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On Nano Plithogenic Neutrosophic Sets and Topology

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***Abstract*— In this paper a new class of set called the Nano Plithogenic Neutrosophic Set was introduced, by integrating plithogenic contradiction modeling with nano approximation concepts under neutrosophic uncertainity. Union, Intersection and Complement were introduced on this new class of set. In addition, Nano Plithogenic Neutrosophic Topology is introduced on the set. Examples are provided to illustrate the proposed concepts. This approach generalizes the existing neutrosophic and nano topological models and provides a flexible tool to deal with complex uncertain information.**

***Keywords*— Neutrosophic Set, Plithogenic Neutrosophic Set, Nano Plithogenic Neutrosophic Set, Nano Plithogenic Neutrosophic Topology, Interior, Closure.**

I. INTRODUCTION

Classical set theory, introduced by Zadeh's predecessors and formalized through crisp sets, plays a fundamental role in mathematics and its applications. In a crisp set, an element either belongs to the set or does not, represented by binary membership values 0 or 1, **fuzzy set theory**, introduced by Zadeh, allows partial membership of elements with degrees ranging between 0 and 1. Fuzzy sets successfully model uncertainty in many practical problems. **Intuitionistic Fuzzy sets**, proposed by Atanassov, incorporate both membership and non-membership degrees, along with a hesitation margin. Subsequently, **Neutrosophic sets**, introduced by Smarandache, generalize Intuitionistic Fuzzy sets by independently defining truth, indeterminacy, and falsity membership degrees. To handle situations involving **multiple attribute values with varying degrees of contradiction**, Smarandache introduced the concept of **Plithogenic sets**. Plithogenic sets incorporate a **contradiction degree** between attribute values and a dominant value, allowing a refined aggregation of information. Plithogenic Neutrosophic sets further enhance this framework by combining Neutrosophic membership with Plithogenic contradiction handling, enabling more realistic modeling of complex multi-criteria environments.

On the other hand, **Nano topology**, introduced as a refinement of rough set theory, studies uncertainty using lower and upper approximations induced by equivalence relations.

Nano topological spaces have been widely investigated in various generalized set frameworks due to their effectiveness in handling boundary regions and indiscernibility among elements. Motivated by the need to integrate **contradiction-aware information modeling** with **approximation-based topological structures**, this paper introduces a **new mathematical framework called the Nano Plithogenic Neutrosophic set**. Furthermore, a **Nano Plithogenic Neutrosophic topology** is developed, and fundamental operations such as union, intersection, complement, interior, and closure are investigated.

II. PRELIMINARIES

Crisp Set : A **crisp set** is a collection of elements in which membership is precisely defined. Each element either belongs to the set or does not belong to it,

Example: Let U={1,2,3,4,} Define Set A={x∈ U | x >2}

Then, A={3,4} where elements 3 and 4 have membership value 1, and elements 1 and 2 have membership value 0.

Fuzzy Set[1]: A **fuzzy set** is characterized by a membership function that assigns to each element a real value between 0 and 1, indicating the degree to which the element belongs to the set

Example : U={a,b,c} A fuzzy set F representing the concept "high performance" may be defined as F={(a,0.8),(b,0.5),(c,0.3)}.Here , element a belongs to the set with a higher degree compared to b and c.

Intuitionistic Fuzzy Set[2] : An **intuitionistic fuzzy set** assigns to each element two independent values: a degree of membership and a degree of non-membership, such that their sum is less than or equal to one.

Example: Let U={a,b} An intuitionistic fuzzy set A is given by

A={(a,0.6,0.2), (b,0.4,0.3)}

Neutrosophic Set [3]: A **Neutrosophic set** generalizes intuitionistic fuzzy sets by associating each element with three independent values: truth membership, indeterminacy membership, and falsity membership, each ranging independently in [0,1]

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Example : Let U ={a,b} A neutrosophic set N can be defined as N={(a,0.7,0.2,,0.1),(b,0.5,0.3,0.2)}.These values represent truth, indeterminacy, and falsity degrees, respectively.

