International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347 -6435 (Online)), Volume 15, Issue 5, May 2026)

A Study of Some Aspects of Statistical Inference for Increasing Failure Rate Distributions

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***Abstract*—** An important concept in the theory of reliability is the increasing failure rate average (IFRA), a specific form of positive aging that is derived from shock models. The issue of testing exponentiality against positive ageing has garnered significant attention from researchers. This paper investigates the distributional properties of a test procedure that is based on U-statistics for the aforementioned issue. The U-statistic is derived from the extrema of subsamples of size two. The proposed test is found to be more effective in terms of Pitman asymptotic relative efficiency. The study also examines the repeated significance tests for testing exponentiality against IFRA. A comparison of two populations with IFRA property is also taken into account and investigated.

***Keywords*—** Asymptotic relative efficiency, U-statistic, subsample, repeated significance test, two-sample problem.

I. Introduction

The problem of testing exponentiality against positive aging is an area of research in reliability and survival analysis received much attention from researchers. The increasing failure rate average (IFRA) class of distributions is one of the significant classes of life distributions that exhibit positive aging properties. The IFRA class of distributions is the smallest class of probability distributions that is closed under the formation of coherent systems. The issue of testing exponentiality against IFRA alternatives is examined in this article. The following is the definition of IFRA:

Definition 1.1: Let Fbe a absolutely distribution function, with $F(0) = 0$. Then, F is an increasing failure rate average distribution if

$$\bar{F}(bx) \geq \{\bar{F}(x)\}^b \qquad x > 0, \quad 0 < b < 1,$$

Where $\bar{F} = 1 - F$ (1.1)

The equality in (1.1) holds if and only if F is an exponential distribution.

Testing exponentiality against positive aging alternatives has been extensively explored in the literature. Significant contributions to this field include the works of Proschan and Pyke [21], Bickel and Doksum [7], Ahmed [1,2] Hollander and Proschan [11], and Koul [12,13]. Nonetheless, tests explicitly developed for the detection of increasing failure rate average (IFRA) alternatives were primarily introduced by Deshpande [8]. Subsequently, Pandit, Shetty, and Bhadri [16] proposed a test statistic grounded in subsample minima to address this problem. More recently, Astagimath and Pandit [5] investigated the testing of exponentiality against IFRA alternatives.

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In the literature, there are many tests available for testing exponentiality against IFR, IFRA, NBU with fixed sample size. However, in many situations, all the observations are not available at the same time. For example, in life testing and clinical trials observations are available over a period of time. In those cases a sequential or multistage test might be appropriate. The one alternative is to develop nonparametric repeated significance test (RST) procedure. Nonparametric RST procedure has been discussed along with several specific RSTs such as tests for population quantile, location, randomness and bivariate independence etc. As RST is no less efficient in asymptotic sense than the fixed sample test and requires less time and cost, the development of RST for any problem of testing is more desirable. Alam and Basu[3] developed RST for NBU alternative. Pandit and Shirke [20] developed an RST for the problem of testing against IFRA alternatives. Pandit and Math [15] studied RST for testing against NBU alternatives. In this paper we have also studied the RST procedure due to Astagimath et.al[4] for testing exponentiality against IFRA alternatives based on a U-Statistic with kernel of degree four, which is proposed for this problem with fixed sample size (Astagimath and Pandit [5]. The estimates of the expected sample size and power of the test procedure are obtained using simulation experiments.

In survival analysis, a problem of interest is selecting a population which possesses more positive ageing property among two populations possessing positive ageing property.

However, the problem of testing whether one distribution possess more 'positive ageing' property than the other distribution has received less attention. Hollander, Park and Proschan[10] is the first procedure developed specifically for detecting NBU-ness property. Pandit and Gudaganavar [18] proposed tests for detecting positive ageing (IFR and NBU ness) property of distributions. Pandit and Gudaganavar [18] proposed a test procedure for selecting a distribution possessing more new better than used at specified age t_0(NBU-t_0) property between two NBU-t_0 populations Pandit and Gudaganavar [19] studied the problem of detecting more NBU-ness property of life distributions. In this paper, we consider a test procedure due to Astagimathet.al[4] to test the hypothesis that the two life distributions are identical against the alternative that one possesses more 'increasing failure rate average (IFRA)' property than the other. In this paper, three papers on one and two-sample problems of testing for IFRA property is reviewed in detail.

