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Some Results on K-Minimally Nonouterplanar on Blict Graphs

Dr. Jayashree. B .Shetty
Associate Prof of Mathematics, Govt. First Grade College Yadgiri.585202

***Abstract*-- In this paper, we obtained the number of lines in the blict graph, when G is a path, Blict graph of a path to be (p-4)-minimally nonouterplanar, when P≥5, For any connected graph G, $B_n(G)$ is notmaximal minimally nonouterplanar and If G is a triangle together with a path of length k adjoined to some point, then $B_n(G)$ is k-minimally nonouterplanar.**

I. Introduction

In this section, the concept of blict graph B_n (G) and blitact of a graph G are given, introduced by Kulli and Biradar [2].

If B = (u_1, u_2,..u_r; r ≥ 2) is a block of G, then we say that point u_1 and block B are 'incident' with each other, as are u_2, and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point, then they are 'adjacent' blocks. The blocks, cut points and lines of a graph are called its 'members'.

The 'Blict graph' $B_n(G)$ of a graph G is the graph whose set of points is the union of the set of blocks, cut points and lines in which two points are adjacent if only if the corresponding blocks and lines of G are adjacent or the corresponding members are incident.

In Figure 1, a graph G and its blict graph B_n (G) is shown.

In 1975, Kulli[1] introduced the idea of minimally nonouterplanar graph.

The innerpoint number i(G) of a planar graph G is the minimum possible number of point not belonging to the boundary of the exterior region in any embedding of G in the plane. Obviously, G is outerplanar if and only if i(G) =0.

A graph is said to be k-minimally nonouter planar if i(G)=k, k≥1.

A 1-minimal nounouter planar graph is maximal minimally nonouter planar if no line can be added without losing minimally nonouter planarity.

The following will be useful in the proof of our results.

Remark 1.[2]

If G = K1,p, P≥ 2 then $B_n(G)$ (or $B_m(G)$ = K_{p+1}, K_{p+1}

Theorem A : [2]

A graph is planar if and only if it has no subgraph homomorphic to kor k_3.

Theorem B:[2]

The blict graph $B_n(G)$ of a graph G is planar if and only if G satisfies the following

1. G is planar
2. The degree of each point of G is at most three.
3. A cut point is not adjacent to other three cut points.
4. A cut point incident with a nonline block B is not adjacent to other two cut points either of one is not incident with B.

If a block B ha two non-adjacent cutpoints then either of one should not be adjacent to other cutpoint which is not incident with B.

Theorem C:[2]

The blict graph $B_n(G)$ of a graph G is minimally nounouter planar if and only if G satisfies following condition.

1. Deg v ≤ 3 for every point v of G and
2. G is a block with exactly two point of degree 3 and these are adjacent.
3. G is a cycle together with an end line adjoined to some point or
4. G is a path of length 4

Theorem D:[2]

Let G be a connected (p,q) graph, then $B_n(G)$=L(G) UK_1 if and only if G is a block.

Theorem E:[2]

The blitact graph $B_m(G)$ of a graph G is minimally nonouter planar if and only if G satisfies the following conditions.

1. Degv ≤ 3 for every point v of G and
2. G is a block with exactly two points of degree 3 and these are adjacent . Or
3. G is a cycle together with an end line adjoined to some point or
4. G is a path of length 4.

Theorem F: [2]

Let G be a connected (p,q) graph, then $B_m(G)$ = L(G) UK_1 if and only if G is a block.

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I. MAIN RESULTS

Some Results On Blict Graph Of A Graph

In the following theorem we obtain the result which determines the number of lines in the blict graph when G is a path.

Theorem 1.

If G is a path of length n, ($n \geq 2$)

Then $|E(B_n(G))|=6(n-1)$

Proof:

We prove the result by induction on n suppose n=2, then G = $K_{1,2,}$ by Remark 1, $B_n(G)$ (or $B_m(G)=K_3$ K_3, which has 6 liens.

Hence the result is true for n=2

Suppose the result is true for n=K-1

Assume G is a path of length K.

Let $e_j=(v_j,v)$ be an endline of G, delete from G the line e_j. Then the resulting graph G is a path of length K-1.

By inductive hypothesis.

$|E(B_n(G))|=6(K-1)$ and

Now again join the line $e_j=(v_j,v)$ to an end line $e_i=(v_i,v_j)$ of G_1 resulting the graph G. The graph $B_n(G)$ is obtained form $B_n(G_1)$ with additional point ej, b_j and v_j, where b_j is an end block incident with the cut point v_j since the line and block coincide in a path. By the definition of the blict graph, the adjacency of additional points with the points of $B_n(G)$ is such that e_j is joined to e_i and both are joined the v_j. In this process 6 new lines are added to $B_n(G_1)$.

The blitact graph $B_m(G)$ is obtained from blict graph $B_n(G)$ by joining the points which are corresponding to the adjacent cut points of G. It is known that the number of cutpoints in a path of length k is equal to (k-1).

