

Analytical Study about Heat and Mass Transfer of Oscillatory Flow of a Fluid

Md Taizuddin Choudhury¹, Zakir Hussain Ahmad², U.K. Srivastava³

¹Department of Mathematics, Rokakhata Model H.S. School, Hakama, Bilasipara, Dhubri, Assam, India.

²Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University, Riyadh, Kingdom of Saudi Arabia.

³Department of Mathematics, R.S.S. College, Chochahan, Muzaffarpur, B.R.A. Bihar University, Muzaffarpur, Bihar, India.

Abstract-- The study of buoyancy effect on fluctuation on three dimensional unsteady flow with heat and mass transfer has been the object of fur flung research due to its possible applications in many modern branches of Science and Technology. The oscillatory free convective flows play an important role in Chemical engineering, turbo machinery and Aerospace technology. In industrial applications many transports exit where the transfer of heat and mass takes place simultaneously because of combined buoyancy effects thermal diffusion and diffusion chemical species. The study of this kind of flows was initiated by Lighthill (1954). Stuart (1955) further extended it to study a two-dimensional oscillatory flow past an infinite plate with fixed suction. Soundalgekar (1979) studies the flow past an infinite normal plate oscillating in its own place and with wall temperature. Messiha (1966) investigated the two-dimensional oscillatory flow when the plate is bounded to a time-dependent suction. Soundalgekar et al (1977a) have also discussed the free convective unsteady flow with mass transfer. Further Vignesam and Soundalgekar investigated the free and forced convective flow with variable temperature.

Keywords--MHD, Visco-elastic, Incompressible fluid, Porous Medium. Unsteady flow.

Continuity Equation:

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (1)$$

Momentum Equations:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} = g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty) + \nu \left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} \right) \quad (2)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{v}}{\partial \bar{z}} = \frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{y}} + \nu \left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} \right) \quad (3)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} = \frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{z}} + \nu \left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right) \quad (4)$$

Energy Equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{z}} = \alpha \left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right) \quad (5)$$

I. INTRODUCTION

Chauhan, D.S. (01) and N. Veerraju (02) are the pioneer workers of the present area. In fact, the present work is an extension of the work done by Siddiqua (03) et al, Prasad, J. S. R. (04) et al, Kumar, J. P. (05) et al, Mamatha, B (06) et al and Adeniyi, A (07). In this paper we have studied analytically about heat and mass transfer of oscillatory flow of a fluids.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Considering the heat and mass transfer flow of a viscous incompressible fluid past an infinite normal porous plate with transverse periodic suction oscillating with time and uniform free stream velocity. The plate is lying vertically on the $\bar{x} - \bar{z}$ plane with \bar{x} - axis taken along the plate in the upward direction. The \bar{y} -axis is taken to the normal to the plane of the plate and directed into the fluid following laminarly with a uniform free stream velocity U . All the fluid properties are assumed constant except that the influence of the density variation with temperature is considered only in the body force term. The prescribed governing equations are given by:

Species Concentration Equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} + \bar{w} \frac{\partial \bar{C}}{\partial \bar{z}} = D \left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} \right) \quad (6)$$

Also, the suction velocity oscillates with time is assumed to be of the form:

$$\bar{V}_w(\bar{z}, \bar{t}) = -v_0 \left[1 + \cos \left(\frac{\pi \bar{z}}{L} - \bar{\omega} \bar{t} \right) \right] \quad (7)$$

Where L and $\varepsilon (<1)$ are the respective wavelength and amplitude of the suction variation defined by (7).

