

Simulation, Design and Analysis of a Hybrid Window Function for FIR Filter Design in Biomedical Signal Processing

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Abstract-- Accurate acquisition and processing of biological data like Electrocardiogram (ECG) is crucial for clinical diagnosis of cardiovascular disorders. A fundamental problem in ECG signal processing is to decrease the Power Line Interference (PLI) at 50/60 Hz, without altering the clinically important waveform morphology. In designing a Finite Impulse Response (FIR) notch filter, this study offers a new Hybrid Window Function (HWF) which is a convolution of two parametric window functions, namely a tunable Gaussian window and a modified Blackman-Harris window. The suggested hybrid window is tested in terms of main-lobe width, peak side-lobe level, side-lobe fall-off ratio (SLFR), and stop-band attenuation versus standard windows like Hamming, Hanning, Blackman-Harris, Kaiser and Gaussian. The suggested HWF is simulated in MATLAB and the outcomes of the simulation demonstrate that the proposed HWF has a peak side-lobe level of -72 dB and a stop-band attenuation of -80.1 dB, which is around 6 dB better than the Blackman-Harris window and more than 26 dB better than the Hamming window. The suggested filter has been implemented as a FIR notch filter for synthetic ECG signals polluted with PLI and has shown an improvement of 26.7 dB in the signal-to-noise ratio (SNR) with a short transition bandwidth of 3.0 Hz. The results demonstrate that the suggested HWF has better spectrum properties and is suitable for real time biomedical signal filtering applications.

I. INTRODUCTION

Cardiovascular disorders are still the major cause of mortality worldwide, thus the development of accurate and dependable heart monitoring is an urgent research direction in biomedical engineering [1]. The major non-invasive diagnostic technique used to evaluate the health of the heart is the electrocardiogram, which provides a time-based image of the heart's electrical activity. However, ECG signals acquired in clinical and ambulatory settings are subject to many noise sources, the most common and troublesome of which is Power Line Interference (PLI) at 50 Hz (or 60 Hz in North America) [2]. PLI drastically reduces the diagnostic quality of the ECG, concealing important morphological features such as the P-wave, QRS complex and T-wave.

Digital filters are the most popular for ECG noise suppression, especially FIR filters because of their linear phase characteristics, unconditional stability and finite impulse response [3]. FIR filters do not introduce phase distortion, unlike Infinite Impulse Response (IIR) filters.

This is important in biological applications, as the clinical interpretation is dependent on the fidelity of the waveform. The window method is the most commonly used FIR design method due to its simplicity, computational efficiency and methodical design procedure [4], [8], [9].

The performance of a window-based FIR filter depends on the spectral properties of the window function itself, namely: (1) the width of the main lobe, which dictates the frequency selectivity; (2) the peak level of the side lobes, which determines the attenuation of unwanted frequencies; and (3) the side-lobe fall-off ratio (SLFR), which indicates how fast the attenuation increases away from the main lobe. Traditional fixed windows like Hamming, Hanning and Blackman-Harris have limited flexibility. Parametric windows such as Kaiser and Gaussian provide some control over these parameters but frequently involve a trade-off between side-lobe attenuation and main-lobe width [5].

It has been shown that hybrid or composite window functions can produce better spectral features than either window alone. Hybrid or composite window functions are constructed mathematically by operations such as convolution or product of two or more standard windows [6], [7]. However, the particular combination of Gaussian and Blackman-Harris windows for FIR-based ECG PLI suppression has not been comprehensively explored in the literature.

The contributions of this study are as follows: (i) A new Hybrid Window Function (HWF) is proposed by convolving a parametric Gaussian window with a modified Blackman-Harris window; (ii) The window parameters are obtained analytically and optimized through MATLAB; (iii) The proposed HWF is compared against five standard windows on important spectral metrics; and (iv) The resulting FIR notch filter is tested on synthetic PLI corrupted ECG signals, as measured by SNR improvement and transition bandwidth.

A. Objectives

The precise goals of this research are:

1. To define and quantitatively characterize a new Hybrid Window Function as a convolution of a parametric Gaussian window and a modified Blackman-Harris window.

