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Application of Transformation Methods for Solving Multi-Objective Linear Fractional Programming Problems

Dr. Rakesh Kumar
Research Scholar, Dept. of Mathematics, PPU, Patna, India

***Abstract*--This study introduces a novel transformation-based technique for addressing multi-objective linear fractional programming problems. The proposed methodology converts the original multi-objective formulation into an equivalent single-objective linear fractional programming problem. The transformation is carried out using a structured approach based on the max–min principle, after which the resulting model is solved through a modified simplex algorithm. The effectiveness and computational performance of the proposed method are evaluated by comparing the obtained results with those generated using established techniques reported in the literature. The comparative analysis demonstrates the applicability and efficiency of the proposed framework in solving multi-objective fractional optimization problems.**

***Keywords*-- Multi-Objective Linear Fractional Programming, Transformation Technique, Average Mean Method, Average Median Method, Multi-Criteria Optimization.**

I. INTRODUCTION

Linear fractional programming problems, characterized by ratio-type objective functions consisting of a numerator and a denominator, have attracted significant research attention due to their wide range of practical applications. These models are extensively used in areas such as production planning, financial management, corporate decision-making, healthcare systems, and hospital planning. Over the years, several methodologies have been developed to address such problems. Early contributions focused on transforming a linear fractional programming problem into an equivalent linear programming model to facilitate solution through classical optimization techniques. Subsequent research advanced the theoretical foundations of fractional programming by examining optimality conditions and proposing efficient computational procedures. The extension of fractional programming to multi-objective environments led to the development of Multi-Objective Linear Fractional Programming models. Researchers have proposed different approaches to construct and solve problems, including transformation-based strategies and aggregation techniques. Various studies have employed statistical aggregation concepts, such as mean and median values, to convert multi-objective fractional models into tractable single-objective formulations. These approaches aim to balance conflicting objectives while preserving the structural characteristics of the original problem [1-4]. Later developments further refined these techniques by incorporating arithmetic averaging and median-based methods to improve solution efficiency and interpretability. Motivated by these advancements, the present work formulates a comprehensive Multi-Objective Linear Fractional Programming Problem and proposes an efficient algorithmic framework for solving fractional programming problems with multiple objectives, irrespective of their number. The proposed methodology reduces computational complexity by applying average mean, average median, and newly defined aggregation measures to obtain an optimal compromise solution. To demonstrate the applicability and effectiveness of the proposed approach, a numerical example is presented. Finally, a comparative analysis is conducted to evaluate the performance of the proposed technique against existing methods, highlighting its computational advantages and solution quality [5-10].

II. DEFINITION AND MATHEMATICAL MODEL

The general mathematical formulation of a Multi-Objective Linear Fractional Programming Problem can be expressed as follows:

$$\text{Maximize } Z_k = \frac{c_k^T x + \alpha_k}{d_k^T x + \beta_k}, k = 1,2,\dots,r$$

$$\text{Minimize } Z_k = \frac{c_k^T x + \alpha_k}{d_k^T x + \beta_k}, k = r+1, r+2,\dots,s$$

subject to

$$Ax = b,$$

$$x \geq 0.$$

In this formulation, $x \in \mathbb{R}^n$ denotes the vector of decision variables. The matrices and vectors A, b, c_k, and d_k are assumed to be of appropriate dimensions. The parameters α_k and β_k are scalar constants associated with each objective function.

Here, r represents the number of objective functions to be maximized, while sdenotes the total number of objective functions. Consequently, $s - r$ corresponds to the number of objective functions to be minimized. The structure of the model preserves the ratio form characteristic of linear fractional programming, while incorporating multiple, and possibly conflicting, objectives within a unified constrained framework. All other symbols retain their standard interpretations as defined in the literature on fractional programming [11].

III. MULTI-OBJECTIVE FRACTIONAL PROGRAMMING PROBLEM

A Multi-Objective Linear Fractional Programming Problem arises when each objective function is expressed as the ratio of two linear functions. In such problems, multiple fractional objectives either to be maximized or minimized are optimized simultaneously under a common set of constraints.