The relevant boundary conditions of the problem are:

$$\begin{aligned} \bar{y} = 0 : \bar{u} = 0, \bar{v} = \bar{V}_w(\bar{z}, \bar{t}), \bar{w} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w \\ \bar{y} \rightarrow \infty : \bar{u} = U, \bar{v} = -\bar{v}_0, \bar{w} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty, \bar{P} = \bar{P}_w \end{aligned} \quad (8)$$

The non-dimensional quantities are introduced as follows:

$$y = \frac{\bar{y}}{L}, z = \frac{\bar{z}}{L}, u = \frac{\bar{u}}{L}, v = \frac{\bar{v}}{v_0}, w = \frac{\bar{w}}{v_0}, t = \bar{\omega} \bar{t}, \omega = \bar{\omega} \frac{L^2}{v}, S = v/D$$

In view of the above dimensionless quantities, equations (1) to (6) become:

$$\text{From 1} \Rightarrow \frac{v_0}{L} \frac{\partial v}{\partial y} + \frac{v_0}{L} \frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (9)$$

$$\begin{aligned} \text{From 2} \Rightarrow \frac{uv_0}{L^2} \omega \frac{\partial u}{\partial t} + \frac{v_0^2}{L} v \frac{\partial u}{\partial y} + \frac{v_0^2}{L} w \frac{\partial u}{\partial z} &= g\beta(\bar{T}_w - \bar{T}_\infty)\theta + g\beta(\bar{C}_w - \bar{C}_\infty)C + \frac{uv_0}{L^2} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \\ \Rightarrow \frac{v}{v_0 L} \omega \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \frac{Lg\beta(\bar{T}_w - \bar{T}_\infty)}{v_0^2} \theta + \frac{Lg\beta(\bar{C}_w - \bar{C}_\infty)}{v_0^2} C + \frac{v}{v_0 L} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \Rightarrow \frac{\omega}{Re} \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= Gr\theta + GmC + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \end{aligned} \quad (10)$$

$$\begin{aligned} \text{From 3} \Rightarrow \frac{uv_0}{L^2} \omega \frac{\partial v}{\partial t} + \frac{v_0^2}{L} v \frac{\partial v}{\partial y} + \frac{v_0^2}{L} w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \rho \left(\frac{v}{L} \right)^2 \frac{\partial p}{\partial y} \frac{1}{L} + \frac{uv_0}{L^2} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \\ \Rightarrow \frac{v}{v_0 L} \omega \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\left(\frac{v}{v_0 L} \right)^2 \frac{\partial p}{\partial y} + \frac{v}{v_0 L} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \\ \Rightarrow \frac{\omega}{Re} \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \frac{1}{Re^2} \frac{\partial p}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \end{aligned} \quad (11)$$

$$\begin{aligned} \text{From 4} \Rightarrow \frac{uv_0}{L^2} \omega \frac{\partial w}{\partial t} + \frac{v_0^2}{L} v \frac{\partial w}{\partial y} + \frac{v_0^2}{L} w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \rho \left(\frac{v}{L} \right)^2 \frac{\partial p}{\partial y} \frac{1}{L} + \frac{uv_0}{L^2} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \\ \Rightarrow \frac{v}{v_0 L} \omega \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\left(\frac{v}{v_0 L} \right)^2 \frac{\partial p}{\partial y} + \frac{v}{v_0 L} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \\ \Rightarrow \frac{\omega}{Re} \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= \frac{1}{Re^2} \frac{\partial p}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{aligned} \quad (12)$$

$$\begin{aligned}
 \text{From 5} \Rightarrow \frac{v_0}{L} \left(\frac{v}{L} \omega \frac{\partial}{\partial t} + v \frac{\partial}{\partial y} + v_0 w \frac{\partial}{\partial z} \right) [(\bar{T}_w - \bar{T}_\infty)\theta + \bar{T}_\infty] \\
 = \frac{\alpha}{L^2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) [(\bar{T}_w - \bar{T}_\infty)\theta + \bar{T}_\infty] \\
 \Rightarrow \frac{v}{v_0 L} \omega \frac{\partial \omega}{\partial t} + \frac{v_0(\bar{T}_w - \bar{T}_\infty)}{L} v \frac{\partial \theta}{\partial y} + \frac{v_0(\bar{T}_w - \bar{T}_\infty)}{L} w \frac{\partial \theta}{\partial z} = \frac{\alpha(\bar{T}_w - \bar{T}_\infty)}{L^2} + \left[\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right], \\
 \Rightarrow \frac{\omega}{Re} \frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{RePr} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right), \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 \text{From 6} \Rightarrow \frac{v_0}{L} \left(\frac{v}{L} w \frac{\partial}{\partial t} + v \frac{\partial}{\partial y} + v_0 w \frac{\partial}{\partial z} \right) [(\bar{C}_w - \bar{C}_\infty)C + \bar{C}_\infty] \\
 = \frac{\alpha}{L^2} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) [(\bar{C}_w - \bar{C}_\infty)C + \bar{C}_\infty], \\
 \Rightarrow \frac{v}{v_0 L} \omega \frac{\partial C}{\partial t} + \frac{v_0(\bar{C}_w - \bar{C}_\infty)}{L} v \frac{\partial C}{\partial y} + \frac{v_0(\bar{C}_w - \bar{C}_\infty)}{L} w \frac{\partial C}{\partial z} = \frac{\alpha(\bar{C}_w - \bar{C}_\infty)}{L^2} + \left[\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right], \\
 \Rightarrow \frac{\omega}{Re} \frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = \frac{1}{ReS} \left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right), \quad (14)
 \end{aligned}$$

