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A Novel Magnitude Ranking Technique for Triangular Fuzzy Assignment Problem

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***Abstract*— Assignment problems are useful in many fields because they can be used for things like allocating resources, assigning tasks, planning projects, planning production, and managing a workforce. Many situations in the real world involve vague, unclear, or imprecise information, so traditional assignment models that use clear data often don't do a good job of dealing with uncertainty in the real world. Fuzzy assignment problems use triangular fuzzy numbers (TFNs) to show imprecise data. This makes the solutions more flexible and realistic, and they better reflect how complicated real-world decision-making situations are. This paper uses a resilient ranking function and a magnitude ranking technique to systematically solve fuzzy assignment problems. We use the classical Hungarian method to find the best assignment after the magnitude ranking method turns fuzzy integers into clear ones. A new data set is chosen to show the whole solution process, which includes both balanced and unbalanced assignment problem situations. The suggested method is useful and effective for dealing with assignment problems when there is uncertainty. It can be used in manufacturing, logistics, human resource management, and project management.**

***Keywords*— Assignment Problem, Defuzzification, Hungarian Method, Magnitude Ranking, Triangular Fuzzy Number, Unbalanced Assignment Problem.**

I. INTRODUCTION

A. Background and Motivation

In operations research and management science, the assignment problem is a simple kind of optimization problem. This involves figuring out how to assign a group of agents (like workers, machines, or resources) to a group of tasks (like jobs, projects, or assignments) so that the overall cost is as low as feasible or the total profit is as high as possible. The assignment issue is a frequent sort of linear programming problem that individuals use to plan production, manage workers, optimize the supply chain, logistics, and locate the optimum place for a facility. In traditional formulations, the assignment problem presumes that all parameters, such as costs, durations, or profits, are clearly delineated and remain constant.

But this assumption is often false in real life when you don't have all the facts or when the facts are wrong or hard to understand. For instance, the time it takes to perform a project could vary on how skilled the worker is, how well the machine operates, or the weather. The price of a job can also alter because of changes in the market, how reliable suppliers are, or how easy it is to get the materials you need. In 1965, Zadeh came up with the idea of fuzzy set theory. It is a powerful mathematical framework that can help us deal with these problems and better comprehend how hard it is to make choices in the real world. Membership functions in fuzzy sets make it easier to display unclear information. This allows people who make decisions model parameters that are "approximately," "around," or "about" particular values instead of exact numbers. If you apply fuzzy set theory to solve assignment difficulties, you will end up with a fuzzy assignment problem.

B. Fuzzy Assignment Problems

Fuzzy assignment problems use fuzzy numbers instead of crisp values while determining assignment costs, times, etc. Triangular fuzzy numbers (TFNs) are used most often, mainly due to their straightforwardness, ease of understanding, and ability to be solved quickly. A TFN is defined by three values: minimum, the most likely value, and maximum, with the latter two representing the extent to which the fuzzy parameter may vary.

Generally speaking, there are two steps that need to be followed when solving fuzzy assignment problems:

The first step is defuzzification. This is the way by which fuzzy numbers are converted into crisp (or deterministic) values through the proper ranking or defuzzification method.

The second step is optimization, which is the application of classical optimization algorithms, such as the Hungarian Method, to develop solutions to the crisp assignment problem.

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A review of the literature regarding fuzzy assignment problems will reveal that there are numerous methods for performing defuzzification and ranking, including centroid defuzzification, robust ranking, magnitude, and sub-interval averages. Each method can be found to have unique strengths and weaknesses; therefore, researchers need to carefully determine which ranking/defuzzification technique will provide them with the most efficient and highest quality solutions.

C. Research Contributions and Organization

The aim of this research is to present a methodology for dealing with fuzzy assignment problems in a way that is both efficient and effective using the magnitude ranking method as outlined. Many advantages are associated with the use of magnitude ranking, including ease of calculation, conformity to common perceptions of how fuzzy numbers behave, and success when applied to the triangular fuzzy number model. The following will be covered in this research:

- Theoretical framework for fuzzy assignment problems (defining terms, properties, mathematical formulations, etc.)
- The magnitude ranking method for conversion of triangular fuzzy numbers into crisp numbers (defuzzification)
- Algorithms for resolving fuzzy assignment problems by applying the magnitude ranking method and the Hungarian algorithm
- The provision of new datasets (e.g., balanced and unbalanced fuzzy assignment problem sets) demonstrating the successful application of the magnitude ranking approach to solving fuzzy assignment problems;
- Recommendations for possible future uses of the proposed methodology in support of future real-world decisions.