Plithogenic Set[5] : A **Plithogenic set** is a generalized set characterized by multiple attribute values and a contradiction degree between each attribute value and a dominant attribute value. The contradiction degree quantifies the level of incompatibility and influences the aggregation process.

Example: Let U={p_1} and consider the attribute "quality" with dominant value *High*. Suppose p_1 has the attribute values *High* and *Medium* with contradiction degree c(Medium,High)=0.4.

If the corresponding neutrosophic values are V_{High}=(0.8,0.1,0.1),V_{Medium}=(0.6,0.2,0.2),then the plithogenic neutrosophic representation of p_1 reflects both values weighted by their contradiction degree.

Plithogenic Neutrosophic Set[6]: A Plithogenic Neutrosophic Set (PNS) P on U is defined as a mapping

P: U $\rightarrow \bigcup_{v_i \in V}\{(v_i, T_{v_i}(x), I_{v_i}(x), F_{v_i}(x))\}$

Where for each x∈U and each attribute value $v_i \in V$,

$T_{v_i}(x)$ represents the truth membership degree

$I_{v_i}(x)$ represents the indeterminacy membership degree

$F_{v_i}(x)$ represents the falsity membership degree

Such that $T_{v_i}(x), I_{v_i}(x), F_{v_i}(x) \in [0,1]$ and the influence of each attribute value is controlled by its contradiction degree with respect to the dominant attribute value.

Example: Let U={ d}

And consider the attribute performance with attribute values

V={High,Medium}

Where High is the dominant attribute value.

Assume the contradiction degree :

c(Medium,High)=0.4

Let the neutrosophic values be

High(d)=(0.8,0.1,0.1)

Medium(d)=(0.6,0.2,0.2)

Then, the plithogenic neutrosophic representation of d incorporates both attribute values, with the contribution of Medium weighted according to its contradiction degree with High.

III. Nano Plithogenic Neutrosohic Set

Let U be a universe and R be an equivalence relation on U . Then for any X$\subseteq U$ we denote

Nano lower approximation : $N_L(X)$

Nano upper approximation : $N_U(X)$

Nano boundary : $N_B(X) = N_U(X) - N_L(X)$

NANO PLITHOGENIC NEUTROSOPHIC SET

Let (U,R) be a nano approximation space. We define Nano Plithogenic Neutrosophic set (NPNS) as

$N_p = (N_L(P), N_U(P), N_B(P))$.

Here each approximation is a plithogenic Neutrosophic Set

i.e., $N_L(P)(x) = \{T_L(x), I_L(x), F_L(x)\}$

$N_U(P)(x) = \{T_U(x), I_U(x), F_U(x)\}$

Where0≤ $T_L \leq T_U \leq 1$,0≤ $I_L \leq I_U \leq 1$, 0≤ $F_L \leq F_U \leq 1$ and contradiction degree c(a,a_d) are preserved.

Operations On Nano Plithogenic Neutrosophic Set

Consider 2 NPNS sets N_P^1 *and* N_P^2 where

$N_P^1 = (N_L^1, N_U^1)$

$N_P^2 = (N_L^2, N_U^2)$

Now NPNS Union $N_P^1 \cup N_P^2 = (N_L^1 \cup N_L^2, N_U^1 \cup N_U^2)$

$T_L = \max(T_{L_1}, T_{L_2})$

$I_L = \frac{I_{L_1}+I_{L_2}}{2}$,

$F_L = \min(F_{L_1}, F_{L_2})$

$T_U = \max(T_U, T_{U_2})$

$I_U = \frac{I_{U_1}+I_{U_2}}{2}$,

$F_U = \min(F_{U_1}, F_{U_2})$

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Now NPNS Intersection

$N_p^1 \cap N_p^2 = (N_L^1 \cap N_L^2, N_U^1 \cap N_U^2)$

$T_L = \min(T_{L_1}, T_{L_2})$

$I_L = \frac{I_{L_1}+I_{L_2}}{2},$

$F_L = \max(F_{L_1}, F_{L_2})$

$T_U = \min(T_{U_1}, T_{U_2})$

$I_U = \frac{I_{U_1}+I_{U_2}}{2},$

$F_U = \max(F_{U_1}, F_{U_2})$

NPNS Complement

$(N_p)^c = ((N_U)^c, (N_L)^c)$

$T^c = F, I^c = 1 - I, F^c = T$

IV. Nano Plithogenic Neutrosophic Topology

Let U be a non-empty universe and P be a plithogenic neutrosophic set on U.