To solve these differential equations, the boundary conditions are assumed as:

$$\begin{aligned}
 y = 0: u = 0, v(z) = -\{1 + \varepsilon \cos(\pi z - t)\}, w = 0, \theta = 1, C = 1 \\
 y \rightarrow \infty: u \rightarrow 1, v \rightarrow -1, w \rightarrow 0, \theta \rightarrow 0, p \rightarrow p_\infty \quad (15)
 \end{aligned}$$

III. HEAT TRANSFER

Since at the boundary the heat exchange between the fluid and the body is only due to conduction, according to Fourier's Law, we have

$$\bar{q}_w = -K \left(\frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=0} \quad (16)$$

Where \bar{y} is the direction normal to the surface of the body. With the help of (19), the coefficient of heat transfer can be calculated in non-dimensional form which is generally known as Nusselt number as follows:

$$\begin{aligned}
 Nu = \frac{\bar{q}_w}{\rho v_0 C_p (\bar{T}_w - \bar{T}_\infty)} &= \frac{K(\bar{T}_w - \bar{T}_\infty)}{\rho v_0 C_p (\bar{T}_w - \bar{T}_\infty)} \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \frac{1}{L} \\
 &= \frac{v}{v_0 L} \frac{K}{\rho v C_p} \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \frac{v}{v_0 L} \frac{\alpha}{v} \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \\
 &= - \frac{1}{RePr} \left(\text{Real part of } \frac{\partial \theta}{\partial y} \right)_{y=0}, \alpha = \frac{K}{\rho C_p} \quad (17)
 \end{aligned}$$

In terms of the amplitude and phase, the Nusselt number can be written as

$$Nu = 1 + \varepsilon |H| \cos(\pi z - t + \Phi_2) \quad (18)$$

Where $|H| = \sqrt{H_r^2 + H_i^2}$, $\tan \Phi_2 = H_i/H_r$,

$$H_r = \left[\frac{\pi}{(C_4^2 + D_4^2)\{(C_1 - \pi)^2 + D_1^2\}} \{ (C_1 - \pi)(D_2 D_4 - C_2 C_4 + C_4(C_1 + \text{RePr}) - D_1 D_4) + D_1(C_2 D_4 - D_2 C_4 + D_4(C_1 + \text{RePr}) + D_1 C_4 + P_1 C_1 - \pi \pi \text{Re} + Q_1 D_1 \pi \text{Re} + P_2 C_1 - \pi \omega - Q_2 D_1 \omega_1 C_1 - \pi + D_1 2 \pi^2 \text{Re}^2 + \omega^2 \text{Pr}) \right]$$

$$H_i = \left[\frac{\pi}{(C_4^2 + D_4^2)\{(C_1 - \pi)^2 + D_1^2\}} \{ -D_1 \{ D_2 D_4 - C_2 C_4 + C_4(C_1 + \text{RePr}) - D_1 D_4 \} + (C_1 - \pi) \{ C_2 D_4 - D_2 C_4 + D_4(C_1 + \text{RePr}) + D_1 C_4 \} - \{ P_1 D_1 \pi \text{Re} - D_1 \omega P_2 + (C_1 - \pi) \text{RePr} Q_1 - (C_1 - \pi) \omega Q_2 \} \frac{1}{\{(C_1 - \pi)^2 + D_1^2\}(\pi^2 \text{Re}^2 + \omega^2) \text{Pr}} \right]$$