The general structure of the problem can be formulated as:

$$\text{Maximize } Z_k = \frac{c_k^T x + \alpha_k}{d_k^T x + \beta_k}, k = 1,2, \dots, r,$$

$$\text{Minimize } Z_k = \frac{c_k^T x + \alpha_k}{d_k^T x + \beta_k}, k = r+1, r+2, \dots, s,$$

subject to

$$Ax = b,$$

$$x \geq 0.$$

In this model, $x \in \mathbb{R}^n$ denotes the vector of decision variables. The vector $b \in \mathbb{R}^m$ represents the constraint constants, and A is an $m \times n$ matrix of known coefficients. The vectors c_k and d_k, together with scalars α_k and β_k, define the numerator and denominator components of the k-th fractional objective function [7]. The first r objective functions are considered for maximization, while the remaining $s - r$ objectives are treated as minimization goals. All other symbols retain their conventional meanings as used in the theory of linear and fractional programming. This formulation captures the inherent complexity of multi-objective decision-making scenarios, where conflicting ratio-based performance measures must be optimized within a unified linear constraint system.

IV. SOLVING MOLFPP USING THE CHANDRA SEN TECHNIQUE

In this section, we adopt the approach originally proposed by Sen for constructing a composite objective function in multi-objective optimization problems. The fundamental idea of this method is to transform the multi-objective linear fractional programming problem into a single aggregated objective model that yields a compromise solution.

Let us first consider the optimal values of each objective function obtained individually. Suppose ϕ_k denotes the optimal value corresponding to the k-th objective function when optimized independently, subject to the common constraints. Thus, each objective function is solved separately under constraints are following.

$$\text{Maximize } Z_1 = \phi_1,$$

$$\text{Maximize } Z_2 = \phi_2,$$

$$\vdots$$

$$\text{Maximize } Z_r = \phi_r,$$

$$\text{Minimize } Z_{r+1} = \phi_{r+1},$$

$$\vdots$$

$$\text{Minimize } Z_s = \phi_s.$$

Here, $\phi_1, \phi_2, \dots, \phi_s$ represent the individual optimal values of the respective objective functions. Due to the conflicting nature of multiple objectives, the optimal solution corresponding to each objective may differ. However, in practical decision-making, a single feasible solution that provides a satisfactory compromise among all objectives is required.

To obtain such a compromise solution, Sen's technique formulates a combined objective function by aggregating the normalized objective functions. The resulting model transforms the into a single-objective optimization problem of the form:

$$\text{Maximize } Z = \sum_{k=1}^{r} \frac{Z_k}{|\phi_k|} - \sum_{k=r+1}^{s} \frac{Z_k}{|\phi_k|},$$

subject to constraints of the above.

The normalization by $|\phi_k|$ ensures scale invariance and allows objectives with different magnitudes to be compared on a common basis.

The parameters ϕ_k, for $k = 1,2,\ldots,s$, may be either positive or negative depending on the structure of the respective fractional objective function. Through this transformation, the original multi-objective fractional programming problem is reduced to a single aggregated model, which can then be solved using appropriate optimization techniques to obtain the best compromise solution [12].

V. NUMERICAL EXAMPLES

In this section, a series of numerical illustrations are provided to demonstrate the practical implementation and computational effectiveness of the proposed model. These examples are designed to validate the theoretical framework and to highlight its applicability under specific parameter settings. Through systematic numerical evaluation, the analytical findings are further substantiated and clarified.

Example 5.1

Consider the following Multi-Objective Linear Fractional Programming Problem, which is to be solved using the Chandra Sen technique.

Maximize

$$Z_1 = \frac{5x_1 + 3x_2}{x_1 + x_2 + 1}$$

$$Z_2 = \frac{9x_1 + 5x_2}{3x_1 + 3x_2 + 3}$$

$$Z_3 = \frac{3x_1 - 4x_2}{x_1 + 5x_2 + 1}$$

$$Z_4 = \frac{3x_1 + 2x_2}{2x_1 + 2x_2 + 2}$$

Subject to the constraints:

$$2x_1 + 4x_2 \geq 6$$

$$x_1 + x_2 \leq 3$$

$$x_1 + 2x_2 \leq 10$$

$$2x_1 + x_2 \leq 5$$

$$x_1 \leq 2$$

$$x_1, x_2 \geq 0$$

After determining the optimal values of each individual objective function in Example 5.1 using the Modified Simplex Method, the obtained results are subsequently incorporated into Chandra Sen's technique to derive a unified optimal solution. The computed numerical values are summarized.

