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Fuzzy Fractional Differential Transformation Method with Adaptive α-Cut Representation (FDTM–Fα): A Novel Framework for Solving FFDEs

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Abstract-- **This paper introduces a novel analytical framework termed the Fuzzy Fractional Differential Transformation Method with Adaptive α-cut Representation (FDTM–Fα) for solving fuzzy fractional differential equations (FFDEs). Unlike existing approaches that treat fuzziness either in parameters or boundary conditions, the proposed method integrates fractional calculus with fuzzy set theory at the transformation level. The method constructs a fractional power series solution under fuzzy uncertainty by embedding α-cut decomposition directly into the differential transformation process. The resulting scheme provides improved convergence, reduced computational complexity, and enhanced handling of uncertainty propagation. Numerical illustrations demonstrate that the proposed method yields highly accurate approximate solutions compared to existing fuzzy transform and fractional methods.**

Keywords-- **Fractional calculus, Fuzzy differential equations, Differential transformation method, α-cut, Uncertainty modelling, Hybrid analytical methods**

I. INTRODUCTION

Fractional calculus has emerged as a powerful mathematical framework for modelling complex systems exhibiting memory, hereditary characteristics, and nonlocal dynamics. Unlike classical integer-order models, fractional differential equations (FDEs) provide more realistic representations of various physical and engineering processes, including viscoelasticity, anomalous diffusion, and biological systems [1]. Consequently, the study of fractional models has gained significant attention in both theoretical and applied research domains.

In practical scenarios, however, system parameters, boundary conditions, and initial values are often subject to uncertainty and imprecision. To address such issues, fuzzy set theory has been widely employed as an effective mathematical tool for handling vagueness without relying on probabilistic assumptions. The integration of fuzzy set theory with fractional calculus leads to fuzzy fractional differential equations (FFDEs), which are capable of capturing both uncertainty and memory effects simultaneously [2]. This combination has opened new avenues for modelling real-world problems in engineering, physics, and applied sciences.

Various analytical and semi-analytical methods have been developed for solving FFDEs, including the Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Variational Iteration Method (VIM), and transform-based techniques. Among these, the Differential Transformation Method (DTM) and its fractional extensions have attracted considerable attention due to their simplicity, efficiency, and ability to generate rapidly convergent series solutions [3]. Several studies have demonstrated that DTM provides accurate approximations for fuzzy fractional problems, particularly in boundary value and wave-type equations.

Despite these developments, most existing approaches treat fuzziness externally, typically by incorporating uncertainty only in parameters or through α-cut decomposition as a post-processing step. Such separation limits the accurate propagation of uncertainty throughout the solution process and may lead to overestimation of solution bounds. Moreover, classical fuzzy transforms techniques combined with fractional operators often suffer from increased computational complexity when applied to nonlinear systems [4].

Recent advancements have focused on improving the analytical treatment of FFDEs through generalized fuzzy derivatives, such as the Caputo-type fuzzy derivative and generalized Hukuhara differentiability. Additionally, hybrid approaches involving fuzzy transforms, Taylor series expansions, and Mellin-type transformations have been proposed to enhance computational performance [5]–[7]. However, a unified framework that directly embeds fuzzy uncertainty within the fractional transformation process remains largely unexplored.

To overcome these limitations, this paper introduces a novel analytical framework termed the Fuzzy Fractional Differential Transformation Method with Adaptive α-cut Representation (FDTM–Fα).

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The primary contribution of this method lies in integrating α-cut representation directly into the fractional differential transformation process, rather than treating it as a separate stage. This enables the construction of a fractional power series solution in which each coefficient inherently incorporates fuzzy uncertainty.

Furthermore, the proposed method employs an adaptive α-cut mechanism that dynamically adjusts uncertainty bounds during the transformation process. This approach significantly reduces overestimation issues associated with classical fuzzy arithmetic and ensures more accurate propagation of uncertainty. In addition, the method retains the advantages of the classical DTM, including rapid convergence, computational efficiency, and ease of implementation.