IV. MASS TRANSFER

The relation between species transfer by convection and the concentration boundary layer may be demonstrated by recognizing that the molar flux associated with species transfer by diffusion, according to Fick's law, it has the form

$$\bar{q}_m = -D \left(\frac{\partial \bar{C}}{\partial y} \right)_{y=0} \quad (19)$$

With the help of (42), the coefficient of mass transfer can be calculated in dimensionless form in terms of Sherwood number as follows:

$$Sh = \frac{\bar{q}_m}{v_0(\bar{C}_w - \bar{C}_\infty)} = \frac{D(\bar{C}_w - \bar{C}_\infty)}{v_0 L(\bar{C}_w - \bar{C}_\infty)} \left(\frac{\partial \bar{C}}{\partial y} \right)_{y=0}$$

$$= \frac{v}{v_0 L} \frac{D}{v} \left(\frac{\partial \bar{C}}{\partial y} \right)_{y=0} = - \frac{1}{\text{ReS}} \left(\text{Real part of } \frac{\partial \bar{C}}{\partial y} \right)_{y=0} \quad (20)$$

In terms of the amplitude and phase, the Sherwood number can be written as

$$Sh = 1 + \varepsilon |M| \cos(\pi z - t + \Phi_3) \quad (21)$$

Where $|M| = \sqrt{M_r^2 + M_i^2}$, $\tan \Phi_3 = M_i / M_r$,

$$M_r = \left[\frac{\pi}{(C_5^2 + D_5^2)\{(C_1 - \pi)^2 + D_1^2\}} \{ (C_1 - \pi)(D_3 D_5 - C_3 C_5 + C_5(C_1 + \text{ReS}) - D_1 D_5) + D_1(C_3 D_5 - D_3 C_5 + D_5(C_1 + \text{ReS}) + D_1 C_5 + P_1 C_1 - \pi \pi \text{Re} + Q_1 D_1 \pi \text{Re} + P_2 C_1 - \pi \omega - Q_2 D_1 \omega_1 C_1 - \pi + D_1 2 \pi^2 \text{Re}^2 + \omega^2 S) \right]$$

$$H_i = \left[\frac{\pi}{(C_5^2 + D_5^2)\{(C_1 - \pi)^2 + D_1^2\}} \{ -D_1 \{ D_3 D_5 - C_3 C_5 + C_5(C_1 + \text{ReS}) - D_1 D_5 \} + (C_1 - \pi) \{ C_3 D_5 - D_3 C_5 + D_5(C_1 + \text{ReS}) + D_1 C_5 \} - \{ P_1 D_1 \pi \text{Re} - D_1 \omega P_2 + (C_1 - \pi) \text{ReS} Q_1 - (C_1 - \pi) \omega Q_2 \} \frac{1}{\{(C_1 - \pi)^2 + D_1^2\}(\pi^2 \text{Re}^2 + \omega^2) S} \right]$$

$$P_1 = C_1 C_2 + D_1 D_2 - C_1(\pi + \text{RePr}), Q_1 = C_1 D_2 - C_2 D_1 + D_1(\pi + \text{RePr})$$

$$P_2 = C_1 D_2 - D_1 C_2 - D_1(\pi + \text{RePr}), Q_1 = D_1 D_2 - C_1 C_1 - C_1(\pi + \text{RePr})$$

V. RESULTS AND DISCUSSION

To get ample geo- physical insight into the problem, the values of Prandtl number (Pr) are chosen for Mercury (Pr=0.025), air at 20°C (Pr=0.71), water (Pr=7.0) and water at 4°C (Pr=11.4). Also, the values of Schmidt number (S) are taken for Hydrogen (S=0.22), Helium (S=0.30), Water vapour (S=0.60), Oxygen (S=0.66) and Ammonia (S=0.78). There are two cases of general interest for Grashoff number $Gr < 0$ due to freezing of the plate and Grashoff number $Gr > 0$ due to heating of plate are being considered.