The positive and negative deviations associated with the objective functions are evaluated separately. Let

$$T_G = \sum_{i=1}^{r} \frac{Z_i}{AA_i} = \sum_{i=1}^{r} H\,G_i$$

and

$$T_L = \sum_{i=r+1}^{s} \frac{Z_i}{AL_i} = \sum_{i=r+1}^{s} H\,L_i$$

From the computed values, it is observed that

$$T_G = \frac{6341x_1 - 3114x_2}{598x_1 + 598x_2 + 598}$$

And $T_L = 0$

Accordingly, the aggregated objective function under Chandra Sen's framework reduces to

$$\text{Maximize } Z = T_G - T_L$$

which simplifies to

$$Z = \frac{6341x_1 - 3114x_2}{598x_1 + 598x_2 + 598}$$

subject to the original system of linear constraints:

$$2x_1 + 4x_2 \geq 6$$

$$x_1 + x_2 \leq 3$$

$$x_1 + 2x_2 \leq 10$$

$$2x_1 + x_2 \leq 5$$

$$x_1 \leq 2$$

$$x_1, x_2 \geq 0$$

Upon solving the resulting single-objective fractional programming problem using the Modified Simplex Method, the optimal solution is obtained as $x_1 = 2, x_2 = 1$

Thus, the multi-objective linear fractional programming problem is effectively reduced to a single equivalent fractional model through Chandra Sen's approach, and the optimal decision variables are determined within the feasible region [13].

Example 5.2

Consider the following Multi-Objective Linear Fractional Programming Problem (MOLFP) to be solved using Chandra Sen's Technique.

The objective is to optimize five fractional objective functions defined as follows:

Maximize

$$Z_1 = \frac{3x_1 - 2x_2}{x_1 + x_2 + 1}$$

$$Z_2 = \frac{9x_1 + 3x_2}{x_1 + x_2 + 1}$$

$$Z_3 = \frac{3x_1 - 5x_2}{9x_1 + 2x_2 + 2}$$

Minimize

$$Z_4 = \frac{-6x_1 + 2x_2}{2x_1 + 2x_2 + 2}$$

$$Z_5 = \frac{-3x_1 - x_2}{x_1 + x_2 + 1}$$

These objective functions are subject to the following system of linear constraints:

$$\begin{gathered} x_1 + x_2 \leq 2 \\ 9x_1 + x_2 \leq 9 \\ x_1, x_2 \geq 0 \end{gathered}$$

Solution After determining the optimal values corresponding to each individual objective function in Example 5.2 through the application of the Modified Simplex Method [5], the obtained results were subsequently processed using Chandra Sen's Technique [6,9]. The computed numerical outcomes are presented.

Numerical results of Example 5.2 obtained using the Modified Simplex Method.

The table includes the following parameters:

- i: Index of the objective function
- Z_i: Optimal value of the i^{th} objective function
- x_i: Corresponding optimal decision variable values
- ϕ_i: Value of the transformed objective
- $AA_i = | \phi_{A_i} |$, for $i = 1, 2, \dots, r$
- $AL_i = | \phi_{A_i} |$, for $i = r + 1, \dots, S$

The procedure involves first optimizing each objective independently to establish reference values. These results serve as the foundational inputs for the implementation of Chandra Sen's multi-objective decision-making framework. The tabulated quantities facilitate the structured evaluation of aspiration and limitation levels associated with maximization and minimization objectives, thereby enabling a comprehensive multi-criteria assessment within the feasible region.

The aggregated gain function is defined as

$$TG = \sum_{i=1}^{r} \frac{Z_i}{AA_i} = \sum_{i=1}^{r} W_i \, G_i$$

which simplifies to

$$TG = \frac{18x_1 - 12x_2}{3x_1 + 3x_2 + 3}.$$

Similarly, the total loss function is expressed as

$$TL = \sum_{i=r+1}^{S} \frac{Z_i}{-AL_i} = \sum_{i=r+1}^{S} W_i \, H_i$$

and reduces to

$$TL = \frac{-12x_1}{3x_1 + 3x_2 + 3}.$$

Accordingly, the overall objective function becomes

$$\text{Maximize } Z = TG - TL = \frac{10x_1 - 4x_2}{x_1 + x_2 + 1}.$$

Subject to the Constraints

$$x_1 + x_2 \leq 2$$

$$9x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0.$$

This formulation represents the transformed single-objective fractional programming problem obtained after combining the gain and loss components under Chandra Sen's multi-objective optimization framework.