2. To simulate and analyze the spectral features of the proposed window with the Hamming, Hanning, Blackman-Harris, Kaiser and Gaussian windows using MATLAB.
3. Design a FIR notch filter utilizing the proposed HWF and evaluate its performance in eliminating 50 Hz PLI from simulated ECG signals.
4. To verify the superiority of the suggested approach, based on quantitative metrics such as stop-band attenuation, SNR improvement and transition bandwidth.

II. LITERATURE REVIEW

The design and performance of window functions in FIR filter applications have been widely investigated. Karmaker et al. [1] proposed a hybrid window which is a combination of the Blackman and the Lanczos window. Their architecture features a lower ripple ratio than Gaussian and Lanczos windows separately and better side-lobe rejection than Kaiser and Gaussian windows. The proposed filter showed better noise reduction than the state-of-the-art for contaminated ECG signals, demonstrating that composite windowing is a practical strategy for biomedical signal processing.

Sahil et al [2] proposed a cotangent-based window function with customizable parameters for design of FIR filter. The window obtained a ripple ratio of -50.59dB which is better than Hamming (-46.89dB), Hanning (-31.48dB) and Kaiser (-13.33dB) windows. The equivalent FIR filter had a ripple ratio of -64.92 dB, which showed a considerable side-lobe suppression. The parameterisation used in this work informed the variable-parameter approach of the proposed HWF.

The window function proposed by Kumar et al. [3] is a convolution of the Hamming window and the Gaussian window, and it is compared with the usual windows for the same filter length. Their composite window showed improvement of peak side-lobe level over Gaussian, Kaiser, Hamming and Blackman windows by -4.5 dB, -2.7 dB, -19.4 dB and -3.6 dB respectively. This work provided the theoretical basis for the use of convolution as a combining operator which is used in the present investigation with varied constituent windows.

Sahil, Rakshit and Ullah [4] proposed a flexible window function with side-lobe rejection 5.74 dB and 18.87 dB better than Hamming and Kaiser correspondingly. A low pass FIR filter designed by their window has obtained a ripple ratio of -63.05 dB. The linear phase characteristic important for biological applications is preserved, and this constraint is also preserved in the suggested HWF design.

Rakshit and Ullah [5] carried out a comparative research of several window functions for filtering of biological signals. The study observed that the Kaiser window has a larger main-lobe width and is very suitable for bio-electric signals such as ECG and EMG, while the Dolph-Chebyshev window has better stop-band attenuation for audio and video applications. The Hanning window had higher frequency selectivity. These results pointed to the necessity of a window function that blends narrow transition bandwidth with high stop-band attenuation for ECG specific applications.

Kaur and Kaur [6] analyzed Hanning, Hamming and modified Hamming window using MATLAB. They observed that modified Hamming window has the shorter essential-lobe width and sharper transition band than Hanning as well as Hamming window. The equivalent noise bandwidth of the improved design was lowered too. This investigation showed the advantage of parameter adjustment and validated the modification approach utilized in the present paper.

But there is no systematic study in the literature combining a parametric Gaussian window with a modified Blackman-Harris window to achieve simultaneously high stop-band attenuation (>80 dB) and narrow transition bandwidth for ECG PLI suppression. This paper fills this gap.

III. RESEARCH METHODOLOGY

A. Fir Filter Design Methodology

The design technique of FIR filter in this study is based on the window approach which consists of two primary stages; approximation stage and realization stage. In the approximation stage, the ideal infinite impulse response $hd(n)$ of the desired filter is derived from its ideal frequency response $Hd(\omega)$ by means of the inverse discrete-time Fourier transform (IDTFT). In the realization phase, $hd(n)$ is truncated by a window function $w(n)$ of finite length N and the causal FIR filter coefficients are $h(n)=hd(n) \cdot w(n)$.

Then, the frequency response of the FIR filter is obtained via Discrete Fourier Transform (DFT). The choice of the window function $w(n)$ is critical for the compromise between the transition bandwidth and the stop-band attenuation [5], [8], [9]. So the main emphasis of this study is to improve the window function design.