The phase, $\tan\Phi_1$, in the main flow direction against Reynolds, Prandtl, Schmidt, Grashoff number for mass transfer and frequency of the fluctuation has been plotted graphically in Fig. 1 due to freezing of the place. An increase in Re leads to a decrease in $\tan\Phi_1$ and it is observed that, the frequency has no effect on $\tan\Phi_1$ in case of large Reynolds number. Increasing Gm, Pr and S result in increase of the phase $\tan\Phi_1$, however, in case of Ammonia $\tan\Phi_1$ fluctuates more in comparison to air at 20°C. The variation of $\tan\Phi_1$ against Re, Pr, S, gm and ω shown in Fig. 2 in case of heating of the plate. The tangent of the phase angle $\tan\Phi_1$ reduces in magnitude for thicker diffusing species and substantial decrease is observed near the plate, whereas by increasing influence of Gm, Re and Pr, the phase $\tan\Phi_1$ increasing substantially. Moreover, there is a phase lead $\Phi_1 \rightarrow 45^\circ$ as $\omega_1 \rightarrow 0^+$.

Due to freezing of the plate Fig. 3 shows the variation of the amplitude $|F|$ of the main flow skin-friction against Re, Pr, S, Gm and ω . Reduction in amplitude $|F|$ is observed near the plate with increasing Re, Pr, S and Gm, but there is a reversed behaviour of $|F|$ occur for large frequency of fluctuation (i.e. $\omega > 2.0$). It is interesting to note that, all curves of $|F|$ are intersect at $\omega = 2.0$.

In Fig. 4, we have studied the effect of Re, Pr, S, GM and ω on the amplitude $|F|$ for externally heated plate. From this figure it is seen that $|F|$ increase with Prandtl number and this behaviour of $|F|$ is opposite to the influence of Reynolds, Schmidt and Grashoff number for mass transfer. Again, substantial decrease occurs in amplitude $|F|$, when $\omega \rightarrow 0^+$.

Fig. 5, depicts the effect of Reynolds, frequency and Prandtl number on amplitude $|H|$ of the rate of heat transfer. This figure clearly shows that the amplitude $|H|$ increases considerably with the increase of Re, Pr and ω . It is remarkable that, the curves of $|H|$ decreases sharply near the plate, and then very sharply increases away the plate. Moreover, the amplitude $|H|$ is more in water at 4°C (Pr=11.4) than in air at 20°C (Pr=0.71).

To study the effects of Re, Pr and ω on the tangent of the phase angle, $\tan\Phi_2$, of the rate of heat transfer has been shown in Fig. 6. It is seen that the phase $\tan\Phi_2$ reduces in magnitude and extent with increase of Re, Pr and ω . Also, substantial increase in $\tan\Phi_2$ is marked for $\omega \rightarrow 0^+$ and when $\omega \rightarrow +\infty$, the phase $\tan\Phi_2$ more fluctuates for air at 20°C and water.

The amplitude $|M|$ of the rate of mass transfer under the influence of Re, S and ω is presented graphically in Fig. 7. It is inferred from this figure that, $|M|$ increases slowly and steadily for water vapour and Ammonia, however, $|M|$ increases for Reynolds number when $\omega \rightarrow +\infty$. It is also observed that, the amplitude $|M| \rightarrow 1$ as $\omega \rightarrow 0^+$.

Fig. 8 demonstrate the variation of the tangent of the phase angle, $\tan\Phi_3$, against Re, S and ω . By the increasing of Reynolds number, the phase $\tan\Phi_3$ decreases for small values of frequency and this effect of Reynolds number on $\tan\Phi_3$ is reversed for large values of frequency. Also, the phase $\tan\Phi_3$, decreases in magnitude for thicker diffusing foreign species and substantial decrease is observed for large values of ω . The phase $\tan\Phi_3$ is more in Ammonia than in water vapour.

