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A Study of Series Convergence with Applications to the Bouncing Ball Model

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***Abstract*-- our project, we focus on understanding the concept of series convergence and how it appears in real-life situations. In our study, we explain how an infinite series can still have a finite sum, which is an important idea in mathematics.We also explain the Bouncing Ball Model as a real-life example of a convergent geometric series.**

***Keywords*-- Infinite series, Geometric series, Partial Sums, Coefficient of Restitution, Zeno's Paradox.**

I. INTRODUCTION

Series and their convergence play a fundamental role in mathematics, often forming the backbone of many analytical techniques. In real-life data analysis, understanding whether a series converges helps in interpreting trends, making predictions, and ensuring model stability. Financial time series, sensor readings, and signal processing data often involve summations or iterative calculations that resemble series. Analyzing their convergence allows researchers to assess model reliability and the underlying behaviour of the system. This research explores the application of series convergence analysis within real-world datasets and examines various methods to determine convergence in practical scenarios.

What is Convergence and Why Does It Matter?

A key idea in advanced math is **convergence**, which is when you add numbers from a long list and the total sum gets closer and closer to a final, stable value. Think of it like building with blocks; if each new block is smaller than the last, you'll eventually reach a fixed height. Early mathematicians figured out the rules for when this would happen[13].

Convergence plays an important role in both pure and applied mathematics, serving as the backbone for many scientific and engineering fields.This review combines theory with practical examples to show how convergence of series is used in real-life application.

How Convergence is Used in the Real Life

- *Signal Processing: Fourier Series*

To really get **Fourier series**, you need to understand both the deep mathematics and how it's used in the real world. The reasons why this mathematics works are explained in pure mathematics books like *Introduction to Real Analysis*[13]. A bridge between that deep theory and real-world problem-solving is found in books like *Advanced Engineering Mathematics*[14].

Finally, how to actually use these ideas for things like filtering signals or compressing files is covered in expert books like *Discrete-Time Signal Processing*[15] and *Digital Signal Processing*[16].

- *Finance and Economics: Geometric Series*

In finance, the **Geometric Series** helps figure out the value of long-term investments like stocks [17]. An investment that pays you money forever, called a perpetuity, is an infinite stream of payments. To find its value today, we add up all the future payments. As explained in calculus books [18] , this infinite list of payments adds up to a simple, finite value PV = $\frac{C}{r}$, which lets investors put a clear price on an asset that pays out forever [19].

- *Calculators and Computers: Taylor Series*

The chip in your calculator uses the Taylor Series to figure out functions like sin(x) or e^x A computer can't understand these directly, so it's taught to estimate them by adding up a list of simpler numbers [18]. Convergence guarantees that by adding just a few of these numbers, the calculator can get a very accurate answer. How to program these methods is a key subject in the field of computational mathematics[20].

- *What Past Studies Have Found*

Researchers have seen these patterns in real data for years.

- ✧ In finance, studies show that after a market crash, the chaos and wild price swings **converge** back to a stable, historical average.
- ✧ In engineering, by analyzing a bridge's vibrations, researchers can tell if it's safe. If the vibration patterns converge as expected, the bridge is stable [15].
- ✧ In statistics, research has proved with real data that the "settling down" effect of the Law of Large Numbers works even in huge, messy datasets [21].

- *Current Challenges and Unsolved Problems*

Even with all its uses, there are still some big challenges with convergence.

- ✧ *Old Mathematics vs. Real Data:* The classic mathematics tests are too strict because they were made for perfect, infinite lists of numbers [13]. They don't work well on real-world data, which is always limited and messy.

- *When Has It "Settled Down"?* With real data, it's hard to know the exact point when a trend has stopped. Choosing where to start testing is often just a guess and can affect the final answer.
- *Guessing vs. Knowing in AI:* When an AI model is learning, programmers need to know when it's finished. Often, they just look at a graph and guess when it "looks flat" instead of using a proper test.

