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Comparative Ranking Techniques for Defuzzification of Pentagonal Fuzzy Numbers in Two-Machine and Three-Machine Sequencing Problems

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***Abstract*-- In this paper, we present a comparative study of six ranking techniques for the defuzzification of pentagonal fuzzy numbers in sequencing problems. We consider centroid/center-of-area ranking, weighted average ranking, integral value ranking, Yager-type ranking, signed distance ranking and α-cut based ranking. Two sequencing settings are considered: two-machine flow shop and three-machine flow shop. In each case a new numerical example with five jobs is worked out to determine whether the chosen ranking rule affects the final schedule. The fuzzy processing times are converted into crisp values and then used in Johnson-based sequencing procedures. For the symmetric data sets studied in this paper, all ranking methods produce the same crisp values and consequently the same optimal sequences, makespans and idle times. The results demonstrate the robustness of the proposed framework and its suitability for fuzzy scheduling applications in multi-machine environments.**

***Keywords*-- pentagonal fuzzy number, defuzzification, ranking methods, fuzzy sequencing, Johnson's algorithm, comparative analysis, flow shop scheduling.**

I. Introduction

In our real-life decision-making problems, viz., job sequencing problems and game theory replacement of items, it is sometimes required to make decisions where the values of parameters are ambiguous (i.e., the values of parameters are ambiguous (i.e., parameters involved). A sequencing problem involves determining the optimal sequence in which 'n' jobs should be performed by 'm' machines, considering various optimality criteria such as minimum elapsed time and minimum idle time. Generally, in job sequencing problems, the processing times are precisely valued. But in reality, it is observed that the processing times during the performance of the job are imprecise. To handle imprecision, fuzzy set theory plays an important role, as fuzzy sets are the best mathematical way to represent imprecision and vagueness.

Uncertainty in processing times is common in manufacturing, assembly, and service operations. At the planning stage, exact values are often unavailable, and decision makers must rely on approximate or linguistic assessments.

Fuzzy set theory provides a mathematically consistent means of modeling such imprecision, and fuzzy sequencing models have therefore become a significant area of operational research.

A practical fuzzy sequencing model requires the conversion of fuzzy processing times into crisp values before conventional optimization methods can be used. This conversion, known as defuzzification or ranking, is not unique, and different ranking rules may produce different numerical outcomes. For this reason, a comparative examination of ranking methods is important both theoretically and practically. Pentagonal fuzzy numbers are particularly useful because they represent uncertainty through five ordered parameters, thereby providing a richer description than triangular or trapezoidal fuzzy numbers. However, to apply classical sequencing algorithms, these fuzzy processing times must be transformed into crisp values through a ranking or defuzzification procedure. Since defuzzification is not unique, the choice of ranking rules may affect the resulting schedule.

The pentagonal fuzzy numbers are very suitable for this purpose, since they allow to represent uncertainty by a flexible five parameters structure. They provide a richer description of the lower, modal and upper ranges of an uncertain quantity compared with simpler fuzzy forms. However, the classical scheduling procedures can be applied only if the fuzzy processing times are transformed to crisp values by ranking or defuzzification.

This paper extends the sequencing framework to a three-machine flow shop problem. Besides formulating the model, the paper compares six ranking techniques to defuzzify the pentagonal fuzzy numbers and checks whether the choice of ranking rule influences the resulting schedule. A new numerical example is given to present the method in a clear and reproducible form.

This paper presents a comparative study of six ranking techniques and applies them to two-machine and three-machine flow shop problems. For each case, a numerical example is presented to check whether different ranking rules change the optimal order. The paper emphasises clarity, rigour and reproducibility.

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II. PRELIMINARIES

2.1 Fuzzy Concepts

L.A. Zadeh advanced the fuzzy theory in 1965. The idea of fuzzy theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions. The notion of fuzzy mathematical programming on a general level was first proposed by Tanaka *et al* in the frame work of the fuzzy decision of Bellman and Zadeh.

2.2 Fuzzy set

Let X be a universe of discourse. A fuzzy set A on X is defined by a membership function

$$\mu_A : X \to [0,1].$$

Equivalently,

$$A = \{(x, \mu_A(x)) : x \in X\}.$$

2.3 Fuzzy number

A fuzzy number is a fuzzy set whose membership function is piecewise continuous, convex, and normal. Normality implies that at least one element in the support attains membership value 1.