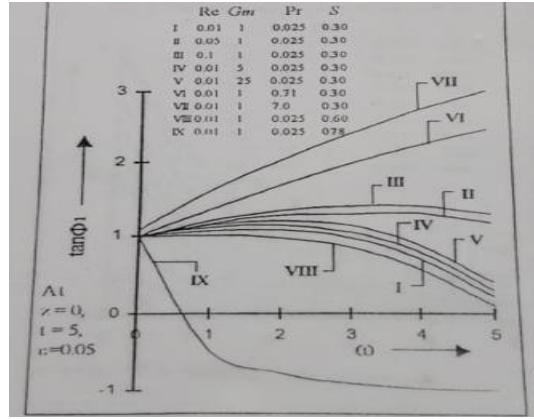
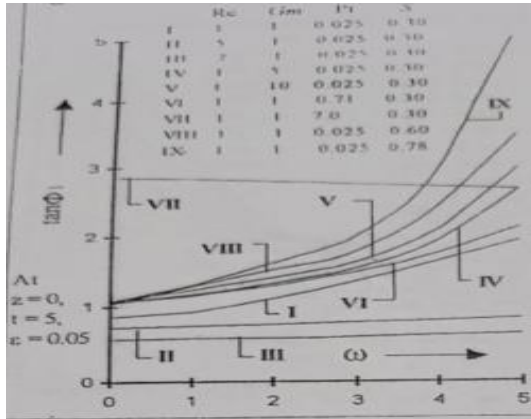


Figure 1: The phase $\tan \Phi_1$ of τ_x Figure 2: The phase $\tan \Phi_1$ of τ_x

when $Gr = 10$

when $Gr = -10$

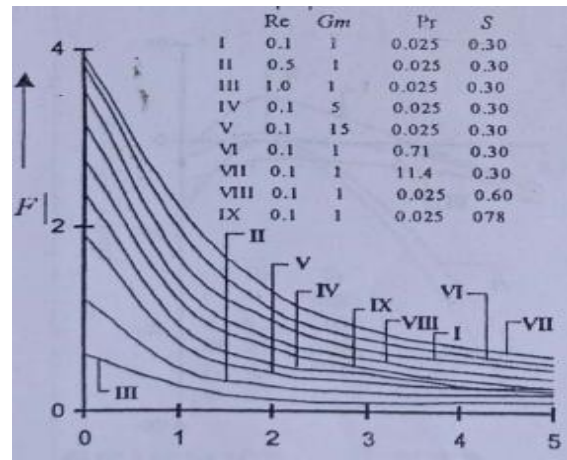
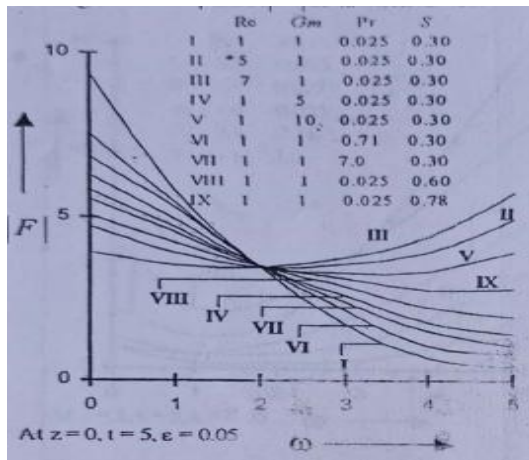