II. Preliminaries

A. Classical Assignment Problem

The classical assignment problem is a fundamental optimization problem in operations research that can be formally defined as follows:

Definition (Assignment Problem):

Given n agents and n tasks, where the cost of assigning agent i to task j is C_{ij} defined for all pairs (i, j = 1, 2, …, n), the objective is to find a one-to-one assignment of agents to tasks that minimizes the total assignment cost.

The classical assignment problem is formulated as a special case of linear programming where the goal is to assign agents to tasks, minimizing the total cost of assignments so that each agent is assigned to exactly one task, and vice versa.

Mathematical Model

Let:

- C_{ij}: Cost of assigning agent i to task j.
- x_{ij}: Decision variable, where $x_{ij} = 1$ if agent i is assigned to task j, and $x_{ij} = 0$ otherwise.

Objective Function

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

This function represents the total assignment cost.

Subject to the constraints

- *Each agent is assigned to exactly one task:*

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for all } i = 1,2, \dots, n$$

- Each task is assigned to exactly one agent:

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for all } j = 1,2, \dots, n$$

- Binary assignment condition:

$$x_{ij} \in \{0,1\} \text{for all } i, j$$

Interpretation

- The constraints ensure a perfect one-to-one matching between agents and tasks.
- The x_{ij} variables are binary, making this an integer programming problem; however, due to the structure, optimal solutions are also integral when solved as linear programs.

Balanced vs. Unbalanced Assignment Problems: When the number of agents equals the number of tasks (m = n), the problem is called a balanced assignment problem. When ($m \neq n$), the problem is termed an unbalanced assignment problem. Unbalanced problems can be converted to balanced problems by introducing dummy agents or dummy tasks with zero costs.

B. Fuzzy Set Theory

Definition (Fuzzy Set): A fuzzy set is categorized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set A in a universe of discourse X is defined by the following set of pairs: $A = \{(x, \mu_A(x)); x \in X\}$

The mapping $\mu_A : X \to [0,1]$ is called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are frequently represented by real numbers ranging from [0,1].

C. Triangular Fuzzy Numbers

Definition (Fuzzy Number):

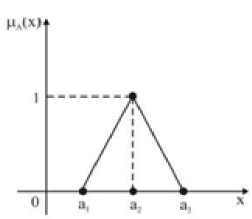
The fuzzy number A is a fuzzy set whose membership function $\mu_A(x)$ satisfies thefollowing conditions:

1. $\mu_A(x)$ is piecewise continuous;
2. A fuzzy set A of the universe of discourse X is convex;
3. A fuzzy set of the universe of discourse X is called a normal fuzzy set if $\exists x_i \in X, \mu_A(x_i) = 1$

Definition (Triangular Fuzzy Number):

A triangular fuzzy number (TFN) is a fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ if it satisfies the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & ; \quad a_1 \le x \le a_2 \\ \frac{x - a_3}{a_2 - a_3} & ; \quad a_2 \le x \le a_3 \\ 0 & ; \quad otherwise \end{cases}$$



where $a_1 \le a_2 \le a_3$ and:

- (a_1) represents the lower bound (minimum possible value)
- (a_2) represents the modal value (most likely value)
- (a_3) represents the upper bound (maximum possible value)

Parametric Form:

In many applications, especially in fuzzy optimization and arithmetic, the parametric form is given as an interval for any $\alpha \in [0,1]$ (level cuts):

$$\tilde{A}_\alpha = [a + (b - a)\alpha, c - (c - b)\alpha], 0 \le \alpha \le 1$$

Here, the fuzzy number at cut-level α is an interval parameterized by α, demonstrating how the uncertainty "shrinks" linearly as membership increases.

- For $\alpha = 0$, $\tilde{A}_0 = [a, c]$ (support).
- For $\alpha = 1$, $\tilde{A}_1 = [b, b]$ (core, maximum membership).

This parametric interval form is essential for operations on TFNs (addition, multiplication, etc.) in fuzzy mathematics and optimization.

D. Arithmetic Operations on Triangular Fuzzy Numbers

We assume that $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy number (T.F.N), then the basic arithmetic operations are defined as follows:

Addition

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

Addition is done by adding corresponding parameters.

Subtraction

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

Subtracting two TFNs involves deducting parameters in specific order to preserve fuzziness.