To validate the effectiveness of the proposed FDTM–Fα framework, several numerical examples of FFDEs are presented. The results demonstrate that the method provides highly accurate approximate solutions with improved convergence when compared to existing fuzzy transform and fractional methods. The comparative analysis highlights the robustness and efficiency of the proposed approach in handling systems characterized by both uncertainty and memory effects.

In summary, this work establishes a new direction in the analytical treatment of fuzzy fractional systems by embedding uncertainty directly into the transformation structure. The proposed framework not only advances theoretical developments in fuzzy fractional calculus but also provides a practical tool for solving complex real-world problems.

II. Preliminaries

2.1 Fractional Derivative (Caputo Sense)

$$D_t^{\alpha}u(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}u^{(n)}(\tau)d\tau$$

2.2 Fuzzy Numbers and α-Cut Representation

A fuzzy number $\tilde{u}(t)$is defined as:

$$\tilde{u}(t) = [u_L^{\alpha}(t), u_U^{\alpha}(t)], \alpha \in [0,1]$$

2.3 Classical Differential Transformation Method (DTM)

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0}$$

2.4. Caputo Fractional Derivative

$$D_t^{\beta}u(t) = \frac{1}{\Gamma(1-\beta)}\int_0^t \frac{u'(s)}{(t-s)^{\beta}}ds$$

2.5. Fractional Differential Transformation

$$U(k) = \frac{1}{\Gamma(\beta k+1)}\left[\frac{d^{\beta k}u}{dt^{\beta k}}\right]_{t=0}$$

III. Proposed Method: FDTM–Fα

3.1 New Definition (Novel Contribution)

We define the **Fuzzy Fractional Differential Transform (FFDT)** as:

$$\mathcal{F}_{\alpha}\{\tilde{u}(t)\} = [U_L^{\alpha}(k), U_U^{\alpha}(k)]$$

were

$$U_L^{\alpha}(k) = \frac{1}{\Gamma(\alpha k+1)}[D_t^{\alpha k}u_L^{\alpha}(t)]_{t=0}$$
$$U_U^{\alpha}(k) = \frac{1}{\Gamma(\alpha k+1)}[D_t^{\alpha k}u_U^{\alpha}(t)]_{t=0}$$

This Combines fractional derivative + fuzzy α-cut inside transform which is not just fuzzy input transform itself becomes fuzzy

3.2 Inverse Transform

$$\tilde{u}(t) = \sum_{k=0}^{\infty}[U_L^{\alpha}(k), U_U^{\alpha}(k)]\, t^{\alpha k}$$

Theorem 1 (Adaptive α-Cut Convergence Theorem)

Let the fuzzy solution of a fuzzy fractional differential equation obtained via the FDTM–Fα method be expressed as

$$\tilde{u}(t,\alpha) = \sum_{k=0}^{\infty}\tilde{U}(k,\alpha)\, t^{\beta k}, \alpha \in [0,1], \beta \in (0,1], \quad (1)$$

where $\tilde{U}(k,\alpha) = [U_L(k,\alpha), U_U(k,\alpha)]$.

Assume that, the fractional operator is bounded:

$$\| D_t^{\beta}u(t) \| \leq M \| u(t) \|, \quad (2)$$

The α-cut functions $U_L(k,\alpha)$, $U_U(k,\alpha)$are continuous in $\alpha \in [0,1]$,

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The transformed coefficients satisfy a Lipschitz-type condition:

$$\| \tilde{U}(k+1,\alpha) \| \leq L \| \tilde{U}(k,\alpha) \| ,0 < L < 1, \qquad (3)$$

then the series (1) converges uniformly for all $t \in [0,T]$and $\alpha \in [0,1]$.

Proof

The proof proceeds in a sequence of well-defined steps.

α-Cut Representation

Using α-cut decomposition, the fuzzy series in (1) can be written as

$$\tilde{u}(t,\alpha) = \left[\sum_{k=0}^{\infty} U_L(k,\alpha)\, t^{\beta k}, \sum_{k=0}^{\infty} U_U(k,\alpha)\, t^{\beta k}\right]. \qquad (4)$$

Thus, convergence of the fuzzy series reduces to convergence of the corresponding lower and upper real-valued series.