Figure 3: The Amplitude $|F|$ of τ_x Figure 4: The Amplitude $|F|$ of τ_x

when $Gr = 10$

when $Gr = -10$

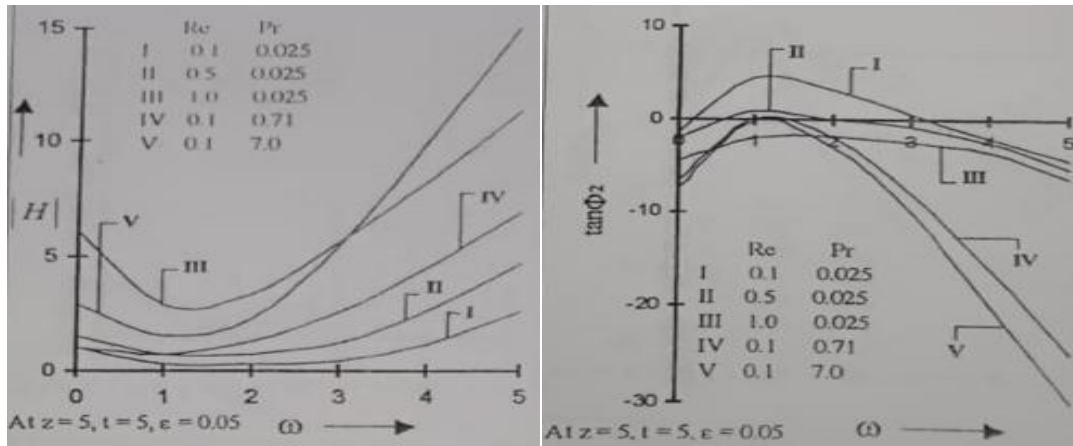


Figure 5: The Amplitude $|H|$ of Nu Figure 6: The phase $\tan\Phi_2$ of Nu

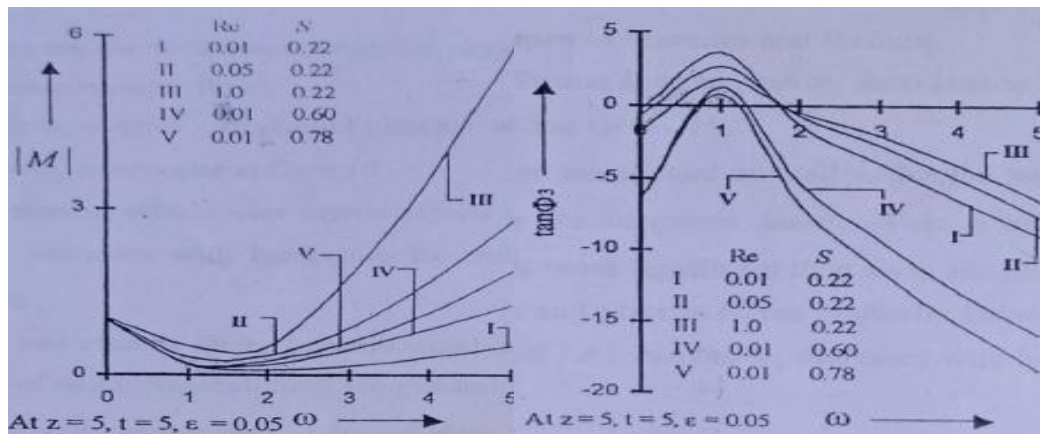


Figure 7: The Amplitude $|M|$ of C Figure 8: The phase $\tan\Phi_3$ of C

VI. CONCLUSIONS

The above study brings out the following results of geophysical interest on the free convective heat and mass transfer flow: When Re is too small, $\tan\Phi_1$ at $Gr = -10$ should increase with an increase in ω and this behaviour of $\tan\Phi_1$ is opposite at $Gr = 10$. When the dominance of viscous effects over inertia effects is very large, it is seen that $|H|$ and $|M|$ increases Re and ω ; while this behaviour is opposite to $\tan\Phi_2$. When the inertial forces and viscous forces become equal in magnitudes, it seems that $|M|$ and $\tan\Phi_3$ of molar concentration are greatest in the presence of Hydrogen; but in the presence of Mercury, $|H|$ of heat transfer is also greatest. When $Re = 1$, the phase $\tan\Phi_1$ of main flow skin-friction increases with increasing GM and ω in presence of Mercury and Helium.

In presence of heavier diffusing species, $\tan\Phi_1$ increases at $Gr = 10$, while this effect of $\tan\Phi_1$ is reversed at $Gr = -10$. Near the plate, Re reduces $\tan\Phi_3$ and as well as for the heavier diffusing species. But away the plate Re increase $\tan\Phi_3$. This is because that the viscous force near the plate is more significant than away the plate. Also, as the inertial forces and viscous forces gradually become comparable in magnitudes, we may say that $|F|$ and $\tan\Phi_2$ decreases with increasing ω .

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