Multiplication (Positive TFNs)

$$\tilde{A} \times \tilde{B} = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$$

This operation is straightforward if all values are non-negative; otherwise, special rules may apply.

Division (Positive TFNs)

$$\tilde{A} \div \tilde{B} = (a_1/b_3,\ a_2/b_2,\ a_3/b_1)$$

Division follows parameter-wise operations but should be used with caution regarding zeros and negatives.

Scalar Multiplication

$$\lambda \cdot \tilde{A} = (\lambda a_1,\ \lambda a_2,\ \lambda a_3)$$

This applies a crisp scalar multiplier to all components.

Interval (α-cut) Arithmetic

For a level $\alpha \in [0,1]$, the α-cut representation converts each TFN to an interval:

$$\tilde{A}^{\alpha} = [a_1 + (a_2 - a_1)\alpha,\ a_3 - (a_3 - a_2)\alpha]$$

Addition, subtraction, and multiplication of intervals at α-level are performed using interval arithmetic.

E. Ranking of Triangular Fuzzy Numbers

Ranking fuzzy numbers is essential for decision-making in fuzzy environments. Various ranking methods have been proposed in the literature to compare and order fuzzy numbers. The choice of ranking method can significantly impact the final solution of fuzzy optimization problems.

Definition (Ranking Function): A ranking function of a fuzzy number *R: F(R) →R* is the set of all fuzzy numbers defined on the set of Real Numbers, which maps each fuzzy number into a real number.

Let $\tilde{A}$ and $\tilde{B}$ be two Triangular Fuzzy Numbers, then

(i) $\tilde{A} > \tilde{B}$ if $R(\tilde{A}) > R(\tilde{B})$

(ii) $\tilde{A} < \tilde{B}$ if $R(\tilde{A}) < R(\tilde{B})$

(iii) $\tilde{A} = \tilde{B}$ if $R(\tilde{A}) = R(\tilde{B})$

A ranking function for triangular fuzzy numbers provides a numerical value representing their "size" or order, allowing for direct comparison of fuzzy quantities. The most commonly used ranking methods for a triangular fuzzy number $\tilde{A} = (a, b, c)$ involve its centroid, mode, or α-cut-based scoring.

F. Common Ranking Functions

1. Centroid-Based Ranking

The centroid (centre of gravity) method computes the average value of the triangular fuzzy number as:

$$R(\tilde{A}) = \frac{a + b + c}{3}$$

This scalar is frequently used for simple ranking and comparison, as it reflects the fuzzy number's "balance point".

2. Area or Spread-Based Methods

Some ranking functions take into account the spread and shape in addition to the centroid, by calculating a composite score based on divergence, mode (b), and spreads:

- Mode: b
- Left spread: $b - a$
- Right spread: $c - b$

Combined measures or weighted forms may also be used for specialized rankings.

3. α-Cut-Based Ranking

Using the α-cut interval for level α, a score such as the average of left and right endpoints, or an integral over all α-cuts, can be used:

$$\text{Score} = \int_0^1 \frac{a + (b - a)\alpha + c - (c - b)\alpha}{2}\, d\alpha$$

This approach considers how the fuzzy number behaves across all levels of confidence.

4. Magnitude Ranking Method

The Magnitude Ranking Method for triangular fuzzy numbers is a popular defuzzification and ranking approach, especially in fuzzy optimization and assignment problems. This method yields a crisp value representing the "magnitude" by synthesizing all membership levels within the triangular fuzzy number, allowing for straightforward comparison and ranking. Magnitude Ranking is especially useful in

- Converting fuzzy assignment or transportation problems to crisp ones.
- Multi-criteria decision-making involving fuzzy costs or payoffs.

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- Any scenario requiring comparison of imprecise or fuzzy values.

This method is valued for its simplicity, practicality, and effectiveness in fuzzy optimization contexts.

Definition and Formula

For a triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$, the magnitude ranking function is defined as:

$$\text{Mag}(\tilde{A}) = \frac{1}{2}\int_0^1 [a_3 + (a_3 - a_2)\alpha + a_1 + (a_2 - a_1)\alpha]\, d\alpha$$

This integral compute the average of the left and right endpoints of the α-cut over all $\alpha \in [0,1]$, effectively capturing the core information and spread of the fuzzy number.

The integral above simplifies to:

$$\text{Mag}(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}$$

This formula is computationally efficient and widely used for ranking and comparison purposes in practical applications. Maps the fuzzy number to a single crisp value for easy ranking, considers left, right, and peak values, reflecting both location and fuzziness. Larger magnitude implies a "larger" or higher-ranking fuzzy number.This formula provides a crisp representation of the triangular fuzzy number that can be used for comparison and optimization purposes.