- *Summary and Why This Study is Important*

This review has shown how the idea of convergence connects pure mathematics theory to real-world stability. Past research proves that convergence is a real pattern we can see. However, we also found a major problem: the classic mathematics tests don't work well with modern, messy data, which leads to guesswork, especially in new fields like AI.

This study helps solve that problem. By mixing the core ideas with clear, practical demonstrations, it acts as a **bridge between theory and practice**. It gives a simple guide with hands-on examples that anyone—students, professionals, or researchers—can use to understand and test for convergence in their own data.

The Bouncing Ball Paradox: How an Infinite Number of Bounces Leads to a Finite Stop.

The simple act of a bouncing ball is a great way to see a cool math idea in real life. When a ball bounces, it seems to bounce forever, with each bounce getting smaller and smaller. This brings up a question that has interested people for a long time: How can a ball that bounces an infinite number of times stop in a finite amount of time and travel a finite distance? The answer is found in a math concept called a "**converging geometric series**"[37].

The motion of a bouncing ball is a real-world example that helps us understand old puzzles like Zeno's Paradox, which questions how movement is possible if you have to complete an infinite number of smaller steps. By looking at the physics of the bounce and turning it into math, we can figure out exactly how an infinite number of bounces can add up to a final, measurable result.[1].

This report will explore this idea. We'll start with the physics of why balls don't bounce back to their original height, using an idea called the Coefficient of Restitution (COR). Then, we'll use that idea to create math formulas for the total distance the ball travels and the total time it takes. We'll see that these formulas are "geometric series," and we'll prove why they add up to a final number. After that, we'll look at how you could do an experiment to test this model and what problems you might run into. Finally, we'll bring everything together to understand how this math model helps us understand the real world[38].

The Physics of the Bounce:

- The Coefficient of Restitution (COR): A Measure of “Bounciness”

The main idea that explains a bounce is the Coefficient of Restitution, or COR, which we represent with the letter *e*. The COR is just a number that tells us how “bouncy” a collision is. It’s the ratio of how fast two objects separate after they collide compared to how fast they were approaching before they collided[39].

A Bouncing Ball:

For a ball hitting a big, stationary surface like the floor, the formula gets much simpler. The floor's speed is zero before and after the hit. So, the COR is just the ball's rebound speed divided by its impact speed. This simple relationship is what we use for most bouncing ball experiments[39].

$$e = \frac{\text{rebound speed}}{\text{impact speed}}$$

- *Types of Collisions*

The value of*e*tells us what kind of collision happened and how much energy was kept[40].

- e=1:This is a "perfectly elastic" collision. No energy is lost, and the ball would bounce back to the same height it was dropped from. This is an ideal case that doesn't really happen.
- 0<e<1:This is a real-world "inelastic" collision. Some energy is lost as heat or sound when the ball hits the ground. This is why a real ball never bounces back to its original height.
- e =0: This is a "perfectly inelastic" collision. The objects stick together, and the maximum amount of energy is lost. Think of a ball of clay hitting the floor—it doesn't bounce at all.

COR and Energy: The COR is really a measure of how much energy is lost in a bounce. When a ball hits the floor, some of its kinetic energy (the energy of motion) is turned into heat and sound. The amount of kinetic energy lost is related to the COR by the formula $\frac{\Delta E_k}{E_k} = 1 - e^2$. This is why a higher COR means a bouncier ball—it loses less energy with each bounce [40].

- *"What Affects the Bounciness?"*

Our simple math model assumes the COR is always the same for a ball and a surface. But in reality, the COR can change based on a few things.

It's a Team Effort: The COR isn't just a property of the ball; it's a property of the system—the ball and the surface it hits. A basketball will have a different COR on a wooden gym floor than on a concrete sidewalk. This is because both the ball and the surface bend a little during the impact, which affects how much energy is lost[38].