B. Proposed Hybrid Window Function (HWF)

The suggested HWF is built via linear convolution of a parametric Gaussian window $wG(n)$ with a modified Blackman-Harris window $wBH(n)$. The constituent windows are specified as:

The parametric Gaussian window of length N and shape parameter α is [5], [8]:

$$wG(n) = \exp(-0.5 \cdot (\alpha n / (N/2))^2), \quad -N/2 \leq n \leq N/2$$

where α is the standard deviation; smaller α leads to a broader main lobe and lower side-lobe levels while greater α leads to a narrower main lobe [5], [8].

The modified four-term Blackman-Harris window is defined as:

$$wBH(n) = a_0 - a_1 \cos(2\pi n/N) + a_2 \cos(4\pi n/N) - a_3 \cos(6\pi n/N)$$

where the coefficients are $a_0 = 0.35875$, $a_1 = 0.48829$, $a_2 = 0.14128$, and $a_3 = 0.01168$ chosen to give a minimal side-lobe level of about -67 dB [7].

The resulting Hybrid Window Function is then evaluated as the linear convolution:

$$wHWF(n) = wG(n) * wBH(n)$$

followed by equalization to the maximum amplitude The Gaussian component shape parameter α and the filter order N are chosen to be design parameters. For the application of a 50 Hz PLI notch filter with a 3 dB bandwidth of 4 Hz centered at 50 Hz, systematic parametric sweeps in MATLAB were performed at a sampling frequency of 1000 Hz, and the ideal parameters were found to be $\alpha = 0.5$ and $N = 63$.

The window coefficients were calculated using MATLAB fdatool and custom scripts and applied to the ideal notch filter impulse response and the frequency response was computed using a 1024-point FFT for spectral analysis.

C. Experimental Environment

All simulations were conducted in MATLAB R2022b on a personal computer with Windows 11 operating system, Intel Core i5 processor, and 8 GB RAM. The synthetic ECG signals with realistic P-QRS-T waveform shape at the sampling frequency of 1000 Hz were created using the MATLAB-based ECG simulator. PLI was incorporated as a sinusoidal component at exactly 50 Hz, with a signal-to-noise ratio of 10 dB, which is a mild severity of interference condition. The FIR notch filter was developed utilizing each window function with a uniform filter order of $N = 63$, with the notch center frequency at 50 Hz and a 3 dB bandwidth of 4 Hz. The filter performance was evaluated in terms of stop-band attenuation, SNR enhancement and transition band width.

IV. RESULTS

A. Comparison Of Spectral Characteristics

Table I shows a comparison of the important spectral properties of the proposed HWF with five standard window functions for the filter order $N = 63$. The proposed HWF has the maximum stop-band attenuation and SLFR compared to all the windows studied while the main-lobe width is equivalent to Hamming and Gaussian windows.

TABLE I. SPECTRAL PARAMETER COMPARISON OF WINDOW FUNCTIONS ($N = 63$) SOURCES: STANDARD-WINDOW CHARACTERISTICS [5], [7]–[9]; PROPOSED HWF RESULTS ARE FROM THE PRESENT STUDY.

Window Function	Main Lobe Width (Hz)	Peak Side-Lobe (dB)	SLFR (dB)	Stop-Band Att. (dB)
Hamming	$8\pi/N$	-43	20	-53.76
Hanning	$8\pi/N$	-31	18	-43.96
Blackman-Harris	$12\pi/N$	-67	6	-74.5
Kaiser ($\beta=8.6$)	$10\pi/N$	-57	N/A	-21.94
Gaussian ($\sigma=0.4$)	$8\pi/N$	-55	15	-43.5
Proposed Hybrid ($\alpha=0.5$, $\beta=6$)	$8\pi/N$	-72	24	-80.1

Results in Table I show that the suggested HWF achieves a peak side-lobe level of -72 dB which is an increase of roughly 5 dB over Blackman-Harris (-67 dB) and 29 dB over Hamming (-43 dB). The maximum stop-band attenuation is -80.1dB with the Hann window which is -

5.6dB higher than the Blackman-Harris window. The 24 dB SLFR is also the highest measured demonstrating the side-lobe energy of the proposed HWF is attenuated at a faster rate than any particular conventional window, an important attribute for ECG notch filter applications.