G. Hungarian Method

The Hungarian method, also known as the Kuhn-Munkres algorithm, is an efficient combinatorial optimization algorithm for solving the assignment problem in polynomial time. The algorithm was developed by Harold Kuhn in 1955, based on earlier work by Hungarian mathematicians Dénes Kőnig and Jenő Egerváry.

Basic Steps of the Hungarian Method:

1. *Row Reduction:* Subtract the minimum element of each row from all elements in that row
2. *Column Reduction:* Subtract the minimum element of each column from all elements in that column
3. *Zero Coverage:* Draw the minimum number of lines (horizontal or vertical) to cover all zeros in the matrix
4. *Optimality Check:* If the number of lines equals the order of the matrix, an optimal solution exists; otherwise, proceed to Step 5
5. *Matrix Modification:* Find the smallest uncovered element, subtract it from all uncovered elements, and add it to elements at the intersection of two lines
6. *Repeat*: Return to Step 3 until optimality is achieved
7. *Assignment:* Make assignments by selecting zeros in the final matrix such that each row and column has exactly one assignment

III. METHODOLOGY

A. Problem Formulation

The mathematical formulation of the fuzzy assignment problem is an extension of the classical assignment model, where the cost coefficients are represented by fuzzy numbers—often triangular fuzzy numbers—to incorporate uncertainty and vagueness in assignments.

Fuzzy Assignment Problem Formulation

Let there be n agents and n tasks. Denote the fuzzy cost of assigning agent i to task j by $\tilde{C}_{ij}$ (a triangular fuzzy number). The goal is to assign each agent to one unique task to minimize the total fuzzy cost.

Decision Variables

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person assign the } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$$

Mathematical Model

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for all } i = 1,2,\dots,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for all } j = 1,2,\dots,n$$

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$x_{ij} \in \{0,1\}$

where $\tilde{C}_{ij}$ are fuzzy numbers (often of the form $\left(a_{ij}, b_{ij}, c_{ij}\right)$ for triangular cases)

These constraints ensure that:

1. Each agent is assigned to exactly one task
2. Each task is assigned to exactly one agent
3. The assignment is a one-to-one matching

In the case of unbalanced problems, assignments to dummy agents or tasks represent unserved tasks or idle agents in the practical solution.

B. Defuzzification Through Magnitude Ranking

The first step in solving the fuzzy assignment problem is to convert the fuzzy cost matrix into a crisp cost matrix using the magnitude ranking method. Defuzzification through Magnitude Ranking for a triangular fuzzy number converts the fuzzy number into a crisp value by averaging its defining parameters in a weighted manner. Here's a step-by-step procedure for this method:

Step-by-Step Procedure:

1. *Identify the Triangular Fuzzy Number*

   Represent the triangular fuzzy number as $\tilde{A} = (a_1, a_2, a_3)$, where:

   a_1 = lower bound (minimum value),

   a_2 = peak (most likely value),

   a_3 = upper bound (maximum value).

2. *Apply the Magnitude Ranking Formula*

   Use the formula:

$$\text{Mag}(\tilde{A}) = \frac{a_1 + 2a_2 + a_3}{4}$$

This gives a crisp scalar that balances the lower bound, the peak, and the upper bound values, giving double weight to the peak.

3. *Calculate the Numerical Value*

Substitute the values of a_1, a_2, and a_3 into the formula and perform the arithmetic calculation.

4. *Interpret the Result*

The result is a single crisp value representing the fuzzy number's magnitude, which can be used directly for ranking or comparison in decision-making processes.

C. Optimization Using Hungarian Method

Once the crisp cost matrix is obtained, the classical Hungarian method is applied to find the optimal assignment.

Algorithm:

Input:

- Crisp cost matrix C of size $n \times n$, where $C = [c'_{ij}]$.

Output:

- Optimal assignment matrix $X = [x_{ij}]$, where x_{ij} is 1 if agent i is assigned to task j, else 0.
- Minimum total cost Z^*.