Let R be an equivalence relation on U. The nano lower approximation $N_L(p)$, nano upper approximation $N_U(p)$ and nano boundary $N_B(p)$ are derived from P.

Let $\tau_{NP} = \{\emptyset_{NP}, U_{NP}, N_L(p), N_U(p), N_B(p)\}$

Then τ_{NP} is called a Nano Plithogenic Neutrosophic Topology on U if

(a) $\emptyset_{NP}, U_{NP} \in \tau_{NP}$

(b) The Union of any two members of τ_{NP} belongs to τ_{NP}

(c)The intersection of any two members of τ_{NP} belongs to τ_{NP}

Then (U, τ_{NP}) is called a Nano Plithogenic Neutrosophic Topological Space.

Note: Element of $\tau_{NP}$are Nano Plithogenic Neutrosophic open sets.

This Topology is generated from plithogenic neutrosophic information under nano approximation.

Example : U=$\{x_1, x_2, x_3\}$

Attribute Quality [Low, Medium, High]
Dominant Attribute Value : High

Contradiction Degrees

c(Medium,High)=0.4

c(Low,High)=0.7

Neutrosophic Set

Element	High	Medium
x_1	(0.8,0.1,0.1)	(0.6,0.2,0.2)
x_2	(0.7,0.2,0.1)	(0.5,0.3,0.2)
x_3	(0.6,0.3,0.1)	(0.4,0.4,0.2)

Plithogenic Neutrosophic Aggregation

P(x)=(1-c)V_{High} +cV_{Medium}

P(x_1)=(0.72,0.14,0.14) , P(x_2) $= (0.62,0.26,0.12)$

P(x_3) $= (0.52,0.34,0.14)$

Hence, Plithogenic Neutrosophic Set

P={(x_1,(0.72,0.14,0.14)), (x_2, $(0.62,0.26,0.12)$), (x_3, $(0.52,0.34,0.14)$)}

Now, Define equivalence Relation R on U

R={$(x_1,x_1),(x_2,x_2),(x_3,x_3),(x_1,x_2),(x_2,x_1)$}

For $\{x_1, x_2, x_3\}$

N_L(P)={(x_1, $(0.62,0.26,0.12)$), (x_2 $(0.62,0.26,0.12)$), (x_3 $(0.52,0.34,0.14)$)}

N_U(P)={(x_1, $(0.72,0.26,0.14)$), (x_2 $(0.72,0.26,0.14)$), (x_3 $(0.52,0.34,0.14)$)}

N_B(P)={(x_1, $(0.10,0,0.02)$), (x_2 $(0.10,0,0.02)$), (x_3 $(0,0,0,)$)}

$\emptyset_{NP} = \{(x_1 (0,1,1)), (x_2 (0,1,1)), (x_3 (0,1,1))\}$

$U_{NP} = \{(x_1 (1,0,0)), (x_2 (1,0,0)), (x_3 (1,0,0))\}$

$\tau_{NP} = \{\emptyset_{NP}, U_{NP}, N_L(p), N_U(p), N_B(p)\}$

Nano Interior:

The nano Interior of a nano plithogenic neutrosophic set P is denoted by NINT(P) and is defined as the largest nano plithogenic neutrosophic open set contained in A

Nano Closure :

The nano closure of a nano plithogenic neutrosophic set P is denoted by NCl(P), is the samllest nano plithogenic neutrosophic closed set containing A

Example :

NINT(P)={(x_1,(0.62,0.26,0.12)), (x_2,(0.62,0.26,0.12)), (x_3,(0.52,0.34,0.14))}

NCl(P)={(x_1,(0.72,0.26,0.14)), (x_2,(0.72,0.26,0.14)), (x_3,(0.52,0.34,0.14))}

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