International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347 - 6435 (Online) Volume 2, Issue 5, May 2014)

Solving Transportation Problems Using Value-Ambiguity Interval Approximations of Fuzzy Numbers

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Abstract- **A new algorithm for finding fuzzy optimal solution of 'Approximations of fuzzy number transportation problem' with the aid of Value-Ambiguity interval approximation (VAIA) of fuzzy number is proposed. The transportation cost, Supply and Demand are represented by VAIA of fuzzy numbers. A Numerical illustration is included for verify the above said notion.**

Keywords-**Fuzzy numbers, approximations of fuzzy numbers, value-ambiguity interval approximations of fuzzy number, fuzzy transportation problem, fuzzy optimal solution.**

I. INTRODUCTION

In some application of fuzzy logic, it is difficult to use general fuzzy numbers therefore, it may be better to use fuzzy numbers with the same type. Some researchers considered a parametric approximation of a fuzzy number such as Nasibor and Peker [12], Ban [1], with respect to the average Euclidean distance. One of the other methods interval approximations of fuzzy numbers which, Chanas [2], Grzegorzewski [6,7,8,9], Roventa and Spirai [16]. which a fuzzy area is converted into one in the interval area.

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost while satisfying fuzzy supply and demand limits. aufman[10] and parra et al. [14] proposed a method for solving fuzzy transportation and also, to find the possibility distribution of the objective value of the transportation problem provided all the inequality constraints are of "≤" types or "≥" types. Chanas et al. [4] developed a method for solving transportation problems with fuzzy supplies and demands via the parametric programming technique using the Bellman-Zadeh criterion [2].

In this Paper, we propose a new algorithm for finding a fuzzy optimal solution for a fuzzy number transportation problem using the value-Ambiguity interval Approximation, where all parameters are VAIA fuzzy numbers.

The section 2 gives the basic concepts and definitions needed for this work. In section 3 the interval approximations namely Value-Ambiguity interval approximations of fuzzy numbers, a new algorithm for solving the fuzzy transportation problem and theorems are presented. In section 4, a relevant numerical example is included. The concluding remarks are presented in the last section.

II. BASICS

In this section, some important definitions, results and notions which are useful to this work are presented.

2.1 Fuzzy Number

A convex and normalized fuzzy set defined on R whose membership function is piecewise continuous is called a fuzzy number.

A fuzzy set is called normal when at least one of its elements attains the maximum possible membership grade.

i.e., for all $x \in R$, $\vee \mu_{\tilde{A}}(x) = 1$,

Where V stand for maximum

A fuzzy set is convex if and only if each of its α-cut is a convex set. Equivalently we may say that a fuzzy set A is convex if and only if

$$\mu_{\tilde{A}}(\lambda r + (1-\lambda)s) \geq \text{Min}\,[\mu_{\tilde{A}}(r), \mu_{\tilde{A}}(s)] \quad \text{for all } r,s \in R^n \text{ and all } \lambda \in [0,1]$$

2.2 Trapezoidal Fuzzy Number

A **Trapezoidal fuzzy number** $\tilde{A} = (a_1, a_2, a_3, a_4)$ is defined by the membership function as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \dfrac{(x-a_4)}{(a_3-a_4)} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

It can be characterized by defining the interval of confidence at level α

Thus for all α ∈ [0, 1]

$\tilde{A}_\alpha = [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4]$.

2.3 Value Of A Fuzzy Number [5]

Let Ã be a fuzzy number with α-cut representation $(A_L(\alpha), A_U(\alpha))$, then the value of Ã is defined as

- $Val(\tilde{A}) = \int_0^1 \alpha[A_U(\alpha) + A_L(\alpha)]\, d\alpha$

2.4 Ambiguity Of A Fuzzy Number [5]

Let Ã be a fuzzy number with α-cut representation $(A_L(\alpha), A_U(\alpha))$, then the ambiguity of Ã is defined as

$$Amb(\tilde{A}) = \int_0^1 \alpha[A_U(\alpha) - A_L(\alpha)]\, d\alpha$$

2.5 Fuzzy Transportation Problem

Consider the fuzzy transportation problem (FTP) having fuzzy costs, fuzzy sources and fuzzy demands,

(FTP) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \times \tilde{x}_{ij}$

Subject to

$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i$, for i = 1,2,…,m …(2.5.1)

$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{b}_j$, for i=1,2,……,n ...(2.5.2)

$\tilde{x}_{ij} \geq 0$, for i = 1,2,………,m and

for i =1,2,………,n …..(2.5.3)

Where

m= the number of supply points;

n=the number of demand points

$\tilde{x}_{ij} \approx (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ is the uncertain of units shipped from supply point i to demand point j.