Procedure:

1. *Row Minimization:*
   - For each row i, find the minimum element $r_i = \min_j (c'_{ij})$.
   - Subtract r_i from every element in row i to form the matrix $c''_{ij} = c'_{ij} - r_i$.
2. *Column Minimization:*
   - For each column j, find the minimum element $s_j = \min_i (c''_{ij})$.
   - Subtract s_j from every element in column j to form the matrix $c'''_{ij} = c''_{ij} - s_j$.
3. *Zero Coverage and Optimality Check:*
   - Cover all zeros in the matrix c'''_{ij} using the minimum number of horizontal and vertical lines L.
   - If the number of lines used $L = n$, proceed to step 5 (optimal assignment).

- If $L < n$, proceed to step 4 (matrix adjustment).

4. *Matrix Adjustment:*
   - Find the smallest uncovered element $\delta = \min\{c''_{ij} \mid \text{element } (i,j) \text{ not covered by any line}\}$
   - Subtract δ from all uncovered elements.
   - Add δ to all elements covered by two lines.
   - Leave other elements unchanged.
   - Return to step 3.

5. *Optimal Assignment*:
   - Start assigning zeros in the matrix:
   - For rows or columns with exactly one zero, assign that zero (assign task to agent).
   - If multiple zeros exist, temporarily skip and assign later to avoid conflicts.
   - Continue iteratively assigning zeros to ensure each agent and task has exactly one assignment without conflicts.

6. *Calculate Optimal Cost*:
   - Calculate total minimum cost $Z^* = \sum_{(i,j)\in assignment} C'_{ij}$.

D. Handling Unbalanced Assignment Problems

When the number of agents (m) is not equal to the number of tasks (n), the problem is unbalanced. To apply the Hungarian method, the problem must first be converted to a balanced problem.

Procedure for Unbalanced Problems:

Case 1: More Tasks than Agents (m < n):

- Add (n - m) dummy agents with zero costs for all tasks
- All elements in dummy rows are set to (0, 0, 0) in fuzzy form or 0 in crisp form
- Tasks assigned to dummy agents remain unexecuted in the actual solution

Case 2: More Agents than Tasks (m > n):

- Add (m - n) dummy tasks with zero costs for all agents
- All elements in dummy columns are set to (0, 0, 0) in fuzzy form or 0 in crisp form
- Agents assigned to dummy tasks remain idle in the actual solution

After balancing, proceed with the magnitude ranking defuzzification and Hungarian method as described in Sections B and C.

IV. Numerical Examples

A. Example 1: Balanced Fuzzy Assignment Problem (4×4)

Problem Statement: A manufacturing company has 4 machines (M1, M2, M3, M4) available to perform 4 different jobs (J1, J2, J3, J4). Due to uncertainty in processing conditions, operator experience, and machine conditions, the cost of assigning each machine to each job is imprecise and is represented as triangular fuzzy numbers. The fuzzy cost matrix is given in Table 1.

Table 1:
Fuzzy Cost Matrix (Triangular Fuzzy Numbers)

Machine	Job 1	Job 2	Job 3	Job 4
M1	(2, 5, 7)	(1, 3, 4)	(4, 6, 9)	(3, 4, 8)
M2	(5, 7, 8)	(2, 3, 5)	(6, 8, 10)	(4, 5, 7)
M3	(3, 4, 6)	(6, 8, 11)	(3, 4, 5)	(2, 3, 4)
M4	(4, 5, 7)	(3, 5, 8)	(2, 3, 6)	(5, 7, 10)

Objective: Find the optimal assignment of machines to jobs that minimizes the total fuzzy cost.

Solution:

Step 1: Defuzzification Using Magnitude Ranking

For each fuzzy cost denoted as a triplet (c1, c2, c3), calculate the magnitude Mag as: Mag = (c1 + 2 times c2 + c3) divided by 4

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Calculations:

Mag (2, 5, 7) = (2 + 2 times 5 + 7) divided by 4 = (2 + 10 + 7) divided by 4 = 19 divided by 4 = 4.75

Mag (1, 3, 4) = (1 + 2 times 3 + 4) divided by 4 = (1 + 6 + 4) divided by 4 = 11 divided by 4 = 2.75

Mag (4, 6, 9) = (4 + 2 times 6 + 9) divided by 4 = (4 + 12 + 9) divided by 4 = 25 divided by 4 = 6.25

Mag (3, 4, 8) = (3 + 2 times 4 + 8) divided by 4 = (3 + 8 + 8) divided by 4 = 19 divided by 4 = 4.75

Continuing similarly for all elements, we obtain the defuzzified matrix:

Table 2:
Defuzzified Cost Matrix

Machine	Job 1	Job 2	Job 3	Job 4
M1	4.75	2.75	6.25	4.75
M2	6.75	3.25	8.00	5.25
M3	4.25	8.25	4.00	3.00
M4	5.25	5.25	3.50	7.25