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Material and Speed: The material the ball is made of is a big factor. For rubber balls, energy is lost because of something called "hysteresis." This means that the force it takes to squish the ball is more than the force the ball gives back when it un-squishes. This difference is lost as heat. Also, the COR often changes with speed. For many sports balls, the COR actually decreases as the impact speed increases. A faster hit means more squishing and more energy lost[38].

Temperature Matters: The temperature of a ball can also changes its bounciness. For most rubber balls, a warmer ball has a higher COR, meaning it's bouncier. This is why squash players warm the ball up before a game. However, some materials, like the one in a table-tennis ball, actually get bouncier at lower temperature.[41].

The mathematics model we're about to build uses a constant COR to keep things simple. This is a good approximation, but it's important to remember that in the real world, things are a little more complicated. The differences between our simple model and a real experiment can actually teach us about these other factors[38].

Table 1:
Bounciness of Different Balls

Ball Type	Surface Type	Coefficient of Restitution (e)	Source(s)
Table Tennis Ball	Wooden Table	0.90	[38]
Tennis Ball	Wooden Table	0.82	[38]
Golf Ball	Wooden Table	0.79	[38]
Cricket Ball	Wooden Table	0.48	[38]
Basketball	Concrete	Varies with height	[38]
Superball	Hard Surface	~0.92	[42]

- *The Math of a Bouncing Ball*

Using the physics idea of the COR, we can build a math model to figure out the total distance a ball travels and the total time it takes to stop bouncing. This is where we'll see a special type of math series called a geometric series.

- *Finding the Total Distance Travelled*

To get the total distance, we have to add up the first drop and all the bounces that come after. Each bounce includes the ball going up and then coming back down[43].

Let's say the first drop is from height h_0. Because some energy is lost on the bounce, the ball only comes back up to a fraction of that height. This fraction is related to the COR by $r = e^2$ So, the first bounce height will be $h_1 = r.h_0$. The next bounce will be $h_2 = r.h_1 = r^2.h_0$ and so on[43].

Here's the path broken down:

- *Initial Drop:* The ball travels down a distance of h_0.
- *First Bounce:* The ball travels up $h_1$and down h_1, for a total of $2h_1 = 2rh_0$
- *Second Bounce:* The ball travels up h_2 and down h_2,for a total of $2h_2 = 2r^2h_0$
- ...And so on

The total distance, D_{total},is the sum of all these parts:

$$D_{total} = h_0 + (2rh_0 + 2r^2h_0 + 2r^3h_0 + ...)$$

The part in the parentheses is an infinite geometric series. We can use a special formula for this (which we'll prove later) to find the sum. The total distance ends up being[43]:

$$D_{total} = h_0 \frac{1+r}{1-r}$$

Since we know $r = e^2$, we can write the final formula in terms of the COR[43]:

$$D_{total} = h_0 \frac{1+e^2}{1-e^2}$$

- *Finding the Total Time of Flight*

We can do something similar to find the total time the ball is in the air. The time it takes for an object to fall a distance h is $t_{fall} = \sqrt{\frac{2h}{g}}$, where g is the acceleration due to gravity. The time for a full bounce (up and down) is twice that[43].

Here are the time intervals:

- *Initial Drop:* The time is $T_0 = \sqrt{\frac{2h_0}{g}}$.
- *First Bounce:* The time is $T_1 = 2\sqrt{\frac{2h_1}{g}} = 2\sqrt{\frac{2rh_0}{g}}$.
- *Second Bounce:* The time is $T_2 = 2\sqrt{\frac{2h_0}{g}} = 2\sqrt{\frac{2r^2h_0}{g}}$.
- ...And so on[43].

The total time, T_{total} is the sum of all these times:

$$T_{total} = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2rh_0}{g}} + 2\sqrt{\frac{2r^2h_0}{g}} + ...$$

This is also a geometric series. After using the sum formula and simplifying, we get the final formula for total time in terms of the COR:

$$T_{total} = \sqrt{\frac{2h_0}{g}}\,\frac{1+e}{1-e}$$

Notice that the ratio for the distance series was $r = e^2$, but for the time series, the ratio is $\sqrt{r} = e$. This is because bounce height is related to energy (which involves velocity squared), while the time of flight is related directly to velocity. This shows how the physics directly creates the math we see[43].