$\tilde{c}_{ij} \approx (c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4)$ is the uncertain cost of shipping one unit from supply point i to the demand point j.

$\tilde{a}_i \approx (a_i^1, a_i^2, a_i^3, a_i^3)$ is the uncertain supply at supply point i and

$\tilde{b}_j \approx (b_j^1, b_j^2, b_j^3, b_j^4)$ is the uncertain demand at demand point j.

The above problem can be put in the table namely, approximation of fuzzy number transportation table as given below,

				Supply
	$\tilde{c}_{11}$	…………	$\tilde{c}_{1n}$	$\tilde{b}_1$
	. . . .	. . . .	. . . .	
	$\tilde{c}_{m1}$	…………	$\tilde{c}_{mn}$	$\tilde{b}_m$
Demand	$\tilde{a}_1$		$\tilde{a}_n$	

III. Interval Approximation Of Fuzzy Numbers

An interval representation of a fuzzy number may have many useful applications. We introduce the notion of an approximation of interval fuzzy number, its value is same the value of a fuzzy number being approximated.

3.1 Value - Ambiguity Interval Approximation (Vaia) Of Fuzzy Number [17]

Let F(R) be a family of fuzzy numbers and $F^I(R)$ be a family of interval fuzzy numbers.

An interval approximation operator V_A is defined as

V_A: F(R) ⟶ $F^I(R)$ in a way that

V_A: Ã→$V_A(\tilde{A}) = [A^-, A^+]$

Where $A^- = 2\int_0^1 \alpha(A_L(\alpha))d\alpha$

$A^+ = 2\int_0^1 \alpha(A_U(\alpha))d\alpha$,

Then the operator V_A is called a value-ambiguity interval approximation operator, Also the interval $[A^-, A^+]$ is called the value – ambiguity interval approximation of Ã.

NOTE:

The operator V_A transforms a family of interval fuzzy numbers.

3.2 Arithmetic Operations

Let $\tilde{A} = (A_L(\alpha), A_U(\alpha))$ and

$\tilde{B} = (B_L(\alpha), B_U(\alpha))$

Also $[\tilde{A}] = [A^-, A^+]$ and $[\tilde{B}] = [B^-, B^+]$ then,

(i) Addition:

$[\tilde{A}] + [\tilde{B}] = [A^- + B^-, A^+ + B^+]$

(ii) Subtraction:

$[\tilde{A}] - [\tilde{B}] = [A^- - B^+, A^+ - B^-]$

International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347 - 6435 (Online) Volume 2, Issue 5, May 2014)

(iii) Scalar multiplication:

For α > 0, α [Ã] = [αA^-,αA^+]

And for α < 0, α [Ã] = [αA^+,αA^-]

(iv) Multiplication: [11]

[Ã] . [$\widetilde{B}$]=[1/2($A^+B^- + A^-B^+$),

1/2($A^-B^- + A^+B^+$)]

(v) Division: [11]

[Ã] / [$\widetilde{B}$]= ($2\frac{A^-}{B^- + B^+}$,$2\frac{A^+}{B^- + B^+}$)

If [$\widetilde{B}$] is positive and (B^-+B^+) ≠ 0

[Ã] / [$\widetilde{B}$]= ($2\frac{A^+}{B^- + B^+}$,$2\frac{A^-}{B^- + B^+}$)

If [$\widetilde{B}$] is negative and (B^-+B^+) ≠ 0

3.3 Illustration

Let [Ã]=[3,7] and $\widetilde{B}$] = [2,6] then

(i) [Ã] + [$\widetilde{B}$] =[3+2,7+6]=[5,13]

(ii) [Ã] - [$\widetilde{B}$] = [3 − 6,7 − 2] = [−3,5]

(iii) [Ã]. [B] = [1/2(7×2+3×6),1/2(3×2+7×6)]
= [1/2(14+18), 1/2(6+42)]
= [16, 24]

(iv) [Ã] / [$\widetilde{B}$] = [2(3/2+6), 2(7/2+6)]
= [6/8, 14/8]

3.4 Transportation Algorithm For Involving Vai-Approximation Of Fuzzy Number [13]

We, now introduce a new algorithm for finding a fuzzy optimal solution for 'Approximation of fuzzy number transportation problem 'using value-ambiguity interval approximation.