In Section 2, a new class of statistics based on U – Statistics with kernel depending on subsample minima is proposed. Section 3 deals RST for testing exponentiality against IFRA alternatives and the comparison of two populations having IFRA property is presented in Section 4.

II. TESTING EXPONENTIALITY AGAINST IFRA ALTERNATIVES BASED ON SUBSAMPLE EXTREMA

2.1. Proposed class of test Statistics

Let $X_1, X_2, ..., X_n$ be a random sample from a continuous probability distribution with distribution function F such that F(0) = 0. The problem is to test

$H_0 : F$ is exponential against $H_1 : F$ is IFRA.

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Let

$$h_b(x_1, x_2, x_3, x_4) = \begin{cases} 1, \text{if } \mathrm{Min}(x_1, x_2) > \mathrm{bMax}(x_3, x_4) \\ 0, otherwise \end{cases},$$

where b is a fixed number such that $0 \leq b \leq 1$.

Now, the U-statistic based on the kernel $h_b(x_1, x_2, x_3, x_4)$, is defined as

$$U(b) = \binom{n}{4}^{-1} \sum h^*{}_b(x_{i_1}, x_{i_2}, x_{i_3}, x_{i_4})$$

where summation is taken over all combinations of integers (i_1, i_2, i_3, i_4) chosen out of integers $(1,2, \ldots, n)$ and $h^*{}_b(x_1, x_2, x_3, x_4)$ is the symmetrized version of $h_b(x_1, x_2, x_3, x_4)$.

Next, we have

$$E_{H_0}(U(b)) = P[Min(x_1, x_2) > bMax(x_3, x_4)]$$

$$= 2\int_0^\infty F(y)\bar{F}^{2b}(y)dF(y)$$

$$= \frac{1}{(b+1)(2b+1)} \text{ under } H_0$$

The asymptotic distribution of the test statistic is given in theorem 1.

Theorem 2.1: Let $X_1, \ldots, X_n$ denote a random sample from population with absolutely continuous c.d.f $F(x)$. Then, the asymptotic distribution of $\sqrt{n}[U(b) - E(U(b))]$ is normal with mean zero and variance $16\xi_1$, where

$$\xi_1 = Cov[h_b^*(X_1, X_2, X_3, X_4)h_b^*(X_1, X_5, X_6, X_7)].$$

$$= \int_0^\infty E^2[h_b(x, X_2, X_3, X_4)]dF(x) - \left[\frac{1}{(b+1)(2b+1)}\right]^2$$

And

$$E[h(x, X_2, X_3, X_4)] = 1 + \frac{b}{2+b} - \frac{2b}{1+b} + \bar{F}^{2b}(x) - \bar{F}^{2b+1}(x)\left(\frac{2b}{2b+1}\right) + \bar{F}^{\frac{2}{b}+1}(x)\left(\frac{2}{2+b}\right)$$

$$-2\bar{F}^{\frac{1}{b}+1}(x)\left(\frac{1}{b+1}\right).$$

2.2. Asymptotic Relative Efficiency

For the purpose of asymptotic relative efficiency (ARE) comparisons, three parametric families of distributions, namely, Weibull, Linear Failure Rate, and Makeham distribution, have been examined, each of these families is characterized by a real-valued parameter, denoted as θ, such that specific values of $\theta = \theta_0$ correspond to distributions under the null hypothesis, while other values correspond to distributions under the alternative hypothesis.

The Pitman ARE's of V(b,K) test of Pandit, Shetty and Bhadri [16] for different values of b=0.1,0.5,0.9 with respect to Deshpande [8] J_b test and of U_b test for different values of b=0.1,0.5,0.9 with respect to Deshpande [8] J_b test are given in the table 2.1 below:

Table 2.1

ARE(V(b,k), J_b) (ARE(U_b, J_b))

b	0.1	0.5	0.9
Weibull	0.8703(1.0230)	0.4461(0.7170)	0.7648(0.6579)
LFR	0.6233(1.1797)	0.4582(1.7295)	0.3711(4.4076)
Makeham	1.0222(1.4293)	1.1937(2.7322)	1.4943(9.4343)

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2.3. Conclusions

This section introduces a novel test for assessing exponentiality against IFRA alternatives, utilizing a U-statistic with a kernel of degree four (Astagimath and Pandit, [5]). An asymptotic comparison in the Pitman sense is conducted with the tests proposed by Deshpande [8] and Pandit et al. [16]. The findings indicate that:

1. The new test demonstrates superior performance when the alternative hypothesis corresponds to the LFR or Makeham distributions.
2. Conversely, for the Weibull alternative, the test developed by Deshpande [8] exhibits better performance.
3. Nonetheless, the new test remains a viable option for the Weibull distribution, as the asymptotic relative efficiencies (AREs) of the new test relative to Deshpande's [8] test are approximately equal to one.

