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# Dynamics Bianchi Type- II Universe with Anisotropic Dark Energy in Lyra Geometry

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***Abstract-*** **In this paper, we have studied the totally anisotropic Bianchi type II universe filled with an anisotropic dark energy within the framework of Lyra geometry. The Einstein's field equations have been solved by applying hybrid expansion law for the average scale factor of the model. It is shown that the universe is early decelerating and late-time accelerating one. The universe is anisotropic throughout its evolution. We have discussed the kinematical and physical behaviors of the model. We have observed that the universe expands forever due to the dominance of dark energy.**

***Keywords*** **- Bianchi II space-time. Anisotropic dark energy. Hybrid expansion law. Early decelerating and late time accelerating universe.**

## I. INTRODUCTION

The recent observational data of high red-shift from Ia supernovac (Riess et al. [1], Perlmutter et al. [2]), comic microwave background (CMB) anisotropy (Netterfield et al. [3]), large scale structure (LSS) (Spergel et al.[4]) have indicated that the present-day universe is undergoing a phase of accelerated expansion. This late-time cosmic acceleration is assumed to be driven by a mysterious fluid, know as dark energy DE, whose origin is still a mystery is modern cosmology. It is believed that the accelerating expansion of the present-day universe in driven by the negative pressure of DE, which tend to increases the rate of expansion. In recent years several sources of DE have been proposed and extensively studied such as cosmological constant (Padmanabhan [5]) quintessence (Martin [6]), techyons (Padmanabhan and Chaudhary [7]), phantom (Alam et al. [8]), K-essence (Chibra et al. [9]), Chaplygin gas (Bento et al. [10]) etc. The DE models have significant importance now as far as theoretical study of the universe is concerned.

At present much interests have been focused on the study of cosmological models with variable equation of state (EoS) parameter $\omega(t)=\frac{p}{\rho}$, where $p$ is the pressure and $\rho$ is the energy density of the matter. The cosmological constant $\wedge$ (or vacuum density) is the most efficient and simplest candidate for explaining the observed accelerated background expansion with EoS parameter $w=-1$, but it needs to be extremely fine tunned to satisfy the current value by DE, which is a serious problem in cosmology.

According to Caldwell et al. [11] the matter with $\omega<-1$ gives rise to Big-Rip type of future singularly. Bamba et al. [12] have presented a review of different DE isotropic cosmologies with early deceleration and late-time acceleration.

The spatially homogeneous and isotropic FRW models are considered to be more suitable to study the large scale structure of the universe. However, it is believed that the early universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the universe with models having anisotropic background. Bianchi I-IV spaces play significant roles for constructing spatially homogenous and anisotropic cosmological models of the universe. Thus it would be worthwhile to explore anisotropic DE models within the framework of Bianchi space-times. Many authors have studied Bianchi type-I in the presence of an anisotropic DE. Rodrigues [13] has constructed a Bianchi type- I CDM cosmological model whose DE component preserves non-dynamical character but yields anisotropic vacuum pressure. Koivisto and Moto [14] have investigated Bianchi type- I cosmological model containing interacting DE fluid with non-dynamical anisotropic EoS and perfect fluid component and have suggested that if the EoS is anisotropic, the expansion rate of the universe becomes direction dependent at late-times and the cosmological models with anisotropic EoS can explain some of the observed anomalies in CMB. Akarsu and Kilinc [15,16] studied Bianchi type-I and III cosmological models filled with DE and perfect fluid. They considered a phenomenological parameterization of minimally interacting DE in terms of its EoS parameter and time-dependence skewness parameters. Samanta [17] has investigated Bianchi type-III cosmological models with anisotropic DE with the assumptions on the anisotropy of fluid, power-law and exponential law in Lyra geometry, Pradhan et al. [18] obtained a new class of LRS Bianchi type-II DE models with variable EoS parameter. Shri Ram et al. [19] have obtained hypersurface homogeneous cosmological models filled with an anisotropic DE in Lyra geomotry by applying a special law of variation for the mean Hubble parameter that gives a negative value of the deceleration parameter.

Singh and Sharma [20] investigated Bianchi Type-II models in the presence of an anisotropic DE in Lyra geometry using power-law form and volumetric expansion law form of the average scale factor. Recently Shri Ram et al. [21] presented a Kantorski-Sachs universe in the presence of anisotropic DE within the framework of Lyra geometer by utilizing a special form of the Hubble parameter that yields a time-varying deceleration parameter.