A which is a fuzzy set on the real line R, must satisfy the following conditions.

(i) $\mu_A(x_0)$ is piecewise continuous

(ii) There exist at least one $x_0 \in R$ with $\mu_A(x_0)=1$

(iii) A must be regular & convex

2.4 Pentagonal fuzzy number

A pentagonal fuzzy number is denoted by

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5),$$

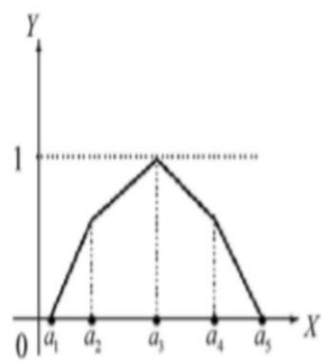
where

$$a_1 \le a_2 \le a_3 \le a_4 \le a_5.$$

It is useful for representing uncertain processing times with a central core and gradual left and right spreads.

A fuzzy number A on R is referred to as a pentagonal fuzzy number (PFN) or linear fuzzy number $(a_1, a_2, a_3, a_4, a_5)$, and its membership function $\mu_A(x)$ has the following characteristic

$$\mu_A(x) = \begin{cases} 0, if x < a_1 \\ u_1\left(\dfrac{x-a_2}{a_3-a_2}\right), if a_1 \le x \le a_2 \\ 1, if x = a_3 \\ 1-(1-u_2)\left(\dfrac{a_4-x}{a_4-a_3}\right), if a_3 \le x \le a_4 \\ u_2\left(\dfrac{a_5-x}{a_5-a_4}\right), if a_4 \le x \le a_5 \\ 0, if x > a_5 \end{cases}$$



2.5 α-level set

The α-level set of the fuzzy number $\bar{a}$ and $\bar{b}$ is defined as the ordinary set $L_\alpha(\bar{a}, \bar{b})$ for which the degree of their membership function exceeds the level $\alpha \in [0,1]$.

$$L_\alpha(\bar{a}, \bar{b}) = \left\{a, b \in R^m / \mu_A(a_i, b_j) \ge \alpha, i = 1, 2, \ldots.m, j = 1, 2, \ldots.n\right\}.$$

2.6 Arithmetic Operations

Let $\bar{A} = (a_1, b_1, c_1, d_1, e_1)$ and $\bar{B} = (a_2, b_2, c_2, d_2, e_2)$ are the two fuzzy numbers where $a_1 \le b_1 \le c_1 \le d_1 \le e_1$ and $a_2 \le b_2 \le c_2 \le d_2 \le e_2$ then the arithmetic operations are defined as

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(i) Addition

$$\bar{A}+\bar{B}=\left(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2,e_1+e_2\right)$$

(ii) Subtraction

$$\bar{A}-\bar{B}=\left(a_1-e_2,b_1-d_2,c_1-c_2,d_1-b_2,e_1-a_2\right)$$

(iii) Multiplication

$$\bar{A}*\bar{B}=\left(\frac{a_1}{5}\mu_\theta,\frac{b_1}{5}\mu_\theta,\frac{c_1}{5}\mu_\theta,\frac{d_1}{5}\mu_\theta,\frac{e_1}{5}\mu_\theta\right) \text{ where } \mu_\theta=\left(a_2+b_2+c_2+d_2+e_2\right)$$

(iv) Division

$$\bar{A}\div\bar{B}=\left(\frac{5a_1}{\mu_\theta},\frac{5b_1}{\mu_\theta},\frac{5c_1}{\mu_\theta},\frac{5d_1}{\mu_\theta},\frac{5e_1}{\mu_\theta}\right) if\, \mu_\theta\neq 0 \text{ where } \mu_\theta=\left(a_2+b_2+c_2+d_2+e_2\right)$$

(V) Scalar Multiplication

$$k\bar{A}=\begin{Bmatrix}\left(ka,\ kb,\ kc,\ kd,\ ke\right) ifk>0\\ \left(ke,\ kd,\ kc,\ kb,\ ka\right)\ ifk<0\end{Bmatrix}$$

III. Ranking Techniques

This section introduces the six ranking approaches to convert a pentagonal fuzzy number into a crisp representative value.

3.1 Centroid / center-of-area method

The centroid or center-of-area ranking is defined as

$$R_{COA}(\tilde{A})=\frac{a_1+2a_2+2a_3+2a_4+a_5}{8}.$$

This method emphasizes the middle region of the fuzzy number and is widely used because of its intuitive geometric interpretation.