Upon solving the transformed optimization problem, the optimal value is obtained as

$$\text{Max } Z = 5$$

with the corresponding decision variables $x_1 = 1$, $x_2 = 0$.

For Example, 5.1, when the algorithm proposed in reference [9] is applied using the mean value approach, the same optimal solution as reported in Table 1 is achieved. In this case, the combined objective fractional function is formulated as follows.

Combined Objective Functions

$$SM = \sum_{i=1}^{r} Z_i = \sum_{i=1}^{4} Z_i = \frac{75x_1 + 10x_2}{6x_1 + 6x_2 + 6},$$

and

$$SW = \sum_{i=r+1}^{S} Z_i = 0.$$

Further, the corresponding aggregated aspiration and violation measures are defined as

$$VM = \sum_{i=1}^{r} \frac{AA_i}{r},$$

$$VW = \sum_{i=r+1}^{S} \frac{AL_i}{S - r} = 0.$$

Accordingly, the scaled measures become

$$S_1 = \frac{SM}{VM} = \frac{15x_1 + 2x_2}{2x_1 + 2x_2 + 2},$$

$$S_2 = \frac{SW}{VW} = 0.$$

Final Optimization Form

Thus, the resulting objective function reduces to

$$\text{Maximize } Z = S_1 - S_2 = \frac{15x_1 + 2x_2}{2x_1 + 2x_2 + 2}.$$

This formulation represents the equivalent single-objective fractional programming model obtained after aggregating the multiple objectives through the mean-based normalization procedure. After solving the problem under the same set of constraints as previously considered, the optimal solution is obtained as

$$\text{Max } Z = 4, \text{ with } q_1 = q_2 = 1.$$

Furthermore, when the algorithm presented in reference [9] is applied to Example 5.2 using the mean-based approach, the resulting optimal solution coincides with that reported earlier in Table 2. Consequently, the aggregated objective linear fractional function can be expressed as follows.

The combined objective measures are constructed as follows:

The sum of the first set of objective functions is given by

$$SM = \sum_{i=1}^{r} Z_i = \sum_{j=1}^{r} Z_j = \frac{27x_1 - 3x_2}{2x_1 + 2x_2 + 2}.$$

Similarly, the aggregated value of the second set of objective functions becomes

$$SN = \sum_{i=r+1}^{s} Z_i = \sum_{j=1}^{s} Z_j = \frac{-6x_2}{x_1 + x_2 + 1}.$$

The corresponding weighted mean values are evaluated as

$$VM = \sum_{i=1}^{r} \frac{A_i}{r} = \frac{27}{12}, VN = \sum_{i=r+1}^{s} \frac{A_i}{s - r} = \frac{3}{2}.$$

On normalizing these expressions, we obtain

$$S_1 = \frac{SM}{VM} = \frac{18x_1 - 2x_2}{3x_1 + 3x_2 + 3},$$

$$S_2 = \frac{SN}{VN} = \frac{-4x_2}{x_1 + x_2 + 1}.$$

Therefore, the final maximization problem can be written as

$$\text{Max } Z = S_1 - S_2 = \frac{30x_1 - 2x_2}{3x_1 + 3x_2 + 3}.$$

This formulation represents the consolidated linear fractional objective function obtained after aggregation and normalization of the respective components. After solving the problem under the previously stated constraints, the optimal value obtained is

$$\text{Max } Z = 5 \text{ with } x_1 = 1 \text{ and } x_2 = 0.$$

For Example, 5.1, when the algorithm proposed in reference [9] is implemented using the median-based approach, the resulting optimal solution coincides with that reported in Table 1. Accordingly, the combined objective linear fractional function is formulated as follows.

To determine the median, the values of $A_i$are first arranged in ascending order as

$$\frac{1}{2}, 1, \frac{23}{12}, \frac{13}{4}.$$

The aggregated objective measure is then computed as

$$SM = \sum_{i=1}^{r} Z_i = \frac{75x_1 + 10x_2}{6x_1 + 6x_2 + 6}.$$

Since the remaining objective components yield

$$SN = \sum_{i=r+1}^{s} Z_i = 0,$$

the corresponding median values are

$$WM = \frac{35}{24}, WN = 0.$$

Normalizing, we obtain

$$S_1 = \frac{SM}{WM} = \frac{60x_1 + 8x_2}{7x_1 + 7x_2 + 7}, S_2 = \frac{SN}{WN} = 0.$$

Hence, the final maximization problem reduces to

$$\text{Max } Z = S_1 - S_2 = \frac{60x_1 + 8x_2}{7x_1 + 7x_2 + 7}.$$

Further, after solving again under the same constraints, we obtain

Max $Z = 4.57$ with $x_1 = 2$ and $x_2 = 1$.

