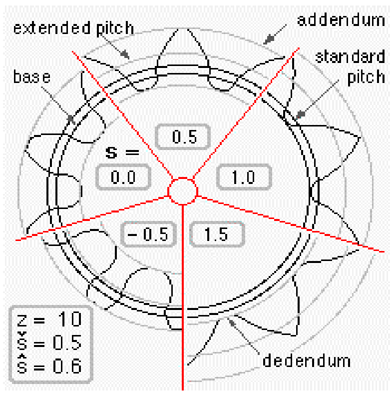

Fig.2 Effect of Profile shift on tooth geometry.

Fig.2 shows the change in shape and size of tooth profile due to different amounts of profile shift. This change affects the performance of gearing in many ways. It can be seen that with positive correction the tooth thickness increases and vice versa. But excessive positive or negative corrections (beyond 1 module) leads to impractical situations.

IV. SALIENT POINTS OF CONTACT

As the gear tooth contact progresses, the salient points of contact along the tooth profile as well as line of action are shown in fig.3 and fig.4 respectively.

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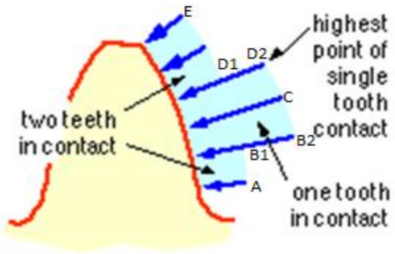
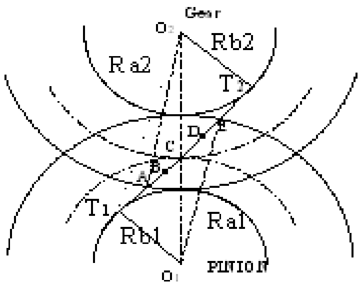


Fig. 3 Salient points of mesh along tooth profile



A – Beginning of Contact
B_1 – End of Two Pair Mesh
B_2 – Beginning of Single Pair in Contact
C – Pitch Point
D_2 – End of Single Pair Mesh
D_1 – Beginning of Two pair Mesh
E – End of Contact

Fig. 4 Salient points of mesh along length of contact.

Though the bending stress can be evaluated when load acts at all these points, only the two important points, i.e, D2 representing HPSTC loading and E representing tip loading are considered in this analysis.

V. ALTERING THE TOOTH-SUM

From the available literature it can be stated that until recently there has been very little investigation or information regarding introducing profile shift by way of altering the tooth-sum among the mating gears. Hence, an approach whereby the tooth-sum is varied for a given center distance and module operating on a different pressure angle is both unique and promising. By employing profile-shift for a given center distance and module, different tooth-sums can be accommodated. As the tooth-sum of a standard gear pair is altered, profile shift is eventually introduced. While doing so the following geometrical modifications are imposed:

1) Modification in size of the base circles which alters the pressure line.
2) Introduction of operating/working pressure angle.
3) Modification in tooth thickness from root to tip.
4) Modification in tooth height.

All these geometrical changes are reflected in the tooth geometry of altered tooth-sum gears, which in turn affects the performance aspects like bending stress, contact stress, contact ratio etc. For a given center distance and module, the size of the base circles of meshing gears change due to alteration in tooth-sum. Thus, the common tangent to the altered base circles change and defines an operating pressure angle α_e, also known as working pressure angle.

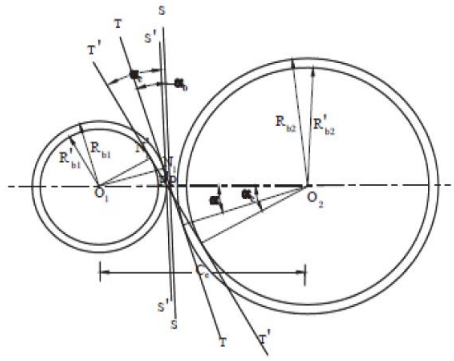


Fig.5 Operting pressure angle.