Therefore $|E(_n(G))|6(k-2)+6=6(k-1)$

Therefore

Hence the result is true for all values of n, $n \geq 2$. This completes the proof of the theorem.

We now deduce the necessary condition for blict graph of a path to be (p-4) – minimally nonouterplanar, when $p \geq 5$.

Theorem .2

If G is a path with $p \geq 5$ points then the blict graph B_n (G) (p-4) minimally nonouterplanar.

Proof:

To prove the result we use mathematical induction on P.

Suppose p = 5. Then the graph G is a path of length four and by Theorem A. (A graph is planar if and only if it has no subgraph homomorphic to k_5 or $k_{3,3}$)

$B_n(G)$ is minimally nonouterplanar. Hence the result is true for p =5.

Suppose p = 6 then the graph G is a path of length five. In the planar embedding of $B_n(G)$, it has a cycle C as a subgraph such that two pairs of non adjacent points of C are joined by a path of length two which produces two inner points in $B_n(G)$. Thus $B_n(G)$ is 2-minimally nonouterplanar. Hence the result is true for p=6.Suppose it is true for p=k. Assume G is a path with P=k+1 points and $B_n(G)$ is m-minimally nonouterplanar. Let be an endpoint of G, delete from G the point v. The resulting graph G1 has k points. By inductive hypothesis B_n (G_1) is (k-4) minimally nonouterplanar.

Let $e_i=(v_i,v_j)$ be an endline of G_1, then bi is an endblock incident with the cut point vi, since the line and block coincide in a path.

The point e_i, b_i in $B_n(G_1)$ are on the boundary of the exterior region ona cycle C, since $B_n(G_1)$ is (k-4) minimally nonouterplanar. Now joint the point v to the point v_j of G_j resulting the graph G.

Let $e_j=(v_j, v)$ be the endline and b_j be the endblock incident with the cutpoint v_j. The graph $B_n(G)$ is obtained from $B_n(G_1)$ with additional points e_j, b_j, and v_j such e_j is joined to e_j and both are joined to v_j and similarly b_j is joined to b_i and both are joined to v_j. Since e_i, b_i are on C the points corresponding to e_i, b_i, v_i and v_j produces a subgraph homeomorphic from k4 which has an inner point.

Therefore m=(k-4)+1

m=k-3

m=(p-1)-3, since p=k+1

m=p-4

Thus $B_n(G)$ is (p-4) – minimally nonouterplanar. This completes the proof of the theorem.

Theorem .3

For any connected graph G, $B_n(G)$ is not maximal minimally nonouterplanar.

Proof:

Suppose $B_n(G)$ is maximal minimally nonouterplanar. Clearly $B_n(G)$ is minimally nonouterplanar.

Then G satisfies the conditions stated in Theorem A (A graph is planar if and only if it has no subgraph homomorphic to k_5 or $k_{3,3}$).

Suppose Δ (G) $\leq$ 3. Now we consider the following cases.

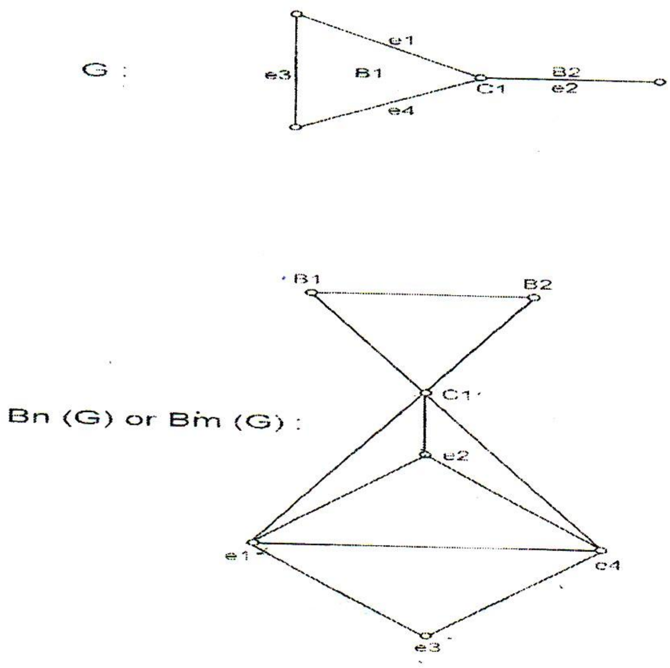

Figure 2.

Case 1:

Suppose G is a block with exactly two points of degree 3 and these are adjacent. Then by theorem B, $B_n(G)$ = L(G) UK1 and by joining the isolated point with some point which is on the exterior region of the subgraph L(G) does not alter the minimally nonouterplanarity of $B_n(G)$, a contradiction.

Case 2:

Suppose G is a cycle together with an endline adjoined to some point. Then the corresponding blict graph $B_n(G)$ has atleast two nonadjacent points (see the points e_i and B_i of Fig.2), whose join does not alter the minimally nonouterplanarity of $B_n(G)$ again a contradiction.