III. Repeated Significance Test for testing Exponentiality against IFRA alternatives

3.1. RST Procedure

Let $(X_1, X_2, \ldots, X_n)$ be a random sample from an absolutely continuous probability distribution with distribution function $F(x)$.

Here, the problem is to test H_0 : F is exponential against H_1 : F is IFRA .

For that an RST procedure for the above problem is studied which is based on the test statistic U_b, the U-statistic due to Astagimath and Pandit[5], considered in section 2.

In this case RST can be constructed as the following using the asymptotic distribution of the statistic U_b

Let N be the target sample size. For $4 \leq k \leq N$ the test statistic is

$$h_N(k) = \underset{4 \leq n \leq k}{Max} \frac{n[U_b^n - [(b+1)(2b+1)]^{-1}]}{4\sqrt{N\xi_1}} \tag{1}$$

Reject H_0 for large values of $h_N(k)$. That is reject H_0, if $h_N(k) > h_{n,\alpha}$, where $h_{n,\alpha}$ is defined as $0 \leq \alpha_N = P_{H_0}\{h_N(k) > h_{n,\alpha}\} < \alpha \leq P_{H_0}\{h_N(k) \geq h_{n,\alpha}\}$
and $h_N = h(U_b^n : n_0 \leq n \leq N)$.

Here n_0 is the initial sample size. Hence operationally, we may proceed as follows:

Compute $h_N(k)$, $k \geq n_0$. If for some k, $h_N(k) > h_{N,\alpha}$, the experimentation is curtailed with rejection of H_0. If no such k exists, the sampling is stopped with the target sample size N and we accept H_0. The asymptotic distribution of $h_N(k)$ is given in the following theorem.

Theorem 2.1: For the Kolmogorov –Smirnov type statistic in (1), for every $0 < \alpha < 1$, $h_{N,\alpha} \to w_\alpha^+$, as $N \to \infty$, where w_α^+ is the upper $100\alpha\%$ points of the distribution

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of w^+ and $w^+ = \underset{0 \leq t \leq 1}{Sup}\, W(t)$. Here $W = \{W(t), 0 \leq t \leq 1\}$ is a standard Brownian motion on $[0,1]$.

3.2. Simulation Study

Monte-Carlo simulation is conducted to study the expected sample size and power of the proposed RST procedure with the target samples sizes $N = 20$. The different alternatives considered for the study are as follows.

1. Weibull distribution:

$$\bar{F}_1(x) = exp(-x^{\theta}), \theta > 1, x > 0 \text{ and } \theta_0 = 1$$

2. Exponentiated exponential distribution (Gupta et al. (1998)):

$$\bar{F}_2(x) = (1 - \ell^{-x})^{\theta}, \theta > 1, x > 0 \quad \text{and} \quad \theta_0 = 1$$

The estimate of the expected sample size (ESS) and estimated power of the proposed test procedures are evaluated. The conclusion subsection gives the conclusions.

3.3. Conclusions:

a) The following observations are made:

1. There is a substantial reduction in the number of observations required to reach a decision to reject the null hypothesis.
2. The estimated power is below the nominal level when the null hypothesis is true and exceeds the nominal level otherwise. In both scenarios, the test achieves power equal to one as the parameter deviates further from the null value. This behavior is considered a desirable characteristic of any robust test.

b) For practical applications, larger values of *b* are recommended.

c) The performance of the newly developed RST presented in this paper surpasses that of the RST proposed by Pandit and Shirke [21].

d) As expected, the performance of the RST varies depending on the specific alternative distribution under consideration.

e) The authors have also examined the performance of the RST for other cases, yielding similar results.

IV. A Test for Comparing Two Populations Having More IFRA Property

4.1. Test Procedure

Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ denote two random samples from continuous life distributions F and G, respectively. Here, a class of tests is studied for testing

$H_0: F = G$ (the common distribution is not specified)

against $H_1: F$ is 'more IFRA' than G.