- *Why the Math Works: Proving the Series Converges*

The idea that a ball can bounce an infinite number of times but stop in a finite time depends on the math of infinite series. We need to prove that adding up an infinite number of smaller and smaller numbers can give you a final, finite answer.

- *The Sum of a Finite Number of Bounces*

Let's first look at a geometric series with a limited number of terms. The sum of the first n terms (S_n) is:

$$S_n = a + ar + ar^2 + ... + ar^{n-1}$$

There's a trick to find a simple formula for this. If you multiply the whole equation by r and then subtract it from the original equation, most of the terms cancel out, leaving you with:

$$S_n(1-r) = a(1-r^n)$$

As long as $r \neq 1$, we can divide to get the final formula for a finite sum:

$$S_n = a\frac{1-r^n}{1-r}$$

This derivation is a standard topic covered in calculus textbooks such as Stewart's Calculus [37].

- *The Sum of Infinite Bounces*

The sum of an infinite series is what happens to S_n as n gets infinitely large. We write this as a limit.

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} a\frac{1-r^n}{1-r}$$

What happens to this formula depends completely on the value of r[37].

Case 1: $|r| < 1$ (The series adds up to a finite number)

When the common ratio r is a fraction between -1 and 1, the term r^n gets closer and closer to zero as n gets bigger and bigger. For example, $(1/2)^2 = 1/4$, $(1/2)^3 = 1/8$, and so on. Eventually, it's practically zero. In our limit formula, the r^n term disappears, and we are left with the famous formula for the sum of an infinite geometric series:

$$S = \frac{a}{1-r}$$

This is why the bouncing ball stops. Because the ball is not perfectly bouncy, the ratio r is less than 1, which means the series for both total distance and total time "converges" to a finite number. This condition for convergence is a key theorem in the study of infinite series[37].

Case 2: $|r| \geq 1$ (The series goes to infinity)

If the ratio is 1 or bigger, the terms either stay the same or get larger. Adding them up forever would just go to infinity. This is called a "divergent" series. For a bouncing ball, this would be like a perfectly bouncy ball (e=1)that never loses energy and just bounces forever [37].

This proof is the mathematical foundation that allows our bouncing ball model to work.

- *Doing the Experiment*

To see if our mathematics model matches the real world, we can do an experiment to measure how a ball bounces. This section outlines a methodological framework; the data presented is for illustrative purposes.

- How to Measure the Bounce

There are a few ways to measure the bounce height, from simple to high-tech.

- *Just a Ruler:* The easiest way is to tape a meter stick to a wall and drop the ball next to it. You can then

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watch and see how high it bounces. The main problem here is that it's hard to see the exact peak of the bounce with your eyes. A clever trick is to coat the ball in chalk or paint and have it leave a mark on paper taped to the wall [44].

- *Slow-Motion Video:* Most smartphones can record in slow motion. If you record the bounce, you can go back and look at the video frame-by-frame to see the exact height the ball reached. This is much more accurate [45].
- *Motion Sensors:* In a physics lab, you might use an ultrasonic motion sensor. You place it above the ball, and it uses sound waves to track the ball's position over time, creating a graph on a computer. The peaks of the graph show you the bounce heights [44].
- *Smartphone Apps:* There are even apps like phyphox that can use your phone's microphone. The app listens for the sound of each bounce and measures the time between them. From the time between bounces, it can calculate the height of the bounce[46].