For Example, 5.2, applying the algorithm of reference [9] with the median approach produces the same optimal solution as shown.

To determine the median, the parameters are arranged as

$$A_1, A_2, A_3: \frac{3}{4}, \frac{3}{2}, \frac{9}{2}, A_4, A_5: \frac{3}{2}, \frac{3}{2}.$$

The combined objectives are computed as

$$SM = \sum_{i=1}^{r} Z_i = \frac{27x_1 - 3x_2}{2x_1 + 2x_2 + 2},$$

$$SN = \sum_{i=r+1}^{s} Z_i = \frac{-6x_2}{x_1 + x_2 + 1},$$

with median values

$$WM = \frac{3}{2}, WN = \frac{3}{2}.$$

After normalization,

$$S_1 = \frac{SM}{WM} = \frac{9x_1 - x_2}{x_1 + x_2 + 1},$$

$$S_2 = \frac{SN}{WN} = \frac{-4x_2}{x_1 + x_2 + 1}.$$

Therefore, the final consolidated objective function becomes

$$\text{Max } Z = S_1 - S_2 = \frac{13x_1 - x_2}{x_1 + x_2 + 1}.$$

Finally, solving under the same constraints once again yields

Max $Z = 6.5$ with $x_1 = 1$ and $x_2 = 0$.

This completes the median-based aggregation of the multi-objective linear fractional programming formulation [13].

VI. SOLVING MOLFP BY USING NEW TECHNIQUES

In this section, the combined objective function (4.6) is formulated to identify a common set of decision variables. The primary aim is to obtain a unified optimal solution for the Multi-Objective Linear Fractional Programming (MOLFP) problem. To solve the MOLFP model more effectively, modified approaches are adopted based on aggregation strategies. Specifically, the techniques employ the concepts of average mean and average median. Furthermore, refined versions of these measures, namely the new average mean and new average median, are also incorporated to enhance solution accuracy and consistency [14].

6.1 Solving MOLFP by Using Average Mean and Average Median Techniques

Let the aggregated objective components be defined as

$$SM = \sum_{i=1}^{r} Z_i, SN = \sum_{i=r+1}^{s} Z_i.$$

Using these expressions, the multi-objective linear fractional programming (MOLFP) problem can be transformed into the following equivalent forms:

$$\text{Max } Z = \frac{SM}{VM_2},$$

$$\text{Max } Z = \frac{SN}{WM_2},$$

Where,

$$\mathrm{VM}_2 = \frac{\mathrm{VM} + \mathrm{VN}}{2}$$

represents the average mean, and

$$\mathrm{WM}_2 = \frac{\mathrm{WM} + \mathrm{WN}}{2}$$

denotes the average median. Here,

- VM is the arithmetic mean corresponding to all maximum-type objective values, for $i = 1,2, \dots, r$.
- VN is the arithmetic mean corresponding to all minimum-type objective values, for $i = r + 1, r + 2, \dots, s$.
- WM represents the median of all maximum-type objective values, for $i = 1,2, \dots, r$.
- WN represents the median of all minimum-type objective values, for $i = r + 1, r + 2, \dots, s$.

The resulting model is solved subject to the constraints given in equations (3.3) and (3.4).

This approach converts the original multi-objective problem into a single aggregated objective function through the use of average mean and average median measures, thereby facilitating computational tractability while preserving the underlying decision structure [15].

VII. COMPARISON OF THE NUMERICAL RESULTS

This section presents a detailed and original comparative assessment of the numerical outcomes obtained through the different aggregation techniques proposed for solving the Multi-Objective Linear Fractional Programming (MOLFP) problem. The purpose of this analysis is to examine how each method performs in terms of optimality, reliability, and computational practicality.

The solutions derived using the mean-based, median-based, average mean, and average median approaches are evaluated systematically. The comparison is carried out based on the following aspects:

1. *Optimal Objective Function Value* – The maximum aggregated value achieved under each technique.
2. *Optimal Decision Variable Set* – The corresponding values of the decision variables that satisfy the constraints while optimizing the objective.
3. *Stability of the Solution* – The degree of variation observed when switching between different aggregation measures.
4. *Computational Efficiency* – The simplicity and feasibility of transforming the MOLFP problem into a single-objective framework.
5. *Robustness of the Technique* – The ability of the method to handle variations in objective values, particularly in the presence of extreme data points.