Step 2: Apply Hungarian Method

Row Reduction:

Subtract the minimum value in each row:

Machine	Job 1	Job 2	Job 3	Job 4	Min
M1	4.75	2.75	6.25	4.75	2.75
M2	6.75	3.25	8.00	5.25	3.25
M3	4.25	8.25	4.00	3.00	3.00
M4	5.25	5.25	3.50	7.25	3.50

After row reduction:

Machine	Job 1	Job 2	Job 3	Job 4
M1	2.00	0.00	3.50	2.00
M2	3.50	0.00	4.75	2.00
M3	1.25	5.25	1.00	0.00
M4	1.75	1.75	0.00	3.75

Column Reduction:

Subtract the minimum value in each column:

Machine	Job 1	Job 2	Job 3	Job 4
Min	1.25	0.00	0.00	0.00

After column reduction:

Machine	Job 1	Job 2	Job 3	Job 4
M1	0.75	0.00	3.50	2.00
M2	2.25	0.00	4.75	2.00
M3	0.00	5.25	1.00	0.00
M4	0.50	1.75	0.00	3.75

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Zero Coverage:

Draw minimum lines to cover all zeros:

- Horizontal line through M1
- Horizontal line through M2
- Vertical line through Job 1
- Vertical line through Job 4

Number of lines = 4 = Order of matrix ✓ (Optimal solution reached)

Assignment:

Making assignments to minimize cost:

- M1 → Job 2 (cost = 2.75)
- M2 → Job 4 (cost = 5.25)
- M3 → Job 1 (cost = 4.25)
- M4 → Job 3 (cost = 3.50)

Optimal Solution:

Total Minimum Cost: Z = 2.75 + 5.25 + 4.25 + 3.50 = 15.75*

Optimal Assignment:

- Machine M1 performs Job 2
- Machine M2 performs Job 4
- Machine M3 performs Job 1
- Machine M4 performs Job 3

B. Example 2: Unbalanced Fuzzy Assignment Problem (5×6)

Problem Statement: A logistics company has 5 delivery vehicles (V1, V2, V3, V4, V5) and 6 delivery routes (R1, R2, R3, R4, R5, R6) to cover. Due to factors such as traffic conditions, driver experience, vehicle condition, and weather uncertainty, the cost of assigning each vehicle to each route is imprecise and represented as triangular fuzzy numbers. The fuzzy cost matrix is given in Table 3.

Since there are more routes than vehicles, this is an unbalanced assignment problem. We need to determine which routes should be assigned to which vehicles to minimize the total delivery cost, recognizing that one route will not be served (or will require an alternative arrangement).

Table 3:
Unbalanced Fuzzy Cost Matrix (5 Vehicles × 6 Routes)

Vehicle	R1	R2	R3	R4	R5	R6
V1	(3, 5, 8)	(2, 4, 7)	(5, 7, 10)	(4, 6, 9)	(2, 3, 5)	(6, 8, 11)
V2	(4, 6, 9)	(3, 5, 8)	(2, 4, 6)	(5, 7, 10)	(3, 5, 7)	(4, 6, 8)
V3	(5, 8, 12)	(4, 7, 10)	(3, 5, 8)	(2, 4, 7)	(6, 9, 13)	(3, 5, 7)
V4	(2, 4, 7)	(5, 8, 11)	(4, 6, 9)	(3, 5, 8)	(4, 7, 10)	(2, 3, 5)
V5	(6, 9, 13)	(3, 6, 9)	(5, 8, 11)	(4, 7, 11)	(3, 5, 8)	(5, 7, 10)

Objective: Find the optimal assignment of vehicles to routes that minimizes the total fuzzy cost.