The operating pressure angle may be larger or smaller than the standard pressure angle depending on whether the teeth alteration is negative or positive. This is shown in fig.5. The operating pressure angle can be calculated using equation (1)

$$\alpha_e = \mathrm{acos}(Zs'\cos(\alpha)/Zs) \qquad -(1)$$

The amount of total profile shift required for accommodating altered tooth-sums operating between specified center distance and module is given by equation (2).

$$X_e = (Z_1' + Z_2')\,(\mathrm{inv}\,\alpha_e - \mathrm{inv}\,\alpha) / (2 \tan\alpha) \qquad -(2)$$

Table 1.
Details of ATS 100 gearing, (GR 1:1)
(Z_s=100, Z_1xZ_2=50x50)

Z_e	Z_s'	Z_1'xZ_2'	α_e, (deg)	X_e	X_1	X_2
- 4	96	48x48	25.564	2.277	1.139	1.139
- 3	97	48x49	24.286	1.659	0.829	0.829
- 2	98	49x49	22.942	1.072	0.536	0.536
-1	99	49x50	21.519	0.518	0.259	0.259
0	**100**	**50x50**	**20.000**	**0.000**	**0.000**	**0.000**
+1	101	50x51	18.361	-0.481	-0.24	-0.24
+2	102	51x51	16.567	-0.920	-0.46	-0.46
+3	103	51x52	14.560	-1.314	-0.657	-0.657
+4	104	52x52	12.237	-1.657	-0.828	-0.828
+5	105	52x53	9.363	-1.938	-0.969	-0.969

Table.1 shows the details of different altered tooth-sum possibilities and its parameters for a tooth-sum 100. It can be seen that for a given center distance 100 mm, it is possible to accommodate a maximum negative teeth alteration of -4 resulting in tooth-sum 96 and a maximum positive teeth alteration of +5 resulting in tooth-sum 105. Teeth alterations beyond this lead to impractical results like a pointed tooth for negative alteration; or a very low pressure angle tooth for positive alteration. For each tooth altered over the standard tooth-sum, the gears will receive profile shifts accordingly.

VI. Effect On Bending Stress

Bending stress in gears depends on the tooth load, tooth height and its root thickness. Bending stress is calculated using the Lewis Equation. For a given material, it determines the load carrying capacity against the tooth breakage or for a given load one can compute the induced bending stress and check for permissible limits using the equation. The failure by bending can be avoided by modifying the tooth geometry. Further, the load shared by the tooth along the path of contact varies from point to point as illustrated in fig.4. The tooth geometry which has short tooth height and thicker root results in reduced bending stress. As profile shift and tooth topping both occurs in altered tooth-sum approach, they contribute to reduction in bending stress. From the study it is understood that negative teeth alteration leads to positive profile shift and vice-versa. Here, negative teeth alteration gears have stronger teeth due to reduced bending stress while positive teeth alteration gears have quieter operation due to increased contact ratio. The bending stress in the gear tooth is induced as the tooth mesh cycle begins. Though it is possible to obtain the bending stress induced at all the salient contact points from A to E, only D2 and E are important. Hence, the bending stresses for only these points are considered. Fig.6 and Fig.7 show the plot of bending stress versus profile shift X_1 induced for altered tooth-sum 100 gears (GR 1:1) having 2mm module and 20 deg pressure angle. The tooth-sum is altered with values of Z_e ranging from -4 teeth to +5 teeth and the resulting total profile shift Xe is taken from Table.1. It can be observed that different tooth-sums have different optimum profile shift co-efficient resulting in minimum bending stress. But, at the same time the bending stress induced on gear tooth has also to be taken care of, hence altered tooth-sum gears having equal bending strength is identified by its profile shift X_1 such that it gives lowest equal bending stress. This occurs for negative teeth alteration with positive profile shift as shown in fig.8 for HPSTC loading and fig. 9 for tip loading.

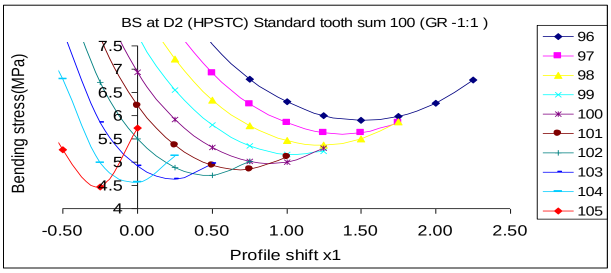


Fig.6 Bending stress BS-HPSTC v/s X_1.

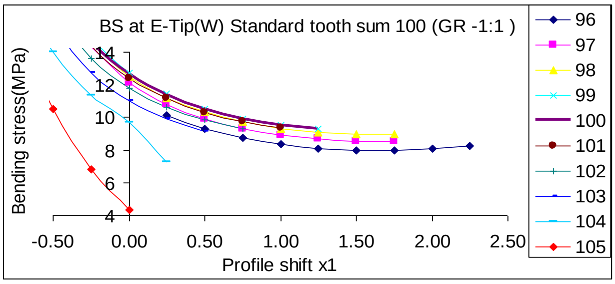


Fig.7 Bending stress BS-Tip v/s X_1.