STEP 1. Construct the approximation of fuzzy number transportation problem by using VAIA of fuzzy number and then, convert it into a balanced one, if it is not.

STEP 2. Subtract each row entries of the approximation of fuzzy number transportation table from the row minimum.

STEP 3. Subtract each column entries of the resulting approximation of fuzzy number transportation table after using the step 2. From the column minimum.

STEP 4. Check if each column fuzzy demand is less than to the sum of the fuzzy supplies whose reduced costs in that column are fuzzy zero. Also, check if each row fuzzy supply is less than to sum of the column fuzzy demands whose reduced costs in that row are fuzzy zero. If so, go to step 7.(such reduced table is called the allotment table).if not, go to step 5.

STEP 5. Draw the minimum number of horizontal lines and vertical lines to cover all the fuzzy zeros of the reduced approximation of fuzzy number transportation table such that some entries of row(s) or / and column(s) which do not satisfy the condition of the step 4.are not covered.

STEP 6. Develop the revised reduced approximation of fuzzy number transportation table as follows:

(i) Find the smallest entry of the reduced fuzzy cost matrix not covered by any lines.

(ii) Subtract this entry from all the uncovered entries and add the same to all entries lying at the intersection of any two lines. And then, go to step 4.

STEP 7. Select a cell in the reduced approximation of fuzzy number transportation table whose reduced cost. Say (m,n). If there are more than one, then select anyone.

STEP 8. Select a cell in the m- row or/and n-column of the reduced approximation of fuzzy number transportation table which is the only cell whose reduced cost is fuzzy zero and then, allot the maximum possible to that cell. If such cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced approximation of fuzzy number transportation table whose reduced cost is fuzzy zero.

STEP 9. Reform the reduced approximation of fuzzy number transportation table after deleting the fully used fuzzy supply points and the fully received fuzzy demand points and also, modify it to include the not fully used fuzzy supply points and the not fully received fuzzy demand points.

STEP 10. Repeat step 7 to step 9 until all fuzzy points are fully used and all fuzzy demand points are fully received.

STEP 11. This allotment gives a fuzzy solution to the given approximation of fuzzy number transportation problem.

Now, we prove the following theorems which are used to derive the solution to a approximation of fuzzy number transportation problem with the aid of VAI-interval approximation, is a fuzzy optimal solution to the approximation of fuzzy number transportation problem.

3.5 Theorem

Any optimal solution to fuzzy problem (I) where

(I) Minimize $\tilde{z}^*= \sum_{i=1}^{m}\sum_{j=1}^{n}(\tilde{c}_{ij} \ominus \tilde{u}_i \ominus \tilde{v}_j)\otimes \tilde{x}_{ij}$

Where$\tilde{c}_{ij}= [C_{ij}^- , C_{ij}^+]$

$\tilde{u}_i= [u_i^-, u_i^+]$

$\tilde{v}_j= [v_j^-, v_j^+]$ and

International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347 - 6435 (Online) Volume 2, Issue 5, May 2014)

$\tilde{x}_{ij}=[x^-_{ij}, x^+_{ij}]$

(i.e) Minimize $\tilde{z}^* = \sum_{i=1}^{m}\sum_{j=1}^{n}([C^-_{ij}, C^+_{ij}] \ominus [u^-_i, u^+_i] \ominus [v^-_j, v^+_j]) \otimes [x^-_{ij}, x^+_{ij}]$ Subject to 2.5.1 to 2.5.3 are satisfied, where $\tilde{u}_i$ and $\tilde{v}_j$ are VAIA of fuzzy numbers, is an optimal solution to the problem (II) Where

(II) Minimize $\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij} \otimes \tilde{x}_{ij}$

Subject to (2.5.1) to (2.5.3) are satisfied

Proof:

Now, $\tilde{z}^* = \sum\sum[C^-_{ij}, C^+_{ij}] \otimes [x^-_{ij}, x^+_{ij}] \ominus [u^-_i, u^+_i] \otimes [x^-_{ij}, x^+_{ij}] \ominus [v^-_j, v^+_j] \otimes [x^-_{ij}, x^+_{ij}]$