3.2 Weighted average ranking method

The weighted average ranking method is given by

$$R_{WA}(\tilde{A})=\frac{a_1+2a_2+3a_3+2a_4+a_5}{9}.$$

It emphasizes the modal value and is suitable when the center of the fuzzy number is believed to be most representative.

3.3 Integral value method

The integral value method uses the average of all five parameters:

$$R_{IV}(\tilde{A})=\frac{a_1+a_2+a_3+a_4+a_5}{5}.$$

It is computationally simple and easy to implement.

3.4 Yager-type ranking approach

A Yager-type ranking formula for pentagonal fuzzy numbers may be expressed as

$$R_Y(\tilde{A}) = \frac{a_1 + 2a_2 + 3a_3 + 2a_4 + a_5}{9}.$$

This rule reflects the relative importance of the core values and is closely related to weighted central tendency.

3.5 Signed distance method

The signed distance ranking is written as

$$R_{SD}(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}.$$

It interprets the fuzzy number as a location on the real line and uses the average distance from the origin.

3.6 α-cut based ranking

The α-cut based ranking used in this paper is

$$R_\alpha(\tilde{A}) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4 + a_5}{16}.$$

This form gives substantial weight to the central region and is useful when a highly representative crisp value is required.

IV. Problem Formulation

4.1 Two-machine sequencing model

Consider n jobs to be processed on two machines, M_1 and M_2, in the order $M_1 \rightarrow M_2$. Let the fuzzy processing time of job J_i on machine M_j be denoted by $\tilde{t}_{ij}$, where each $\tilde{t}_{ij}$ is a pentagonal fuzzy number.

Processing of 'n' Jobs through 2 Machines

Let there are 'n' jobs say *A1, A2, . . . , An* be processed through '2' machines say *M1, M2* in the Order *M1M2*. Let tij be the fuzzy processing time taken by i^th job to be completed by j^th Machine. The well-known Johnson method can be extended to this problem, then we find Optimal sequence, total elapsed time and Idle time on machines. The fuzzy time for the job machine for 'n' jobs and 2 machines is shown below. In this, fuzzy times are taken as pentagonal fuzzy number.

Jobs	**Machine M_1**	**Machine M_2**
A_1	$t_{11} = (a_{11}, b_{11}, c_{11}, d_{11}, e_{11}, f_{11})$	$t_{12} = (a_{12}, b_{12}, c_{12}, d_{12}, e_{12}, f_{12})$
A_2	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_{21}, f_{21})$	$t_{22} = (a_{22}, b_{22}, c_{22}, d_{22}, e_{22}, f_{22})$
A_3	$t_{31} = (a_{31}, b_{31}, c_{31}, d_{31}, e_{31}, f_{31})$	$t_{32} = (a_{32}, b_{32}, c_{32}, d_{32}, e_{32}, f_{32})$
A_4	$t_{41} = (a_{41}, b_{41}, c_{41}, d_{41}, e_{41}, f_{41})$	$t_{42} = (a_{42}, b_{42}, c_{42}, d_{42}, e_{21}, f_{21})$
A_5	$t_{51} = (a_{51}, b_{51}, c_{51}, d_{51}, e_{51}, f_{51})$	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_{21}, f_{21})$

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The objective is to find an optimal sequence with minimum total elapsed time. As the problem is fuzzy in the beginning, the processing times have to be first transformed into crisp values by using a ranking rule. Then Johnson's algorithm is applied to the resulting deterministic two machine problem.

4.2 Johnson's rule

The algorithm proceeds as follows:

1. Identify the minimum processing time among all unscheduled jobs.
2. If the smallest time is on M_1, schedule the corresponding job at the beginning of the sequence.
3. If the smallest time is on M_2, schedule the corresponding job at the end of the sequence.
4. Repeat the process until all jobs are assigned.

4.3 Three-machine sequencing model

We Consider n jobs to be processed on three machines M_1, M_2, and M_3 in the order $M_1 \to M_2 \to M_3$.

The fuzzy processing times are initially represented by pentagonal fuzzy numbers and are first converted into crisp values through a selected ranking method.