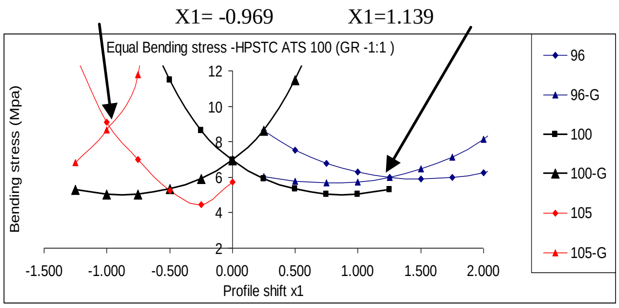


Fig.8 Equal Bending stress BS-HPSTC v/s X_1.

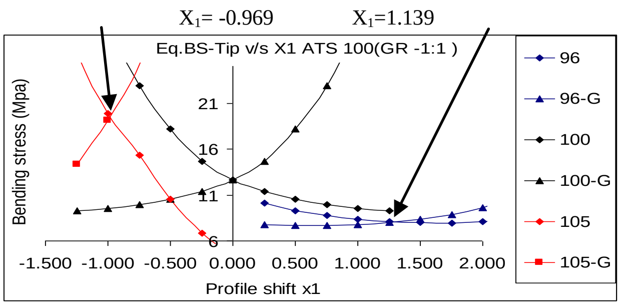


Fig.9 Equal Bending stress BS-Tip v/s X_1.

VII. Effect On Contact Ratio

Contact ratio of a gear pair is the ratio of the length of the path of contact to the base pitch and is affected only when the tooth profile is modified. The tooth load shared by the gear tooth is shown in the fig.10.

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In normal contact ratio gears (CR<2) the tooth load on the gear tooth is half the full load (shared by two pairs of gear teeth) for some time and full load (shared by one pair of teeth) for the remaining duration of engagement. While in high contact ratio gears (CR>2) the tooth load on the gear tooth never exceeds half the full load throughout the mesh cycle, because minimum two pairs of teeth will always be in mesh.

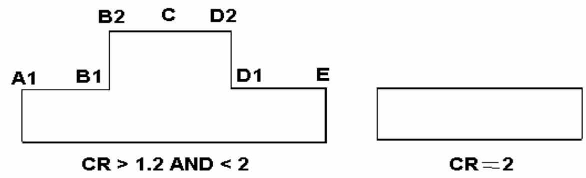

Fig 10. Basic load sharing a long the length of contact.

The effect of varying the tooth-sum on contact ratio is studied. For the aforesaid gears the geometrical dimensions for both standard and altered tooth-sum gears are calculated and contact ratio is determined. Fig.11 shows the variation of contact ratio for different values of Z_e altered over tooth-sum 100; it is observed that the contact ratio increases with increase in teeth alteration. For altered tooth-sum gears of GR 1:1, the value of contact ratio peaks when the resulting profile shift is distributed equally among the mating gears ($X_1=X_2=X_e/2$) in which case the length of approach is equal to length of recess. Thus, by altering the tooth-sum, HCR gearing can be achieved for positive values of Z_e. This is impossible in standard gears.

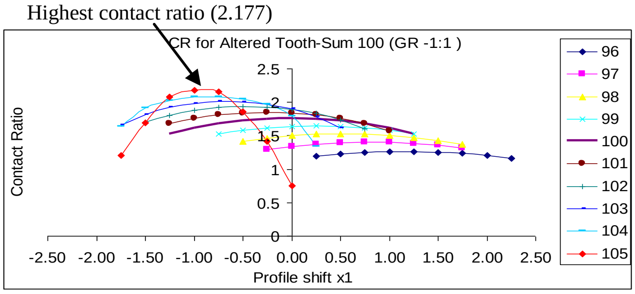

Fig 11- Contact ratio v/s X_1

VIII. RESULTS AND DISCUSSION

Analytical results are presented for involute spur gears of tooth-sum 100, 2 mm module, 20 deg pressure angle and 10 mm face width considering a static load of 9.81 N/mm. From the plots in fig.6 and fig.7 it can be seen that there is a change in bending stress corresponding to teeth alteration. For each teeth altered, the bending stress decreases with increase in profile shift upto certain value of X_1, beyond which it again increases. This means that each tooth-sum has an optimum profile shift coefficient that results in lowest bending stress.

But the bending stress induced on pinion should not be the lonely criteria; other aspects like bending stress induced in the mating gear tooth and contact ratio also needs to be considered here.

Though many combinations of altered tooth-sum and profile shifts are possible, conditions to avoid interference or undercutting, minimum top land thickness of 0.4m, minimum contact ratio of 1.2 and location of pitch point within the length of contact are required to identify the practically operable domain.