In this paper, we obtain a Bianchi type-II cosmological models in the presences of an anisotropic dark energy within the framework of Lyra geometry. The outline of the paper is as follows, In Sect. 2, the metric of the totally anisotropic Bianchi type-II and the field equations are described. Section 3 deals with the solution of the filed equations by utilizing the HEL for the average scale factor, which describes a unified description of early decelerating and late-time accelerating universe. We also study the kinematical and physical features of the cosmological model in Sect. 4. Finally, we summarize the conclusions in the last Sect. 5.

## II. Bianchi Type – II Metric and Field equations-

We consider the totally anisotropic Bianchi type- II space-time in the form

$$ds^2 = dt^2 - A^2(dx - zdy)^2 - B^2dy^2 - C^2dz^2 \qquad (1)$$

where A(t), B(t) and C(t) are cosmic scale functions.

The energy-momentum tensor $T_\mu^\upsilon$ of an anisotropic fluid can be written in the diagonal form as

$$T_\mu^\upsilon = diag[T_1^1, T_2^2, T_3^3, T_4^4] = diag[-p_x, -p_y, -p_z, \rho] \qquad (2)$$

Where $\rho$ is the energy density of the fluid; $p_x$, $p_y$ and $p_z$ are pressures on $x$, $y$ and $z$-axes respectively. The parameterization of deviation from isotropy by introducing skewness parameter $\delta$ i.e. is the deviation from $\omega$ on $x-$axis only, the energy-momentum tensor can be written as

$$T_\mu^\upsilon = diag\ [-(w+\delta), -\omega, -\omega, 1]\rho \qquad (3)$$

Sen [22], Sen and Dunn [23] proposed a scalar-Tensor theory of gravitation and constructed analogue of Einstein is field equation based on Lyra gromentry.

The Einstein's field equation are given by

$$R_{\upsilon\mu} - \frac{1}{2}Rg_{\upsilon\mu} + \frac{3}{2}(\phi_\mu\phi_\nu - \frac{1}{2}g_{\mu\nu}\phi_\alpha\phi^\alpha) = -T_{\mu\nu} \qquad (4)$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the energy-momentum tensor the anisotropic fluid and $\phi_\mu$ is the time –like displacement field vector given as $\phi_\mu = (0,0,0,\psi(t))$, $\psi(t)$ being the time dependent gauge function.

In comoving coordinate system, the field equations for the totally anisotropic Bianchi type-II space-time yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{3}{4}\frac{A^2}{B^2C^2} + \frac{3}{4}\psi^2 = -(\omega+\delta)\rho \qquad (5)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{1}{4}\frac{A^2}{B^2C^2} + \frac{3}{4}\psi^2 = -\omega\rho \qquad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{A^2}{B^2C^2} + \frac{3}{4}\psi^2 = -\omega\rho \qquad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{1}{4}\frac{A^2}{B^2C^2} + \frac{3}{4}\psi^2 = \rho \qquad (8)$$

Using Bianchi identities to (4) and assuming that the matter field is conserved separately, we obtain

$$\dot{\psi} + \psi\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \qquad (9)$$

Now dot denotes derivative with respect to lime t.

Now we define some parameters for the Bianchi type-II model (1) which are important tools in cosmological observations. The average scale factor and spatial volume are defined as

$$a^3 = ABC, \quad V = a^3. \qquad (10)$$

The physical parameters like expansion scalar $\theta$, shear scalar $\sigma^2$ are defined as follows:

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \qquad (11)$$

$$\sigma^2 = \frac{1}{2}\sum_{\mu=1}^{3}(H_\mu{}^2 - 3H^2) \qquad (12)$$

where H is the mean Hubble parameter and $H_\mu(\mu = 1,2,3)$ represent the directional Hubble parameters in the direction of $x$, $y$ and z axes respectively givens as

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$$H = \frac{1}{3}(H_1 + H_2 + H_3), \qquad (13)$$

$$H_1 = \frac{\dot{A}}{A},\; H_2 = \frac{\dot{B}}{B},\; H_3 = \frac{\dot{C}}{C}. \qquad (14)$$

The anisotropy Parameter $A_m$ of the expansion is given as

$$A_m = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2. \qquad (15)$$

For isotropic behavior of cosmological model $A_m = 0$. An important observational quantity is the deceleration parameter q defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \qquad (16)$$

The sign of q indicates whether the model inflates or not. The positive value of q corresponds to standard decelerating model whereas the negative sign indicates inflation.

## III. Solutions Of The Field Equations

In this section, we obtain the exact solutions of the field equation (5)-(9) for the scale factors A, B, C, and the physical parameter $\rho, \omega, \delta$ and $\beta$. Subtracting (6) from equation (7), we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \qquad (17)$$

Equation (17) on integration, provides

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_1}{a^3} \qquad (18)$$

where $k_1$ is an integration constant.