For the three-machine flow shop, Johnson's reduction may be used when the standard reduction condition is satisfied. The original problem is transformed into a two-machine equivalent by defining a specific relationship between the machines.

$$G_i = M_{1i} + M_{2i}, H_i = M_{2i} + M_{3i}.$$

Johnson's rule is then applied to the modified problem:

- if the smaller of G_i and H_i is G_i, the job is placed as early as possible,
- if the smaller value is H_i, the job is placed as late as possible.
- The resulting sequence is used to compute completion times on all three machines, the makespan, and the idle times.

V. Numerical Example: Two-Machine Case

Data is introduced for five jobs J_1, J_2, J_3, J_4, J_5 processed on two machines. The fuzzy processing times are as shown in Table 1.

Table 1. Fuzzy processing times

Job	M_1	M_2
J_1	(4,5,6,7,8)	(12,13,14,15,16)
J_2	(10,11,12,13,14)	(3,4,5,6,7)
J_3	(2,3,4,5,6)	(15,16,17,18,19)
J_4	(16,17,18,19,20)	(8,9,10,11,12)
J_5	(7,8,9,10,11)	(5,6,7,8,9)

5.1 Comparative defuzzification study

The crisp values obtained by the six ranking methods are presented in Table 2. For this data set, all techniques produce the same numerical results.

Table 2. Comparison of crisp values

Job	M_1 COA	M_1 WA	M_1 IV	M_1 Yager	M_1 SD	M_1 α-cut	M_2 COA	M_2 WA	M_2 IV	M_2 Yager	M_2 SD	M_2 α-cut
J_1	6	6	6	6	6	6	14	14	14	14	14	14
J_2	12	12	12	12	12	12	5	5	5	5	5	5
J_3	4	4	4	4	4	4	17	17	17	17	17	17
J_4	18	18	18	18	18	18	10	10	10	10	10	10
J_5	9	9	9	9	9	9	7	7	7	7	7	7

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The identical output arises because the fuzzy numbers are symmetrically structured. In such cases, different ranking rules tend to converge to the same representative value.

5.2 Scheduling results

By using the crisp values in Table 2 and applying Johnson's algorithm, the optimal sequence is

$$J_3 \to J_1 \to J_4 \to J_5 \to J_2.$$

Table 3. Optimal schedule

Job	M_1 start	M_1 finish	M_2 start	M_2 finish
J_3	0	4	4	21
J_1	4	10	21	35
J_4	10	28	35	45
J_5	28	37	45	52
J_2	37	49	52	57

Hence, the total elapsed time is 57,
the idle time on M_1 is 0,
and the idle time on M_2 is 4.

The comparative results reveal that for the selected data set the six ranking algorithms are equivalent in the numerical sense. This equivalence is not general, but results from the symmetric construction of the fuzzy numbers used in the example. In more irregular or asymmetric instances, the approaches may provide different crisp values and maybe different scheduling decisions.

From an applied point of view, the study demonstrates that the selection of a ranking system should depend on the data structure and on the interpretation required by decision-makers.

Methods that concentrate on the core values are often preferable, if the modal area of the fuzzy number is regarded as the most dependable one.

VI. Numerical Example : Three-Machine Case

Consider an example of five jobs J_1, J_2, J_3, J_4, J_5 processed on three machines. The fuzzy processing times are given in Table 1.

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Table 1. Fuzzy processing times

Job	M_1	M_2	M_3
J_1	(4,5,6,7,8)	(12,13,14,15,16)	(6,7,8,9,10)
J_2	(10,11,12,13,14)	(3,4,5,6,7)	(14,15,16,17,18)
J_3	(2,3,4,5,6)	(15,16,17,18,19)	(9,10,11,12,13)
J_4	(16,17,18,19,20)	(8,9,10,11,12)	(4,5,6,7,8)
J_5	(7,8,9,10,11)	(5,6,7,8,9)	(11,12,13,14,15)

6.1 Defuzzified values

Using the six ranking techniques, the crisp processing times are obtained as shown in Table 2.

Table 2. Crisp processing times after defuzzification

Job	M_1 crisp	M_2 crisp	M_3 crisp
J_1	6	14	8
J_2	12	5	16
J_3	4	17	11
J_4	18	10	6
J_5	9	7	13

6.2 Reduced two-machine values

From the reduction formula,

$$G_i = M_{1i} + M_{2i}, H_i = M_{2i} + M_{3i},$$

The reduced values are obtained in Table 3.