$\approx \tilde{Z} \ominus \sum[1/2(u^+_i x^-_{ij} + u^-_i x^+_{ij}), 1/2(u^-_i x^-_{ij} + u^+_i x^+_{ij})]$

$\ominus \sum[1/2(v^+_j x^-_{ij} + v^-_j x^+_{ij}), 1/2(v^-_j x^-_{ij} + v^+_j x^+_{ij})]$

$\approx \tilde{Z} \ominus \sum_{i=1}^{m}[(u^-_i, u^+_i) \otimes (x^-_{ij}, x^+_{ij})] \ominus \sum_{j=1}^{n}[(v^-_j, v^+_j) \otimes (x^-_{ij}, x^+_{ij})]$

$\approx \tilde{Z} \ominus \sum_{i=1}^{m} \tilde{u}_i \otimes \tilde{b}_i \ominus \sum_{j=1}^{n} \tilde{v}_j \otimes \tilde{a}_i$

From (2.5.1) and (2.5.2) since

$\sum_{i=1}^{m} \tilde{u}_i \otimes \tilde{b}_i$ And $\sum_{j=1}^{n} \tilde{v}_j \otimes \tilde{a}_i$ are independent of $\tilde{x}_{ij}$, for all I and j.

We can conclude that any optimal solution to the problem (II) is also a fuzzy optimal solution to the problem (I). Hence the theorem.

3.6 Theorem

If { $\tilde{x}_{ij}$, i=1,2,…….,m and j=1,2,….n } is a feasible solution to the problem(II) and $([C^-_{ij}, C^+_{ij}] \ominus [u^-_i, u^+_i] \ominus [v^-_j, v^+_j]) \geq \tilde{0}$, for all I and j where $\tilde{u}_i$ and $\tilde{v}_j$ are some value- ambiguity interval approximation of fuzzy numbers, such that Minimum

$\sum_{i=1}^{m}\sum_{j=1}^{n}([C^-_{ij}, C^+_{ij}] \ominus [u^-_i, u^+_i] \ominus [v^-_j, v^+_j]) \otimes [x^-_{ij}, x^+_{ij}]$ subject to (2.5.1) to (2.5.3) are satisfied, is fuzzy zero, then { $\tilde{x}_{ij}$, i=1,2,…….,m and j=1,2,….n } is a fuzzy optimum solution to the problem(II).

Proof:

From the theorem 3.5, the result follows. Hence the theorem.

IV. Numerical Example

The value-ambiguity of interval approximation of trapezoidal fuzzy number by using transportation problem.

(P1) Consider the following fuzzy transportation problem.

				Supply
(1,2,3,4)	(1,3,4,6)	(9,11,12,14)	(5,7,8,11)	(1,6,7,11)
(0,1,2,4)	(-1,0,1,2)	(5,6,7,8)	(0,1,2,3)	(0,1,2,3)
(3,5,6,8)	(5,8,9,12)	(12,15,16,19)	(7,9,10,12)	(5,10,12,17)
Demand				
(5,7,8,10)	(1,5,6,10)	(1,3,4,6)	(1,2,3,4)	

The corresponding approximation of fuzzy transportation problem(P1) by using the above discussed notion, we have (P2):

				Supply
(5/3,10/3)	(7/3,14/3)	(31/3,38/3)	(19/3,9)	(13/3,26/3)
(2/3,8,3)	(-1/3,4/3)	(17/3,22/3)	(2/3,7/3)	(2/3,7/3)
(13/3,20/3)	(7,10)	(14,17)	(25/3,32/3)	(25/3,41/3)
Demand				
(19/3, 26/3)	(11/3, 22/3)	(7/3, 14/3)	(5/3,10/3)	

It is noted that by using the fuzzy arithmetic [13]

Total Demand=Total supply=19 and hence

The given problem is balanced one.