In order to obtain a consistent solution of the field equation, we assume that

$$A^m = BC \qquad (19)$$

where m is a positive constant. We further assume that

$$B = A^{\frac{m}{2}} D, \qquad C = A^{\frac{m}{2}} D^{-1} \qquad (20)$$

Substituting for B and C in equation (18), we get

$$\frac{\dot{D}}{D} = \frac{K}{a^3} \qquad (21)$$

where K is an arbitrary constant. From (10) and (19), we get

$$A = V^{\frac{1}{m+1}} = a^{\frac{3}{m+1}} \qquad (22)$$

We can determine the scale factors A,B and C if the average scale factor a is known function of time. Singh and Sharma [20] have presented the solutions of field equations (5)-(9) by using the power-law and exponential law forms of the average scale factor a(t). Here we obtain exact solutions of the field equations by utilizing the hybrid expansion law (HEL) for the average scale factor of the form

$$a(t) = k\; t^{\alpha} e^{\beta t} \qquad (23)$$

where $k > 0$, $\alpha \geq 0$ and $\beta \geq 0$ are constant. Akarsu et al. [24] proposed this generalised form the average scale factor which is a combination of power-law and exponential-law cosmologies in a unified way. This law leads to the power-law cosmology for $\beta = 0$ and the exponential-law cosmology for $\alpha = 0$. Kumar [23] has studied the dynamics of Bianchi type-V model by considering HEL for the average scale factor. Using this law, Shri Ram and Chandel [26] discussed a magnetized string cosmological model in f(R,T) gravity theory. Chandel and Shri Ram [27] investigated a Bianchi type – V early decelerating and late-time accelerating cosmological model with perfect fluid and heat conduction using HEL.

From (22) and (23), we get the solution for the scale factor A as

$$A = \left(t^{\alpha} e^{\beta t}\right)^{\frac{3}{m+1}} . \qquad (24)$$

Substituting (23) in equation (21) and integrating, we obtain

$$D = K_1 \exp\left\{-\frac{K}{3}(3\beta)^{3\alpha-1}\overline{(1-3\alpha, 3\beta t)}\right\} \qquad (25)$$

where $\Gamma$ denotes the lower incomplete gamma function and $K_1$ is a constant of integration. Without loss of generality, we take $K_1 = 1$ substituting (25) in equation (20), we obtain the expressions of the scale factors B and C as

$$B = (t^{\alpha} e^{\beta t})^{\frac{3m}{2(m+1)}} \exp\left\{-\frac{k}{3}(3\beta)^{3\alpha-1}\overline{(1-3\alpha, 3\beta t)}\right\}, \qquad (26)$$

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$$C=(t^{\alpha}e^{\beta t})^{\frac{3m}{2(m+1)}}\exp\left\{\frac{k}{3}(3\beta)^{3\alpha-1}\overline{(1-3\alpha,3\beta t)}\right\}. \quad (27)$$

For the average scale factors A, B and C to be realistic, we must have $\alpha\le\frac{1}{3}$. Using (24), (26) and (27) in equation (9) and integrating, we obtain the gauge function $\beta$ as

$$\psi=\frac{\psi_o}{t^{3\alpha}e^{3\beta t}} \quad (28)$$

where $\psi_o$ is an arbitrary constant.

The directional Hubble parameters and the average Hubble parameter are obtained as

$$H_1=\frac{3}{m+1}\ \left(\frac{\alpha}{t}+\beta\right), \quad (29)$$

$$H_2=\frac{3m}{2(m+1)}\left(\frac{\alpha}{t}+\beta\right)+\frac{K}{t^{3\alpha}e^{3\beta t}}, \quad (30)$$

$$H_3=\frac{3m}{2(m+1)}\left(\frac{\alpha}{t}+\beta\right)-\frac{K}{t^{3\alpha}e^{3\beta t}}, \quad (31)$$

$$H=\frac{\alpha}{t}+\beta. \quad (32)$$

The expansion scalar ($\theta$), shear scalar ($\sigma$) and the anisotropy parameter $(A_m)$ are obtained as

$$\theta=3\left(\frac{\alpha}{t}+\beta\right),$$

$$\sigma^2=\frac{3(m-2)^2}{2(m+1)^2}\left(\frac{\alpha}{t}+\beta\right)^2+\frac{2K^2}{t^{6\alpha}e^{6\beta t}}, \quad (33)$$

$$A_m=\frac{3}{2}\frac{(m-2)^2}{(m+1)^2}+\frac{2K^2}{\left(\frac{\alpha}{t}+\beta\right)^2 t^{6\alpha}e^{6\beta t}}. \quad (34)$$