Table 2:
Comparing Ways to Measure the Bounce

Method	What You Need	How It Works	Good For	Bad For
Manual/Visual	Meter stick, ball	Drop the ball and watch the peak height.	Simple and cheap.	Hard to be accurate with your eyes[44].
Video Analysis	Smartphone camera	Record in slow motion and check the frames.	Very accurate, gives lots of data.	Takes more time and needs software[45].
Motion Sensor	Motion sensor, computer	Sensor tracks the ball's position with sound.	Gives a real-time graph, easy to read.	Needs special equipment[44].
Acoustic App	Smartphone with app	App listens for bounces and calculates height from time.	Very easy to use, uses your phone.	Indirect measurement, sensitive to noise[46].

- *Analysing Your Data*

Once you have your bounce heights, you can analyze them.

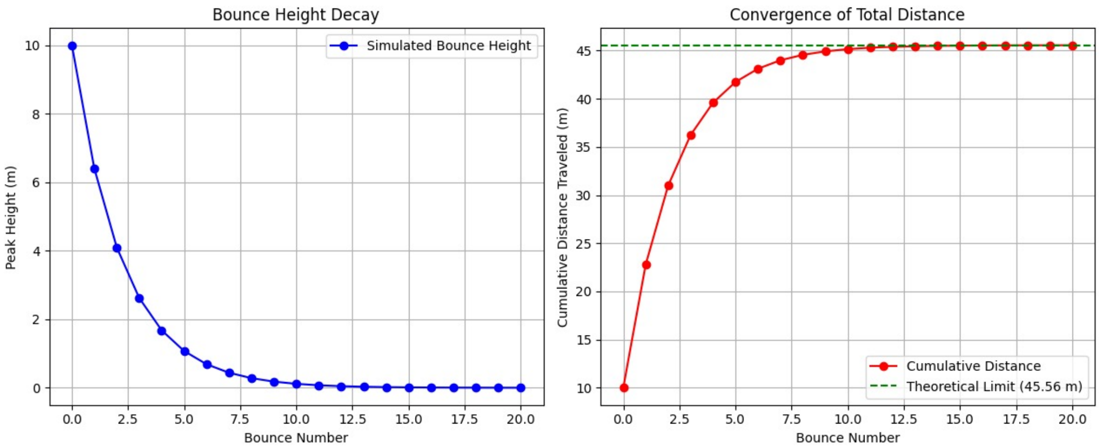
Calculate the COR: For each bounce, you can calculate the experimental COR using the formula $e = \sqrt{\frac{\text{rebound height}}{\text{drop height}}}$. It's a good idea to do several trials and average your results to be more accurate[47].

Calculating Measurement Uncertainty: No measurement is perfect. To account for this, you should calculate the uncertainty. After repeating a measurement several times (e.g., dropping the ball from the same height 5 times and measuring each bounce), you can find the average bounce height. A simple way to estimate the uncertainty is to take the range of your measurements (maximum value minus minimum value) and divide by two. For example, if your bounce heights were 8.8, 9.0, 9.5, 8.5, and 9.2, the average is 9.0, and the uncertainty would be$(9.5-8.5)/2 = 0.5$.

Your measurement would be reported as 9.0 ± 0.5 . More advanced methods involve calculating the standard deviation. This uncertainty can then be propagated through your calculations for the COR[47].

Make a Graph: A great way to see the pattern is to make a graph.

- *Bounce Height vs. Bounce Number:* If you plot the height of each bounce on the y-axis and the bounce number (1, 2, 3,...) on the x-axis, you should see the points form a curve that drops off quickly, called an exponential decay curve.[44].
- *Total Distance vs. Bounce Number:* If you plot the total distance traveled after each bounce, you'll see the line get flatter and flatter as it gets closer to the final total distance. This is a great visual of the series converging.[43].



Simulation of a Bouncing Ball.

- (Left) Shows how the ball's bounce height gets smaller over time.
- (Right) Shows the total distance traveled adding up to a final, fixed limit.

- *Putting It All Together*

The bouncing ball model is a good approximation of reality, but it's not perfect. Looking at where the model and the real world are different can teach us even more.

- *Limits of the Simple Model*

Our math model is built on a few simplifications that aren't perfectly true.