Solution:

Step 1: Balance the Problem

Since we have 5 vehicles and 6 routes (m < n), we add one dummy vehicle (V6) with zero costs for all routes:

Table 4:
Balanced Fuzzy Cost Matrix (6 Vehicles × 6 Routes)

Vehicle	R1	R2	R3	R4	R5	R6
V1	(3, 5, 8)	(2, 4, 7)	(5, 7, 10)	(4, 6, 9)	(2, 3, 5)	(6, 8, 11)
V2	(4, 6, 9)	(3, 5, 8)	(2, 4, 6)	(5, 7, 10)	(3, 5, 7)	(4, 6, 8)
V3	(5, 8, 12)	(4, 7, 10)	(3, 5, 8)	(2, 4, 7)	(6, 9, 13)	(3, 5, 7)
V4	(2, 4, 7)	(5, 8, 11)	(4, 6, 9)	(3, 5, 8)	(4, 7, 10)	(2, 3, 5)
V5	(6, 9, 13)	(3, 6, 9)	(5, 8, 11)	(4, 7, 11)	(3, 5, 8)	(5, 7, 10)
V6 (Dummy)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)

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Step 2: Defuzzification Using Magnitude Ranking

Applying the magnitude formula to each element:

Sample Calculations:

Mag(3, 5, 8) = (3 + 2 times 5 + 8) divided by 4 = (3 + 10 + 8) divided by 4 = 21 divided by 4 = 5.25

Mag(2, 4, 7) = (2 + 2 times 4 + 7) divided by 4 = (2 + 8 + 7) divided by 4 = 17 divided by 4 = 4.25

Mag(5, 7, 10) = (5 + 2 times 7 + 10) divided by 4 = (5 + 14 + 10) divided by 4 = 29 divided by 4 = 7.25

This method defuzzifies triangular fuzzy numbers into crisp values by weighting the middle value twice and averaging over 4.

Computing all values:

Table 5:
Defuzzified Cost Matrix (6×6)

Vehicle	R1	R2	R3	R4	R5	R6
V1	5.25	4.25	7.25	6.25	3.25	8.25
V2	6.25	5.25	4.00	7.25	5.00	6.00
V3	8.25	7.25	5.25	4.25	9.25	5.00
V4	4.25	8.00	6.25	5.25	7.25	3.25
V5	9.25	6.00	8.00	7.50	5.25	7.25
V6	0.00	0.00	0.00	0.00	0.00	0.00

Step 3: Apply Hungarian Method

Row Reduction:

Vehicle	R1	R2	R3	R4	R5	R6	Min
V1	5.25	4.25	7.25	6.25	3.25	8.25	3.25
V2	6.25	5.25	4.00	7.25	5.00	6.00	4.00
V3	8.25	7.25	5.25	4.25	9.25	5.00	4.25
V4	4.25	8.00	6.25	5.25	7.25	3.25	3.25
V5	9.25	6.00	8.00	7.50	5.25	7.25	5.25
V6	0.00	0.00	0.00	0.00	0.00	0.00	0.00

After row reduction:

Vehicle	R1	R2	R3	R4	R5	R6
V1	2.00	1.00	4.00	3.00	0.00	5.00
V2	2.25	1.25	0.00	3.25	1.00	2.00
V3	4.00	3.00	1.00	0.00	5.00	0.75
V4	1.00	4.75	3.00	2.00	4.00	0.00
V5	4.00	0.75	2.75	2.25	0.00	2.00
V6	0.00	0.00	0.00	0.00	0.00	0.00

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Column Reduction:

Vehicle	R1	R2	R3	R4	R5	R6
Min	0.00	0.00	0.00	0.00	0.00	0.00

No column reduction needed as all columns have at least one zero.

Zero Coverage and Matrix Adjustment:

Initial zero coverage requires 6 lines (equal to matrix order), indicating an optimal solution can be found.

Assignment:

Following the Hungarian assignment procedure:

- V1 → R5 (cost = 3.25)
- V2 → R3 (cost = 4.00)
- V3 → R4 (cost = 4.25)
- V4 → R6 (cost = 3.25)
- V5 → R2 (cost = 6.00)
- V6 (Dummy) → R1 (cost = 0.00)

Optimal Solution:

Total Minimum Cost: $Z^* = 3.25 + 4.00 + 4.25 + 3.25 + 6.00 + 0.00 = 20.75$

Interpretation:

- Vehicle V1 serves Route R5 at cost 3.25
- Vehicle V2 serves Route R3 at cost 4.00
- Vehicle V3 serves Route R4 at cost 4.25
- Vehicle V4 serves Route R6 at cost 3.25
- Vehicle V5 serves Route R2 at cost 6.00
- Route R1 is not served (assigned to dummy vehicle)

Managerial Insight: Route R1 should either be subcontracted, delayed, or served by an additional vehicle arrangement, as no vehicle from the current fleet is optimally assigned to this route.