Consider the parameter,

$$\gamma(F, G) = \gamma(F) - \gamma(G)$$

where $\gamma(F) = 2\int_0^{\infty} F(x)\, \overline{F}^{2b}(x) dF(x)$ and $\gamma(G) = 2\int_0^{\infty} G(x)\, \overline{G}^{2b}(x) dG(x)$.

Here, $\gamma(F)$ and $\gamma(G)$can be considered as the measure of degree of the IFRA-ness. Astagimath et.al [4] test used this measure as basis for their test statistic. If F, G belongs to IFRA, then $\gamma(F) > 0$ $(\gamma(G) > 0)$ and $\gamma(F, G)$can be taken as a measure by which F is 'more IFRA' than G. Under $H_0, \gamma(F, G) = 0$ and it is strictly greater than zero under H_1.

An unbiased estimator for $\gamma(F, G)$, which is defined as

$$U_{m,n} = U_m - U_n$$

where U_m and U_n are U-statistics with kernels of degree 4 which are defined as

$$h_{1b}(x_1, x_2, x_3, x_4) = \begin{cases} 1 & if\ Min(x_1, x_2) > bMax(x_3, x_4) \\ 0 & Otherwise \end{cases}$$

and

$$h_{2b}(y_1, y_2, y_3, y_4) = \begin{cases} 1 & if\ Min(y_1, y_2) > bMax(y_3, y_4) \\ 0 & Otherwise \end{cases}$$

respectively.

Asymptotic normality of the test $U_{m,n}$

In this subsection, we study the asymptotic distribution of $U_{m,n}$. For that define

$$\xi_1(F) = E[\psi_1(X_1)]^2 - [\gamma(F)]^2,$$

where $\psi_1(x) = E[h_{1b}(x, X_2, X_3, X_4)]$

Next, $\xi_1(G)$ is defined as

$$\xi_1(G) = E[\psi_1^*(Y_1)]^2 - [\gamma(G)]^2,$$

where $\psi_1^*(y) = E[h_{1b}(y, Y_2, Y_3, Y_4)]$

The asymptotic normality of the test $U_{m,n}$is presented in the following theorem.

Theorem: The asymptotic distribution of $\sqrt{N}\left[U_{m,n} - \gamma(F, G)\right]$ is normal with mean zero and variance given by $\sigma^2\left(U_{m,n}\right) = \sigma_1^2 + \sigma_2^2$, where $\sigma_1^2 = \frac{16\xi_1(F)}{\lambda}$ and $\sigma_2^2 = \frac{16\xi_1(G)}{1-\lambda}$, where $\xi_1(F)$and$\xi_1(G)$ are as defined above.

Under $H_0: F = G = F_0$, then $\xi_1(F) = \xi_1(G) = \xi_1(F_0)$

The approximate α-level test rejects H_0in favour of H_1, if $\frac{\sqrt{N}\,U_{m,n}}{\sigma^2(U_{m,n})} > Z_\alpha$, where Z_α is the upper α-percentile point of standard normal distribution. Since, $\gamma(F, G) > 0$ under H_1 and from the asymptotic normality of $U_{m,n}$, the test based on $U_{m,n}$is consistent against the alternative F is 'more IFRA than' G.

4.2. Consistent Estimator of $\sigma^2\left(U_{m,n}\right)$:

Here, we find a consistent estimator for $\sigma^2\left(U_{m,n}\right)$, which is necessary to implement the test procedure developed here. For that let,

$$\hat{\xi}_1(F) = \frac{1}{m-1}\sum_{i=1}^{m}[\psi_1(X_i) - U_m]^2,$$

where $\psi_1(X_i) = \frac{1}{m-1}\sum_i h_1(X_i, X_j, X_k, X_l)$

and the summation $\sum_i$extends over all possible $i < j \neq k \neq l < m$.

We define $\hat{\xi}_1(G) = \frac{1}{n-1}\sum_{j=1}^{n}[\psi_1^*(Y_j) - U_n]^2$,

where $\psi_1^*(Y_j) = \frac{1}{n-1}\sum_j h_1(Y_j, Y_{i,}, Y_{k,} Y_l)$ and the summation $\sum_j$is over all possible $j < i \neq k \neq l < n$.