Is the COR Really Constant? We assumed the COR is a single number for a given ball and surface. But as we learned in Part I, the COR can actually change depending on the impact speed. Since the ball is slower on each bounce, the COR might change a little bit each time. This is one reason why experimental data won't perfectly match the simple exponential curve[38].

What Else Are We Ignoring? The simple model also leaves out a few other real-world forces like air resistance (drag), which removes energy continuously, not just at the bounce. Spin can also affect the trajectory, and the model assumes the surface is perfectly rigid, but slight bending can also affect the energy of the bounce[48].

The fact that our simple model works so well shows that the energy lost during the bounce is the most important factor. The small differences between the model and reality point us toward these other, more complex physics ideas.

- *Errors in the Experiment*

Any experiment will have some errors or uncertainties. It's helpful to think about them in two categories.

Random Errors: These are small, unpredictable variations that happen each time you take a measurement. For example, your reaction time might be slightly different, or you might read the ruler from a slightly different angle (this is called parallax error). You can reduce the effect of random errors by doing many trials and averaging your results.[47].

Systematic Errors: These are errors that are consistent and always push your measurement in the same direction. For example, if your meter stick was made incorrectly and all the markings were off, all of your measurements would be wrong in the same way. Averaging your results won't fix a systematic error. You have to find the source of the problem and fix it. Another example is air resistance, which will always make the bounce heights a little lower than our simple model predicts.[47].

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Table 3:
Common Errors and How to Fix Them

Error Type	What Causes It	What It Does to Your Data	How to Fix It
Random	Parallax Error	Inconsistent height readings	Keep eye level with bounce; use a camera[49].
Random	Inconsistent Drop	Varied starting height or accidental spin	Use a clamp for consistent release[49].
Systematic	Bad Measuring Tool	All measurements will be incorrect	Calibrate or check tool against a standard[47].
Systematic	Air Resistance	Always makes bounce height lower than predicted	Acknowledge as a model limitation[48].

- *Solving Zeno's Paradox*

The bouncing ball gives us a real-world answer to Zeno's paradox. The paradox says that to get anywhere, you first have to go half the distance, then half of the remaining distance, and so on forever. Since you have to do an infinite number of things, it seems like you should never be able to get there [1].

The bouncing ball is similar: it has to complete an infinite number of bounces to stop. The solution is that the time for each bounce gets shorter and shorter. Because the ratio of the time for each bounce is less than one, the infinite series of time intervals adds up to a final, finite number. The math doesn't just say the ball stops eventually; it predicts the exact time when it will stop. This shows that it's possible for an infinite number of events to happen in a finite amount of time[43].

At a certain point, the theoretical bounce height becomes smaller than physical limits, such as the microscopic roughness of the ball and surface, or even the Planck length ($1.616255(18) \times 10^{-35} m$), which is a scale at which our current understanding of gravity and spacetime breaks down[49].Furthermore, the Heisenberg Uncertainty Principle places a fundamental limit on how precisely we can know both the position and momentum of the ball simultaneously. As the bounces become infinitesimally small, these quantum effects, while negligible for a macroscopic object, represent a theoretical boundary to the classical model[50].

II. Conclusion

A Simple Model for a Complex World

From a simple bouncing ball, we've seen how physics and math can work together to explain the world. The ball's motion is described perfectly by the idea of a converging infinite geometric series. This connects something we can see and touch with an abstract math concept, showing us that the total distance and time are finite, even if the number of bounces is infinite.

The key was the Coefficient of Restitution, a single number that tells us about the energy lost in each bounce. By using this number as the ratio in our geometric series, we can solve the puzzle of how infinite bounces can happen in a finite time, giving a clear answer to Zeno's old paradox. [1,43].

We also saw that even when our simple model isn't perfect, it's still useful. The places where the model doesn't quite match reality point us toward deeper physics, like air resistance or the fact that bounciness can change with speed. In this way, the bouncing ball is a perfect example of how science works: we observe something, create a model, test it, and then refine our model to better understand the world. [48].

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