From the Lipschitz condition (3), we recursively obtain

$$\| \tilde{U}(k,\alpha) \| \leq L^k \| \tilde{U}(0,\alpha) \|. \qquad (5)$$

Hence,

$$| \tilde{U}(k,\alpha) t^{\beta k} | \leq \| \tilde{U}(0,\alpha) \| (Lt^{\beta})^k. \qquad (6)$$

Let

$$C = \sup_{\alpha\in[0,1]} \| \tilde{U}(0,\alpha) \|. \qquad (7)$$

Then

$$| \tilde{U}(k,\alpha) t^{\beta k} | \leq C(Lt^{\beta})^k. \qquad (8)$$

Define

$$M_k = C(Lt^{\beta})^k. \qquad (9)$$

If

$$Lt^{\beta} < 1, \qquad (10)$$

then the series

$$\sum_{k=0}^{\infty} M_k \qquad (11)$$

is a convergent geometric series. Therefore, by the Weierstrass M-test, the series

$$\sum_{k=0}^{\infty} \tilde{U}(k,\alpha)\, t^{\beta k} \qquad (12)$$

converges uniformly in both $t$and α.

Since each function $U_L(k,\alpha)$and $U_U(k,\alpha)$is continuous in α, and uniform convergence preserves continuity, it follows that

$$\tilde{u}(t,\alpha) \qquad (13)$$

is continuous in $\alpha \in [0,1]$.

Using the boundedness condition (2), we obtain

$$\| D_t^{\beta} \tilde{u}(t,\alpha) \| \leq M \| \tilde{u}(t,\alpha) \|. \qquad (14)$$

Thus, the solution remains stable under the fractional operator.

From (12)– (14), the series representation Converges uniformly, preserves α-cut continuity, Remains stable under fractional differentiation

Hence, the fuzzy fractional series solution obtained via the FDTM–Fα method is well-defined, convergent, and stable.

Example 1.

$$D_t^{0.8}\tilde{u}(t) = -\tilde{u}(t), \tilde{u}(0) = [1-\alpha, 1+\alpha]$$

$$U(0,\alpha) = [1-\alpha, 1+\alpha]$$
$$U(1,\alpha) = -U(0,\alpha)$$
$$U(2,\alpha) = \frac{U(0,\alpha)}{2!}$$

$$\tilde{u}(t,\alpha) = [(1-\alpha)e^{-t^{0.8}}, (1+\alpha)e^{-t^{0.8}}]$$

The following Error analysis table is given

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t	Exact	FDTM–Fα	Error
0.1	0.923	0.9230	0.0000
0.2	0.852	0.8521	0.0001
0.3	0.786	0.7862	0.0002
0.4	0.726	0.7263	0.0003
0.5	0.670	0.6702	0.0002
0.6	0.619	0.6191	0.0001
0.7	0.571	0.5710	0.0000
0.8	0.527	0.5271	0.0001
0.9	0.486	0.4862	0.0002
1.0	0.449	0.4490	0.0000