Then, from Puri and Sen [22] $\hat{\xi}_1(F)$ and $\hat{\xi}_1(G)$ are consistent estimators of $\xi_1(F)$ and $\xi_1(G)$ respectively and hence the consistent estimator $\hat{\sigma}^2(U_{m,n})$ of $\sigma^2(U_{m,n})$ is obtained by replacing $\xi_1(F)$ and $\xi_1(G)$by $\hat{\xi}_1(F)$ and $\hat{\xi}_1(G)$respectively in the expression$\sigma^2(U_{m,n})$. That is,

$$\hat{\sigma}^2(U_{m,n}) = \frac{16N\hat{\xi}_1(F)}{\lambda} + \frac{16N\hat{\xi}_1(G)}{1-\lambda}, \qquad N = m+n.$$

By Slutsky's theorem, we have $\frac{\sqrt{N}U_{m,n}}{\hat{\sigma}^2(U_{m,n})}$ is asymptotically $N(0,1)$ under H_0.

4.3. Asymptotic Relative Efficiency

We study the asymptotic relative efficiency of $U_{m,n}$, relative to the $V_{k,n}$ test statistic given by Hollander, Park and Proschan(1986) for the two pairs of distributions $(F_{i,\theta}, G)$.

Here, we assume that G is an exponential distribution with mean one. We denote $F_{1,\theta}$ as survival function for Weibull distribution and $F_{2,\theta}$ as survival function for Linear failure rate distribution and $F_{3,\theta}$ as survival function for Makeham distribution.

The ARE's of the proposed tests with respect to the J_b test of Deshpande [8] and **V(b,k)** test of Pandit et.al(2008) for Weibull distribution with parameter θ, Linear failure rate distribution and Makeham distribution are presented below in table 4.1.

Table 4.1

AREs of $(U_{m,n}, J_b)$ [ARE$(U_{m,n}, V(b,k)$]

B	0.1	0.5	0.9
Weibull	1.0230 [1.1755]	0.7170[1.6073]	0.6579[1.1625]
LFR	1.1797[1.8927]	1.7295[1.9428]	2.0994[3.4463]
Makeham	1.4293[1.3983]	2.7322[2.2888]	9.4343[2.0555]

Next, we compute the efficiency of the two sample test based on $U_{m,n}$ proposed in section 4.1, by specifying the common null distribution in the null hypothesis as F_θ with $\theta \geq 1$ and considering sequence of alternatives $(F_{\theta\phi_N}, F_\theta)$, where$\phi = 1 + \frac{a}{\sqrt{N}}$, a being

arbitrary positive constant. Note that as $N \to \infty$, the sequence of alternatives converges to the null hypothesis. The efficacy of the $U_{m,n}$ test is given by

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$$eff(U_{m,n}) = \frac{[\gamma'(F_{\theta\phi_N}, F_\theta)]^2}{\sigma_0^2(U_{m,n})}$$

where $\sigma_0^2(U_{m,n})$ is null asymptotic variance of $\sqrt{N}\,U_{m,n}$ and $\gamma'(F,\ G) = \left[\frac{d\gamma(F_{\theta\phi_N}, F_\theta)}{d\phi}\right]_{\phi=1}$

The sequence of alternatives considered here are $(F_{1,\theta\phi_N}, F_{1,\theta})$, $(F_{2,\theta\phi_N}, F_{2,\theta})$ and $(F_{3,\theta\phi_N}, F_{3,\theta})$.

The Asymptotic relative efficiencies of the proposed test $U_{m,n}$ relative to the test due to Hollander, Park and Proschan [10] $V_{k,n}$ for the various alternatives are evaluated and the observations are presented.

4.4. Some Remarks:

1.The Asymptotic relative efficiencies of the proposed test with respect to the test due to Deshpande (1983) and Pandit et.al(2008) are computed for three pairs of distributions (F_θ, G) with G is exponential with mean one and F_θ as Weibull, Linear failure rate and Makeham distributions.

2. It is observed that the proposed test performs better for the alternatives considered F_θ is either Weibull, Linear failure rate and Makeham distributions when G is exponential.

3. The asymptotic efficacies of the test proposed are evaluated for three pairs of distributions $(F_{1,\theta\phi_N}, F_{1,\theta})$, $(F_{2,\theta\phi_N}, F_{2,\theta})$ and $(F_{3,\theta\phi_N}, F_{3,\theta})$ with F_1, F_2, F_3 as Weibull, linear failure rate and Makeham distributions respectively.

4. Hence, if the data under consideration is exactly IFRA, the new test proposed would be a better choice.

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