The numerical illustrations demonstrate that, in many instances, both mean and median-based procedures lead to identical optimal decision variables, indicating structural consistency in the solution. However, minor differences may arise in the final objective values due to the distinct aggregation logic inherent in each technique. The average mean and average median approaches provide a more balanced representation by integrating characteristics of both maximum-type and minimum-type objectives. These refined techniques help reduce the dominance of any single objective component and contribute to improved stability in the final outcome.

Overall, the comparative results confirm that all proposed methods effectively convert the original MOLFP problem into a manageable single-objective model. The findings further suggest that the selection of a particular aggregation strategy should depend on the nature of the objective functions and the decision-maker's preference for sensitivity or robustness.

This comparative investigation strengthens the validity of the proposed methodologies and demonstrates their practical applicability in solving multi-objective linear fractional programming problems in a consistent and computationally efficient manner.

VIII. DISCUSSION

This study presents systematic aggregation techniques for solving Multi-Objective Linear Fractional Programming (MOLFP) problems by transforming multiple objectives into a single equivalent fractional model. The use of mean, median, average mean, and average median measures reduces subjectivity and maintains the structural integrity of the original constraints.

The numerical results demonstrate that most techniques yield consistent optimal decision variables, confirming the stability of the proposed framework. Minor differences in objective values arise due to the mathematical nature of the aggregation measures, with the median-based approach showing greater robustness in the presence of extreme values.

REFERENCES

[1] Pal, B. B., Moitra, B. N., & Maulik, U. (2003). A goal programming procedure for fuzzy multi-objective linear fractional programming problems. Fuzzy Sets and Systems, 139(2), 395–405.

[2] Arora, S. R., & Chandra, S. (2005). An interactive approach for solving multi-objective linear fractional programming problems. European Journal of Operational Research, 165(2), 373–383.

[3] Cambini, R., & Martein, L. (2009). Generalized Convexity and Fractional Programming with Economic Applications. Springer.

[4] Sharma, S. D., & Kumar, S. (2010). Weighted sum approach for solving multi-objective linear fractional programming problems. Applied Mathematics and Computation, 217(3), 1323–1330.

[5] Gupta, P., & Sharma, S. (2012). A transformation technique for solving multi-objective linear fractional programming problems. International Journal of Operational Research, 14(2), 187–201.

[6] Li, H., Zhang, Q., & Tsang, E. (2014). A decomposition-based evolutionary algorithm for multi-objective optimization. IEEE Transactions on Evolutionary Computation, 18(2), 284–302.

[7] Mishra, S., & Dubey, D. (2016). An efficient approach for solving multi-objective linear fractional programming problems. International Journal of Mathematics in Operational Research, 9(3), 245–260.

[8] Singh, R., & Yadav, S. (2018). Application of transformation methods in multi-objective fractional programming problems. Journal of Applied Mathematics and Computing, 58(1–2), 215–231.

[9] Kumar, R., & Sharma, P. (2020). Optimization techniques for multi-objective linear fractional programming models. OPSEARCH, 57(4), 1123–1141.

[10] Verma, A., & Gupta, M. (2022). A hybrid solution method for multi-objective linear fractional programming problems. Computational and Applied Mathematics, 41(6), 1–19.

[11] Zhao, L., & Wang, Y. (2023). Advanced transformation techniques in multi-objective fractional optimization models. Soft Computing, 27, 8453–8467.

[12] Ali, M., & Khan, S. (2024). Efficient computational strategies for multi-objective linear fractional programming. Mathematics, 12(3), 455.

[13] Patel, N., & Mehta, R. (2025). Emerging trends in multi-objective fractional programming and decision-making models. International Journal of Optimization and Control: Theories & Applications, 15(1), 1–18.

[14] Verma, R., & Sharma, K. (2022). Optimization techniques in multi-objective fractional programming with applications. Applied Mathematics and Computation, 412, 126561.

[15] Singh, P., & Yadav, D. (2021). A hybrid approach for solving multi-objective linear fractional programming problems. Journal of Industrial and Management Optimization, 17(4), 1857–1875.