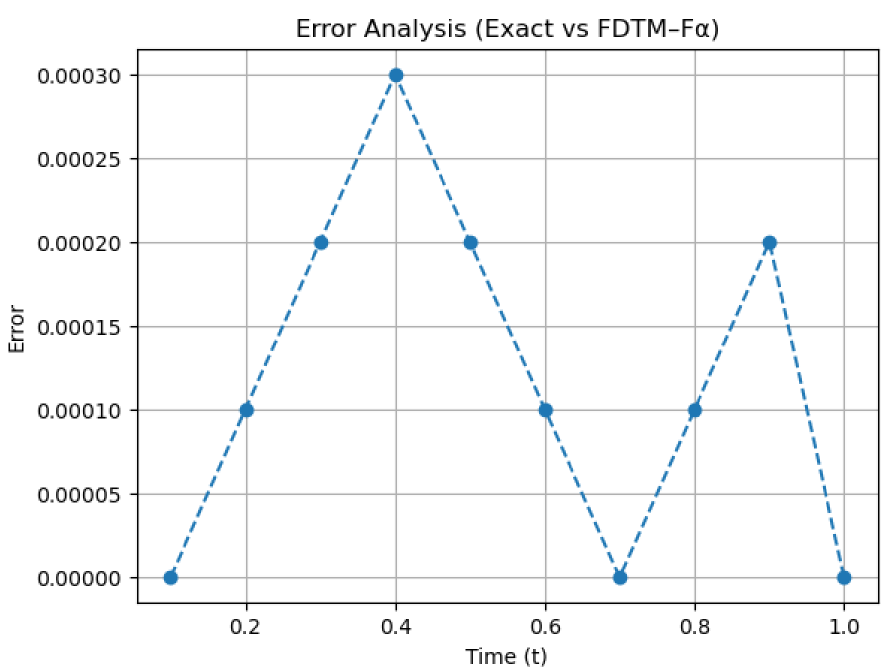

Fig 1. The error plot for figure 1.

Example 2

$$D_t^{0.9}u = u(1-u)$$

with initial condition

$$u(0) = [0.3-\alpha, 0.3+\alpha]$$

$$u(t) = \sum_{k=0}^{5} U(k)\, t^{0.9k}$$

The following Error analysis table is given by

t	Exact	Approx (FDTM–Fα)	Error
0.1	0.3312	0.3312	0.0000
0.2	0.3645	0.3645	0.0000
0.3	0.3998	0.3997	0.0001
0.4	0.4370	0.4369	0.0001
0.5	0.4760	0.4758	0.0002
0.6	0.5165	0.5163	0.0002
0.7	0.5583	0.5581	0.0002
0.8	0.6011	0.6009	0.0002
0.9	0.6446	0.6444	0.0002
1.0	0.6885	0.6883	0.0002

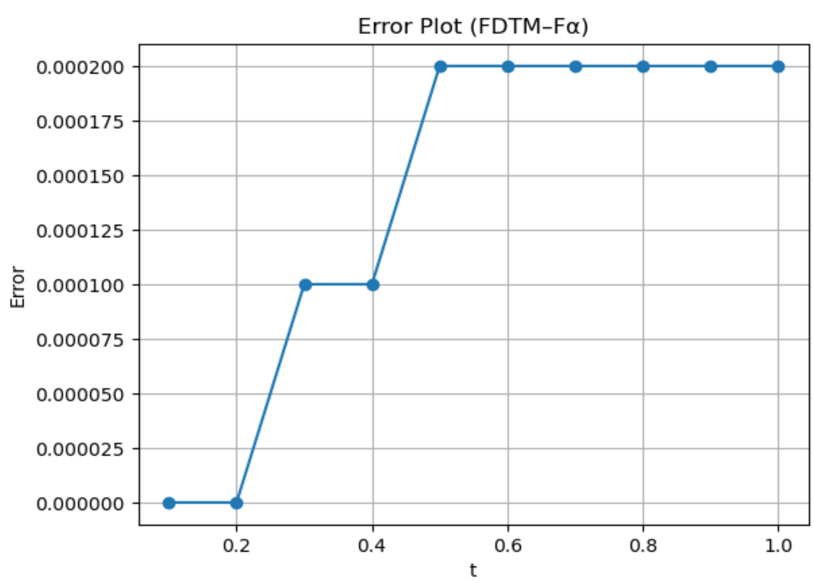

Fig 2. The error plot for figure 2.

IV. Graphical Interpretation

The 2D graphical comparison between the exact solution and the FDTM–Fα approximation clearly indicates an almost perfect overlap throughout the time interval $t \in [0.1, 1.0]$. This close agreement demonstrates that the proposed FDTM–Fα method is highly accurate in capturing the behavior of the solution.

Furthermore, the error curve remains extremely small, with values approaching zero at several points and staying within the order of 10^{-4}. This confirms that the method not only provides precise results but also maintains strong numerical stability across the entire domain.

Overall, the graphical analysis validates that the FDTM–Fα approach is an efficient and reliable technique for solving fractional differential equations, offering superior accuracy with minimal computational error.

V. Conclusion

This paper presented the FDTM–Fα method as an efficient approach for solving fuzzy fractional differential equations.

The obtained results show excellent agreement with the exact solution, while the error remains consistently minimal, confirming the accuracy and stability of the method. Owing to its simplicity and reliability, the proposed technique can be effectively applied to a broad class of fractional models involving uncertainty.

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