Now, using the step 2 to the step 3 of the following reduced approximation fuzzy number transportation table is

				Supply
0	(-1.3,3)	(7,11)	(3,7.4)	(4.3,8.6)
(-0.7,2.9)	0	(4.3,7.6)	(-0.7,1)	(0.6,2.3)
0	(0.4,5.7)	(7.4,12.7)	(1.7,6.6)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Now, column wise reduce the approximation of fuzzy number transportation table is

				Supply
0	(-1.3,3)	(-0.6,6.7)	(2,8.1)	(4.3,8.6)
(-0.7,2.9)	0	0	0	(0.6,2.3)
0	(0.4,5.7)	(-0.2,8.4)	(0.7,7.3)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

using the step 5, we have

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International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347 - 6435 (Online) Volume 2, Issue 5, May 2014)

				Supply
0	(-1.3,3)	(-0.6,6.7)	(2,8.1)	(4.3,8.6)
(-0.7,2.9)	0	0	0	(0.6,2.3)
0	(0.4,5.7)	(-0.2,8.4)	(0.7,7.3)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Therefore the smallest entry is (-1.3,3)

Now, using step 6 and step 4, we have

				Supply
0	0	(-3.6,8)	(3.3.5.1)	(4.3,8.6)
(0.6,3.2)	0	0	0	(0.6,2.3)
0	(1.7,2.7)	(-3.2,9.7)	(-2.3,8.6)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

using the step 5, we have

				Supply
0	0	(-3.6,8)	(3.3.5.1)	(4.3,8.6)
(0.6,3.2)	0	0	0	(0.6,2.3)
0	(1.7,2.7)	(-3.2,9.7)	(-2.3,8.6)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Here the smallest entry is (-3.6,8)

Now, using step 6 and step 4, we have, and using step 5

				Supply
0	0	0	(-4.7,8.7)	(4.3,8.6)
(-3,11.2)	(-3.6,8)	0	0	(0.6,2.3)
0	(1.7,2.7)	(-11.2,13.3)	(-10.3,12.2)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Here the smallest entry is (-11.2,13.2)

Now using the step 6 and step 4, we have

				Supply
(-11.2,13.3)	0	0	(-15.9,22)	(4.3,8.6)
(-14.2,24.5)	(-3.6,8)	0	(-11.2,13.3)	(0.6,2.3)
0	(-11.6,13.9)	0	0	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Now step 4 is satisfied

Now, using the allotment rules of the fuzzy transportation problem, we have the allotment

				Supply
	(3.6,7.3)	(0.7,1.3)		(4.3,8.6)
		(0.6,2.3)		(0.6,2.3)
(6.3,8.6)		(-0.6,1.7)	(1.6,3.3)	(7.3,13.6)
Demand				
(6.3,8.6)	(3.6,7.3)	(2.3,4.6)	(1.6,3.3)	

Therefore the fuzzy optimal solution for the given approximation of fuzzy number transportation problem is

$\tilde{x}_{12}$=(3.6,7.3) ; $\tilde{x}_{13}$=(0.7,1.3) ; $\tilde{x}_{23}$=(0.6,2.3)

$\tilde{x}_{31}$=(6.3,8.6) ; $\tilde{x}_{33}$=(-0.6,1.7) ; $\tilde{x}_{34}$=(1.6,3.3)

With the fuzzy objective value

$$\tilde{Z} = [(\tilde{c}_{12} \otimes \tilde{x}_{12}) + (\tilde{c}_{13} \otimes \tilde{x}_{13}) + (\tilde{c}_{23} \otimes \tilde{x}_{23}) + (\tilde{c}_{31} \otimes \tilde{x}_{31}) + (\tilde{c}_{33} \otimes \tilde{x}_{33}) + (\tilde{c}_{34} \otimes \tilde{x}_{34})]$$

$\tilde{Z}$=[(3.6,7.3)⊗(2.8,4.6) +(0.7,1.3)⊗ (10.3,12.6)+ (0.6,2.3) ⊗(5.6,7.3)+(6.3, 8.6) ⊗ (4.3,6.3)+(-0.6,1.7) ⊗ (14,17)+(1.6,3.3) ⊗(8.3,10.6)

=(18.8)+(41.92)+(11.79)+(10.07)+(10.25)+(24.12)

$\tilde{Z}$=116.95

But it is noted that the value $\tilde{Z}$ = 132 for the problem (I) [13], which Shows that our proposed method is more effective than the other.

V. Conclusion

In this work, the optimal solutions for approximations of fuzzy number transportation problem are obtained in terms value-Ambiguity Interval Approximations of fuzzy numbers. A new algorithm for finding the above notion is proposed. This is a systematic procedure, easy to understand and apply. It can also be verified that this proposed algorithm is more effective than the previous procedures involved in the fuzzy numbers.

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