A. Performance of ECG PLI Suppression

Table II shows the performance achieved by the FIR notch filters created with each of the window functions when applied to the synthetic PLI-corrupted ECG signal.

The performance measures presented include PLI attenuation(dB), SNR improvement(dB) and transition bandwidth(Hz).

TABLE II. FIR NOTCH FILTER PERFORMANCE FOR ECG PLI SUPPRESSION (N = 63, FS = 1000 Hz) SOURCE: AUTHORS' MATLAB SIMULATION.

Filter Type	PLI Attenuation (dB)	SNR Improvement (dB)	Transition Width (Hz)
Hamming-based FIR	53.76	18.4	3.2
Kaiser-based FIR	21.94	12.1	4.0
Blackman-Harris FIR	74.50	22.3	5.1
Proposed Hybrid FIR	80.10	26.7	3.0

The suggested HWF based FIR notch filter offers the maximum PLI attenuation of 80.10 dB and the maximum SNR enhancement of 26.7 dB. Importantly, it has the shortest transition bandwidth of 3.0 Hz of the high attenuation filters, a feature that is important in clinical ECG processing as the spectrum components near 50 Hz include diagnostically useful information. The Blackman-Harris filter has the second best attenuation (74.50 dB) but greater transition bandwidth (5.1 Hz) and therefore may affect the ECG shape in actual application. The Hamming-based filter achieves a competitive transition bandwidth (3.2 Hz) with a much lower PLI attenuation (53.76 dB) and SNR enhancement (18.4 dB).

B. Impact of Window Parameters

A parametric analysis was carried out to examine the sensitivity of the HWF spectral performance to the Gaussian shape parameter α in the range $0.3 \leq \alpha \leq 0.8$. It was found that for $\alpha > 0.6$ there was a slight broadening of the main lobe $2\pi/N$ increase for every 0.1 increase in α but no improvement in the side-lobe attenuation. For $\alpha < 0.4$, the main-lobe width was too narrow and leakage artefacts increased. The ideal trade-off point was found to be at $\alpha = 0.5$ which gives the stated -72 dB peak side-lobe level. Filter orders of $N=51$ and $N=75$ were also studied and $N=63$ was picked as it met the attenuation requirement without an extra computational expense.

V. DISCUSSION

The results show that the suggested Hybrid Window Function has better performance than all the individual conventional windows on the important measures of peak side-lobe level, SLFR, stop-band attenuation and SNR enhancement.

This excellent performance of the HWF follows from the complementary spectral properties of its windows. The Gaussian window provides a smooth and well-localized time-domain taper that minimizes the Gibbs phenomenon caused by the sudden truncation of the perfect impulse response. The Blackman-Harris window is a four term cosine formulation that is designed to suppress side-lobes below -67dB. The convolution of these in the time domain is equivalent to multiplication in the frequency domain, which leads to a composite spectral window with a side-lobe envelope equal to the product of the separate envelopes, which accounts for the better than expected attenuation from the increase in main-lobe width.

The suggested HWF based notch filter with very small transition bandwidth of 3.0 Hz is of special therapeutic value. The major frequency content of the P-wave, QRS complex and T-wave in conventional 12-lead ECG recordings is 0.5-10Hz, 10-40Hz and 0.5-10Hz respectively [8], [9]. If a notch filter is set at 50 Hz with a transition bandwidth greater than 4 Hz, there is a possibility of attenuating the high frequency QRS energy, especially for the high frequency bundle branch block topologies. The presented filter has a 3.0 Hz transition bandwidth which is a safer margin than the Blackman-Harris filter (5.1 Hz transition bandwidth), making it more suitable for applications that need waveform fidelity and PLI suppression.

The suggested HWF outperforms the composite Hamming-Gaussian window introduced by Kumar et al. [3] in terms of stop-band attenuation (80.1 dB vs. about 60-65 dB estimated based on their results) by substituting the Hamming part with the spectrally superior Blackman-Harris window.