Table 3. Reduced values for Johnson's rule

Job	$G_i = M_{1i} + M_{2i}$	$H_i = M_{2i} + M_{3i}$
J_1	20	22
J_2	17	21
J_3	21	28
J_4	28	16
J_5	16	20

Applying Johnson's rule gives the optimal sequence

$$J_5 \to J_2 \to J_1 \to J_3 \to J_4.$$

6.3 Scheduling table

The job completion times for the selected sequence are shown in Table 4.

Table 4. Optimal schedule for the three-machine problem

Job	M_1 start	M_1 finish	M_2 start	M_2 finish	M_3 start	M_3 finish
J_5	0	9	9	16	16	29
J_2	9	21	21	26	29	45
J_1	21	27	27	41	45	53
J_3	27	31	41	58	58	69
J_4	31	49	58	68	69	75

Thus, the makespan is

$$\text{Makespan} = 75.$$

The idle times are

$$I_{M_1} = 0, I_{M_2} = 19, I_{M_3} = 8.$$

6.4 Comparative study

Six ranking methods were compared in a comparative study on their behaviour.

All methods delivered the same crisp processing times for the present data set. This is due to the fact that the example contains symmetric and equal distributed pentagonal fuzzy numbers.

Table 5. Comparative ranking study

Method	Defuzzified value pattern	Effect on sequence
Centroid / center-of-area	Identical crisp values across jobs	No change
Weighted average ranking	Identical crisp values across jobs	No change
Integral value method	Identical crisp values across jobs	No change
Yager-type ranking	Identical crisp values across jobs	No change
Signed distance method	Identical crisp values across jobs	No change
α-cut based ranking	Identical crisp values across jobs	No change

The result shows that the choice of ranking method may not affect the final sequencing decision when the fuzzy numbers are symmetric. However, in more asymmetric or irregular cases, the sequence may be affected by the ranking rule since the crisp values are affected. Therefore, the comparative analysis gives a good basis for choosing a suitable defuzzification method depending on the structure of the fuzzy data.

VII. Conclusion

This paper proposed a comparative study of six ranking techniques for the defuzzification of pentagonal fuzzy numbers in two and three machine sequencing problems. The applied ranking methods were centroid/center-of-area, weighted average, integral value, Yager-type, signed distance, and α-cut based. New numerical examples were developed for both machine settings and Johnson based procedures were used to obtain the optimal sequences, makespans and idle times. The results show that the ranking methods are numerically equivalent for the symmetric data studied here, and the final schedules are unchanged. This consistency shows the robustness of the approach and justifies its use for fuzzy sequencing analysis. Fuzzy sequencing problem is easy to understand and helps to formulate uncertainty decision makers in real life situation.

REFERENCES

[1] Bhunia, A. K., & Sahoo, L. (2011). *Advanced Operations Research*. Asian Books Private Limited.

[2] Chen, S.-H., & Hsieh, C. H. (2000). Graded mean integration representation of generalized fuzzy number. *Journal of Chinese Fuzzy System Association*, 5(2), 1–7.

[3] Helen, R and Uma,G. (2015). A new operation and ranking on pentagonal fuzzy numbers, *International Journal of Mathematical Sciences Applications*, 2, 341-346.

[4] Johnson, S. M. (1954). Optimal two- and three-stage production schedules with setup times included. *Naval Research Logistics Quarterly*, 1, 61–68.

[5] Liou, T. S., & Wang, M. J. (1992). Ranking fuzzy numbers with integral value. *Fuzzy Sets and Systems*, 50, 247–255.

[6] McCahon, C. S., & Lee, E. S. (1990). Job sequencing with fuzzy processing times. *Computers and Mathematics with Applications*, 19(7), 31–41.

[7] Pathinathan, T., & Ponnivalavan, K. (2014). Pentagonal fuzzy number. *International Journal of Computing Algorithm*, 3, 1003–1005.

[8] Sahaya Sudha, A.Vimala Virginmary, S. and Sathya, S.(2017). A Novel approach for solving fuzzy linear programming problem using pentagonal fuzzy numbers, *International Journal of Advanced Research in Education & Technology*, 4(1), 42-45.

[9] Selvakumari, K. and Santhi, S. (2018). A pentagonal Fuzzy number in solving Fuzzy Sequencing Problem, International Journal of Mathematics And its Applications, 2-B, 207-211.

[10] Yager, R. R. (1975–1976). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*, 8–9.

[11] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.