The deceleration parameter (q) has the value given as

$$q=-1+\frac{\alpha}{(\alpha+\beta t)^2} \quad (35)$$

For the present model, the energy density ($\rho$), deviation free EoS parameter ($\omega$) and the skewness parameter $\delta$ are obtained as

$$\rho=\frac{9m(m+4)}{(m+1)^2}\left(\frac{\alpha}{t}+\beta\right)^2-\frac{1}{4}\frac{1}{t^{6\alpha\frac{(m-1)}{m+1}}e^{6\beta\frac{(m-1)t}{(m+1)}}}+\frac{(3\psi_o^{\,2}-4K^2)}{4t^{6\alpha}e^{6\beta t}}, \quad (36)$$

$$\omega=-\frac{1}{\rho}\left[\frac{9(m^2+2m+4)}{4(m+1)^2}\left(\frac{\alpha}{t}+\beta\right)^2-\frac{3\alpha(m+2)}{2(m+1)t^2}+\frac{3\psi_o^2+4K^2}{4t^{6\alpha}e^{6\beta t}}+\frac{1}{4t^{\frac{6\alpha(m-1)}{m+1}}e^{\frac{6\beta(m-1)t}{m+1}}}\right], \quad (37)$$

$$\delta=\frac{1}{\rho}\left[\frac{-9(m-2)}{2(m+1)}\left(\frac{\alpha}{t}+\beta\right)^2+\frac{3\alpha(m-2)}{2(m+1)t^2}+\frac{1}{t^{\frac{6\alpha(m-1)t}{m+1}}e^{\frac{6\beta(m-1)t}{m+1}}}\right]. \quad (38)$$

## IV. Some Physical Features of the Model

We observe that the spatial volume of the model is zero at the time $t=0$. At this epoch the energy density, expansion scalar and shear scalar are infinite. Therefore, the present model has a big-bang singularity at $t=0$. As time increases, the energy density decreases and ultimately attains a constant value as $t\to\infty$. The expansion scalar and shear scalar are decreasing functions of time which assume constant value for late times.

Equation (34) shows that the anisotropy parameter is infinite at $t=0$ and assumes a constant value $\frac{3}{2}\left(\frac{m-2}{m+1}\right)^2$ as $t\to\infty$. This means that the anisotropy in the model is maintained throughout its evolution.

From (37) we find that the time-dependent EoS parameter is infinite at $t=0$ and a decreasing function of t which assumes a constant value

$$\omega=-1+\frac{2(m-2)}{m(m+4)} \quad (39)$$

as $t\to\infty$. It is worthwhile to note that $\omega$ tends to -1 as $t\to\infty$ for $m=2$. From this we can infer that the universe for $m=2$ is late-time accelerating due to the dominance of cosmological constant as source of DE. The skewness parameter, being infinite at $t=0$, tends to a constant for late-time.

Equation (35) gives the variation of deceleration parameter with time. We observe that the universe evolves with variable deceleration parameter and the transition from deceleration to acceleration takes place at time

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$$t = \frac{\sqrt{\alpha} - \alpha}{\beta} \qquad (40)$$

Which restrict $\alpha$ in the range $0 < \alpha < 1$. As $t \to \infty$, q ~ -1 which shows the inflationary behavior of the universe. For sufficiently large times, we fluid that $H \sim \beta$ which indicates that the universe expands forever with the dominance of DE.

The gauge function $\psi(t)$, being infinite initially, dies out as $t \to \infty$. The gauge function contributes significantly to energy density and the EoS parameter. The concept of Lyra manifold is meaningful only for finite time, but dose not remain for very large time.

The present model is new and different than the models obtained by Sing and Sharma [20].

## V. Conclusion

A new anisotropy Bianchi type-II with variable EoS parameters has been investigated within the frame work of Lyra geometry. The anisotropy DE model is based on exact solutions of the Einstein's filed equations for the totally anisotropy Bianchi type-II space-times filled with prefect fluid with variable EoS parameter $\omega$. The exact solutions of the Einstein's field equations have been obtained by utilizing the HEL for the average scale factor of the model which correspond to an early decelerating and late-time accelerating universe. The universes has a signature flep at a finite time.

The anisotropy parameter $A_m$, which is infinite at the initial singularity, tends to a constant as $t \to \infty$. Therefore the model dose not approach isotropy large-time. In the derived model, $\omega$ is obtained as constant as time-varying which ultimately tends to a constant as time tends to infinity.

The skewness parameter $\delta$ tends to a constant for large-time. Therefore the anisotropy in the pressures on coordinate axes persists for large time.

For sufficiently large time $H \sim \beta$ and $q \sim -1$, which indicate that the universe expands forever due to the dominance of DE.

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