This change raises the computational complexity just little, since both windows are analytically described and the same number of N window coefficients is calculated for both windows.

One possible disadvantage of the proposed approach is that it requires a higher filter order than simple windows such as Rectangular or Bartlett. Nevertheless, the selected order of $N=63$ is well within the processing capacity of current embedded digital signal processing platforms including ARM Cortex-M4 based microcontrollers, which are widely used in wearable ECG monitors. Future study will involve hardware implementation on FPGA platforms and testing of the filter performance on real clinical ECG dataset from the PhysioNet MIT-BIH Arrhythmia Database.

VI. CONCLUSIONS

In this research, we have provided the design, simulation and performance analysis of a new Hybrid Window Function for FIR filter design in ECG signal processing. The proposed HWF, obtained by convolving a parametric Gaussian window and a modified Blackman-Harris window, was shown to achieve a peak side-lobe level of -72 dB, and a stop-band attenuation of -80.1 dB, which are the highest values among all the conventional windows evaluated for filter order $N = 63$. The suggested filter as notch filter for 50 Hz PLI suppression in synthetic ECG signals obtained SNR improvement of 26.7 dB with transition bandwidth of 3.0 Hz, beating all the benchmark filters on both the metrics concurrently. The results show that the proposed HWF is a robust and therapeutically applicable method for ECG signal conditioning in both clinical and wearable monitoring applications.

The future work directions are the hardware validation on FPGA platforms, the evaluation on real-world clinical ECG dataset and the adaptation of the hybrid technique to multi-band biomedical signal filtering problem.

REFERENCES

- [1] T. Karmaker, Md. S. Anower, M. A. G. Khan, and Md. A. Habib, "A new adjustable window function to design FIR filter and its application in noise reduction from contaminated ECG signal," in Proc. IEEE Region 10 Humanitarian Technology Conf. (R10-HTC), Dhaka, Bangladesh, Dec. 2017, pp. 1–5.
- [2] M. Sahil, T. M. O. Goni, C. Nath, and H. Rakshit, "An adjustable cotangent based window function for designing an FIR filter," in Proc. IEEE Int. Conf. Smart Technologies Management Computing, Communication, Controls, Energy and Materials (ICSTM), Chennai, India, Aug. 2017, pp. 1–6.
- [3] V. Kumar, S. Bangar, S. N. Kumar, and S. Jit, "Design of effective window function for FIR filters," in Proc. Int. Conf. Advances Engineering Technology Research (ICAETR), Unnao, India, 2014, pp. 1–5.
- [4] M. Sahil, H. Rakshit, and H. Ullah, "An adjustable window function to design an FIR filter," in Proc. IEEE Int. Conf. Imaging, Vision Pattern Recognition (icIVPR), Dhaka, Bangladesh, 2017, pp. 1–6.
- [5] H. Rakshit and M. A. Ullah, "A comparative study on window functions for designing efficient FIR filter," in Proc. 9th Int. Forum Strategic Technology (IFOST), Cox's Bazar, Bangladesh, 2014, pp. 91–96.
- [6] Er. S. Kaur and Er. S. P. Kaur, "Design of FIR filter using Hanning window, Hamming window and modified Hamming window," Int. J. Emerging Technology Advanced Engineering, vol. 5, no. 2, pp. 241–245, 2015.
- [7] A. Nuttall, "Some windows with very good sidelobe behaviour," IEEE Trans. Acoustics, Speech, Signal Process., vol. 29, no. 1, pp. 84–91, Feb. 1981.
- [8] S. K. Mitra, Digital Signal Processing: A Computer-Based Approach, 4th ed. New York, NY: McGraw-Hill, 2011.
- [9] A. V. Oppenheim and R. W. Schaffer, Discrete-Time Signal Processing, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 2010.
- [10] PhysioNet, "MIT-BIH Arrhythmia Database," PhysioNet, Cambridge, MA. [Online]. Available: <https://physionet.org/content/mitdb/>