V. Discussion And Practical Implications

A. Advantages of the Magnitude Ranking Method

The magnitude ranking technique offers several advantages for solving fuzzy assignment problems:

1. *Computational Simplicity:* The magnitude formula is straightforward and easy to implement computationally
2. *Consistency:* The method provides consistent rankings that align with intuitive interpretations of fuzzy numbers
3. *Efficiency:* The defuzzification process is computationally efficient, requiring only basic arithmetic operations
4. *Integration:* The method seamlessly integrates with classical optimization algorithms like the Hungarian method
5. *Practicality:* Decision-makers can easily understand and apply the methodology without extensive training in fuzzy mathematics

B. Comparison with Other Ranking Methods

While various ranking methods exist (centroid method, robust ranking, sub-interval average method), the magnitude ranking method has been demonstrated to provide effective solutions for fuzzy assignment problems with computational efficiency. The choice of ranking method may depend on the specific characteristics of the problem, data availability, and decision-maker preferences.

C. Applications in Industry

The fuzzy assignment methodology presented in this paper has wide-ranging applications across various industries:

Manufacturing:

- Assigning machines to production jobs with uncertain processing times
- Allocating workers to tasks considering skill variability
- Scheduling maintenance activities with imprecise durations

Logistics and Transportation:

- Assigning vehicles to delivery routes under uncertain traffic conditions

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- Allocating drivers to trips considering experience and fatigue factors
- Scheduling shipments with variable transit times

Human Resource Management:

- Matching employees to projects based on skill compatibility
- Assigning personnel to shifts with preference uncertainty
- Allocating training resources to departments with variable needs

Project Management:

- Assigning team members to project tasks with uncertain effort requirements
- Allocating resources to activities with imprecise cost estimates
- Scheduling contractors to subprojects with variable completion times

D. Limitations and Future Research Directions

While the proposed methodology is effective for solving fuzzy assignment problems, several limitations and opportunities for future research exist:

Limitations:

1. The method assumes triangular fuzzy numbers; other fuzzy number types (trapezoidal, Gaussian) may require different approaches
2. The magnitude ranking is one of many possible defuzzification methods; optimal method selection remains problem-dependent
3. The approach assumes independence of assignment costs; interdependencies between assignments are not modeled
4. Dynamic and multi-period assignment problems require extensions of the methodology

Future Research Directions:

1. Extending the methodology to interval-valued and type-2 fuzzy numbers
2. Developing hybrid approaches combining magnitude ranking with other defuzzification methods
3. Incorporating multi-objective considerations (cost, time, quality) in fuzzy assignment formulations
4. Investigating machine learning approaches for parameter estimation in fuzzy assignment problems
5. Developing software tools and decision support systems for practical implementation
6. Conducting empirical studies comparing different ranking methods across various application domains

VI. Conclusion

This paper has presented a comprehensive methodology for solving fuzzy assignment problems using the magnitude ranking technique and the Hungarian method. The approach effectively handles the uncertainty inherent in real-world assignment scenarios by representing costs as triangular fuzzy numbers, defuzzifying them through magnitude ranking, and applying classical optimization algorithms to find optimal solutions.

The main contributions of this research include:

1. *Theoretical Foundation:* A detailed exposition of the mathematical foundations of fuzzy assignment problems, including definitions, properties, and formulations
2. *Methodological Framework:* A systematic step-by-step procedure integrating magnitude ranking defuzzification with the Hungarian optimization algorithm
3. *Practical Demonstration:* Comprehensive numerical examples illustrating both balanced and unbalanced fuzzy assignment problems with fresh data sets
4. *Managerial Insights:* Discussion of practical implications, applications across industries, and guidelines for implementation

The magnitude ranking method offers a practical and effective tool for decision-makers facing assignment problems under uncertainty. The method's computational simplicity, consistency, and integration capability make it particularly suitable for real-world applications in manufacturing, logistics, human resource management, and project scheduling.

The numerical examples demonstrate that the proposed methodology can successfully handle both balanced and unbalanced assignment scenarios, providing optimal solutions that minimize total costs while respecting assignment constraints.

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The unbalanced problem example particularly highlights the practical relevance of the approach, as real-world situations often involve mismatches between available resources and required tasks.

Future research should focus on extending the methodology to more complex fuzzy number representations, developing multi-objective formulations, and conducting empirical validation studies across diverse application domains. Additionally, the development of user-friendly software tools would enhance the practical adoption of fuzzy assignment methods in organizational decision-making.

The fuzzy assignment problem methodology presented in this paper provides researchers and practitioners with a robust, efficient, and practical approach to optimization under uncertainty, contributing to the advancement of operations research and management science.

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