Thus for any connected graph G, $B_n(G)$ is not a maximal minimally nonouterplanar.

This completes the proof of the theorem.

Theorem .4

If G is a triangle together with a path of length K adjoined to some point, then $B_n(G)$ is k-minimally nonouterplanar.

Proof:

Suppose G is a triangle together with a path of length one adjoined to some point. Then by Theorem C.[2]

The blict graph $B_n(G)$ of a graph G is minimally nounouterplanar if and only if G satisfies following condition.

1 Deg v $\leq$ 3 for every point v of G and
2 G is a block with exactly two point of degree 3 and these are adjacent.
3 G is a cycle together with an end line adjoined to some point or
4 G is a path of length 4.

$B_n(G)$ is minimally nonouterplanar. When the length of the path is extended by ka (k>1) which is adjoined to some point of the triangle, then the proof of the theorem will be similar to that of Theorem 3.2.2, and hence we omit the proof.

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Theorem 5.

A graph G has a planar blict graph if and only if it has no subgraph homeomarphic to $K_{3.3}$ or $K_{1,4}$ and also to G_1 or G_2 of Fig 3 with respect to the cutpoints.

Proof:

Let G be a graph with a planar blict graph. We now show that all graphs homeomorphic to K $_{3.3}$ or K_{14} and to G1 or G2 with respect to the cut points have nonplanar blict graphs. It follows from theorem B since graphs. Homeomorphic to $K_{3,3}$ are nonplanar, graphs homemorphic to $K_{1,4}$ have a point of degree 4, graph s homemorphic to G1 with respect to the cutpoints have a cutpoint which is adjacent to other 3 ouptoints, graphs homemorphic to G2 with respect to the cutpoints have a cutpoint incident with as nonline block B and is adjacent to the other 2 cut points either of one is not incident with B.

Conversely, suppose that G contains no subgraph hemeoorphic to $K_{3.3}$ or $K_{1,4}$ and also G_1 or G_2 with respect to the cut points. Its implies that G has no subgraphs homeornorphic to $K_{3.3}$ or K_5. Then by theorem A, G is planar.

Now assume Δ (G) = 4. Then G has a point of degree 4 then it has a subgraph homeomorphic to $K_{1,4}$ a contradiction. Thus degree of each point is at most three.

Let v be the cutpoint of degree 3. We consider the following cases.

Case 1:

Suppose v lies on 3 blocks such that G is $K_{1,3}$ together with the paths of length ≥ 1 adjoined to each endpoint. Then G has a subgraph homemorphic to G, with respect to the cutpoints, a contradiction.

Case 2:

Suppose v lies on 2 blocks such that G is cycle together with a path Pm (m≥2) is adjoined at v and suppose a path pn(n≥1) is adjoined at a point u (u=v) on the cycle. Then G has a subgraph homemorphic to G_2, again a contradiction.

From the above cases, we conclude that (i) a cutpoint is not adjacent to other three cutpoints (ii) a cutpoint incident with a nonline block B is not adjacent to other 2 cutpoints either of one is not incident with B and (iii) if a block B has non adjacent cut points then either of one is not adjacent to the other cutpoint which is not incident with B. thus theorem B implies that ‘G has a planar blict graph.

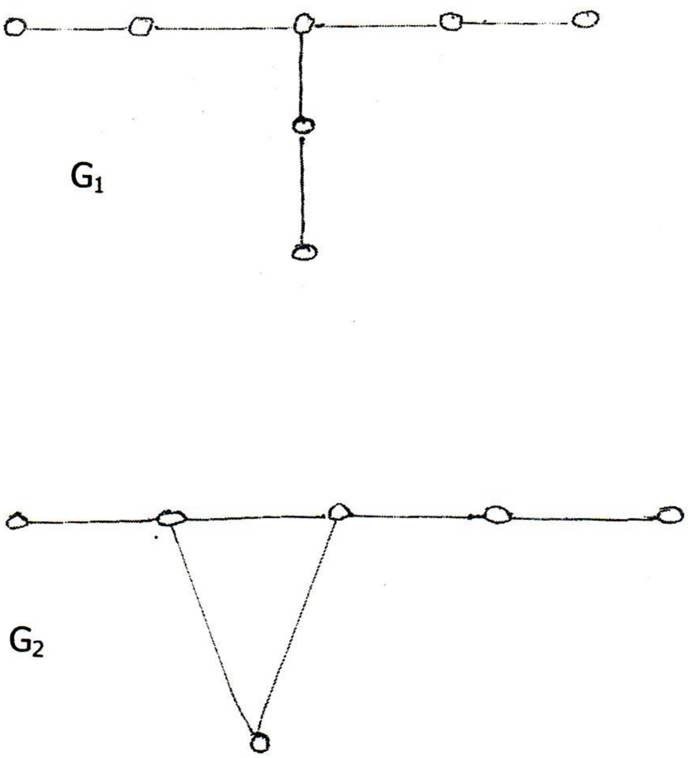

Figure 3

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