Table 2.
Results of ATS 100 gears, GR 1:1

For Minimum Bending stress (BS) condition	Zs= 100(50x50) Zs'= 96(48x48) X_1= 1.139		Ze= -4 X_e= 2.277 X_2= 1.139
Parameter	BS-HPSTC (Mpa)	BS-Tip (MPa)	CR
ATS gear	6.108	8.193	1.266
Standard gear	6.94	12.66	1.755
% change	-12%	-35.28%	-27.86%

For Highest Contact ratio (CR) condition	Zs= 100(50x50) Zs'= 105(52x53) X_1= -0.969		Ze= 5 X_e= -1.938 X_2= -0.969
Parameter	BS-HPSTC (Mpa)	BS-Tip (MPa)	CR
ATS gear	8.768	19.218	2.177
Standard gear	6.94	12.66	1.755
% change	26.34%	51.80%	24.05%

The plots in fig.8 and fig.9 helps in identifying the profile shift for situations of equal bending stress, the plot in fig.11 helps to identify highest contact ratio. Table.2 shows the summary of results. It can be seen that for a standard tooth-sum 100, minimum bending stress can be obtained with negative teeth alteration of -4 resulting in tooth-sum 96 (Z_1=48, Z_2=48) having positive profile shift of X_1=1.139, X_2=1.139, total profile shift being X_e=2.277. With this alteration, a bending stress reduction of 12% and 35.28% is obtained when load is acting at the HPSTC and at the tip respectively. Similarly, a high Contact ratio of 2.177 (a 24.05% increase) is achieved with positive teeth alteration of +5 resulting in tooth-sum 105 (Z_1=52, Z_2=53) having negative profile shift of X_1= -0.969, X_2= -0.969, total profile shift being X_e= -1.932.

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Fig.12 shows the comparison between the profiles of STS tooth and ATS tooth which helps in understanding the reasons for reduction in bending stress. As is seen, the root thickness increases for negative teeth alteration and vice versa, while tooth topping takes place for both positive and negative teeth alterations. The amount of change in tooth root thickness and amount of topping are different for counter teeth alterations.

This study successfully proves that by implementing the altered tooth-sum concept, it is possible to obtain a reduction in bending stress. Based on this, it is learnt that by negative teeth alteration with positive profile shift, 35.28% reduction in bending stress for tip load condition is achieved and by positive teeth alteration with negative profile shift 24.05% increase in CR is achieved as compared to standard gears for tooth-sum 100.

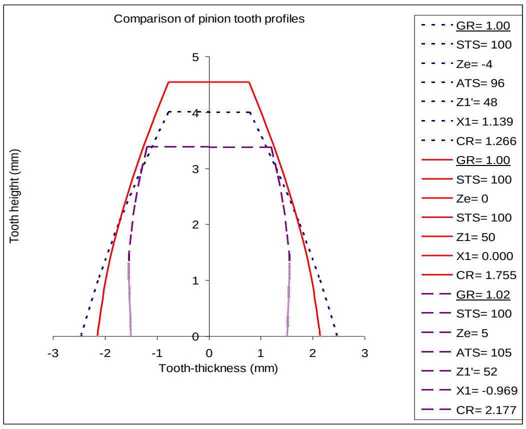

Fig.12 Tooth profiles of altered tooth-sum pinions.

IX. CONCLUSION

The tooth numbers of a standard tooth-sum gears operating between a specified center distance and module can be varied with negative or positive teeth alteration either to have a reduced equal tooth bending stress or maximum contact ratio respectively. This affects the load shared among the mating gears. In order to enhance the bending load capacity of the gear tooth, altering the tooth-sum which operates on different pressure angle while in mesh was considered. Both positive and negative teeth alteration was provided on a tooth-sum 100 with 2 mm module and 20 deg pressure angle gears.

The effect of altering the tooth-sum on the bending stress and contact ratio was studied. It was found that by altering the tooth-sum and judicious distribution of resulting profile shift among the mating gears, it is possible to design gears either to have a lower tooth bending stress by negative teeth alteration or high contact ratio by positive teeth alteration. Thus, this method of altering the tooth-sum among mating gear pair helps in designing gears that can have either lower bending stress or high contact ratio. Considering the material needs, heat treatment and vibration aspects, altered tooth-sum design can be traded off for

a. Increased load carrying capacity.
b. High stiffness.
c. Quiet operation.
d. Longer life.
e. Reduced cost.
f. Size and weight reduction and
g. No structural changes.

As the above benefits can be derived from the proposed analysis without resorting to any structural changes, it may be concluded that altered tooth-sum concept can be considered as a novel, promising and unique alternative approach to gear design.

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