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Numerical Determination and Analysis of Magnetohydrodynamic Mixed Convection in A Rectangular Enclosure with Heating and Cooling.

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***Abstract*--This study investigates the numerical determination and analysis of three dimensional laminar steady flows for magnetohydrodynamic (MHD) mixed convection within a rectangular enclosure subjected to combined heating and cooling effects. The enclosure is heated partially from the bottom surface while the side walls are subjected to cooling through convectional openings, with additional airflow induced by a fan located at the center of the domain. The working fluid considered is electrically conducting, and its thermo-physical properties are assumed to remain constant throughout the analysis. A uniform magnetic field is applied along the x-axis direction in order to study the influence of electromagnetic forces on the velocity and temperature fields within the enclosure. The physical problem is formulated mathematically using governing equations for conservation of mass, momentum, and energy, together with appropriate boundary conditions. These equations are transformed into their dimensionless form and discretized using the Finite Difference Method (FDM). Numerical simulations are carried out in MATLAB to obtain velocity profiles, temperature distributions, and the effects of key non dimensional parameters such as Reynolds number, Richardson number, Grashoff number, Hartmann number and Eckert number. The computational results are presented in both tabular and graphical form to illustrate the dependence of flow and heat transfer characteristics on these controlling parameters. The findings indicate that variations in magnetic field strength, buoyancy forces, and inertial effects significantly modify both velocity and temperature fields inside the enclosure. Enhanced Hartmann number is observed to suppress fluid motion, while increased Reynolds number promotes stronger convective transport. Temperature distribution is strongly influenced by heat absorption and viscous dissipation effects represented by Eckert number. The outcomes of this work are expected to provide useful insights for the design and optimization of MHD-based microchannel cooling systems, solar pond technology, and other thermal management applications where combined convection and electromagnetic interactions are important.**

***Keywords*--Magnetohydrodynamics-MHD Richardson number Hartmann number Reynolds number Eckert number Heat absorption coefficient Magnetic field**

***Abbreviations*--MHD - Magnetohydrodynamic, FDM - Finite Difference Method, MATLAB–Matrix laboratory, PDE-Partial deifferential equation, ODE-Ordinary differential Equation**

I. INTRODUCTION

Heat transfer is the exchange of thermal energy between physical systems. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the intervening medium through which the heat is transferred. Three fundamental modes of heat transfer are conduction, radiation and convection. Conduction is the transfer of heat through an intervening matter without bulk motion of the matter. Radiation is the transmission of heat energy through space without the necessary presence of matter. Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. It can be classified as natural (free) or forced convection depending on how the fluid motion is initiated. Irrespective of the classification, the fluid motion through convection enhances heat transfer with the higher the velocity the faster the heat transfer**.** The high velocity fluid results in a decreased thermal resistance across the boundary layer from the fluid to the heated surface. This in turn increases the amount of heat that is carried away by the fluid

Convection current is significant in the boiling water in a kettle or pot. The lower parts are warmed first hence the water at the bottom end becomes less dense and rises to the top while the denser cool water on the surface goes to the lower bottom because of its high density. It can be noted that the difference in temperature brings about difference in density of fluid and the state conveys buoyant forces.

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The forces created make the denser parts of the water on the surface to move to the bottom end. This is upheld by conduction and convection which is created by a safe working temperature.

The motivation of this study was to determine the effects of various parameters on temperature distribution, velocity profiles, and heat transfer within a rectangular enclosure subjected to continuous heating and cooling.

Magneto hydrodynamic (MHD) is the branch of science, which deals with the flow of electrically conducting fluids in electric and magnetic fields. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents give rise to mechanical forces, which modify the fluid. However, MHD is usually regarded as a very contemporary subject, probably the largest. Advance towards an understanding of such phenomena comes from the fields of astrophysics and geophysics.

Mixed convection arises when both natural and forced convection contribute to heat transfer. The interaction between buoyancy forces and pressure-driven forces depends on flow characteristics, geometry, and temperature gradients. Situations where either mechanism alone is insufficient often require their combination. Examples include nuclear reactor cooling and electronic equipment thermal management.

The objectives of this research incude; to formulate a mathematical model of MHD mixed convection flow in a rectangular enclosure; to determine the flow parameters that affect the flow *(Re, Ec & Ha)*; to determine the effects of the flow parameters on temperature distribution and the velocity profiles within the enclosure and to determine the effects of the heat absorption coefficient on the temperature distribution

The contributions of this manuscript are: a) In engineering sectors to help in predicting and controlling the flow behavior in various MHD applications, such as liquid metal pumps, fusion reactor blankets, and metallurgical processes. b) In pharmacy, the the study is applied to understand and control fluid flow in processes like purified water distribution systems, suspension preparation, and industrial mixing. It determines if fluid flow is laminar (smooth, organized) or turbulent (chaotic, irregular), which affects factors like microbial control by influencing biofilm formation in pipes and ensuring orderly particle suspension in products. This understanding helps prevent quality issues and ensures product integrity.

II. RELATED WORKS/LITERATURE REVIEW

Magneto hydrodynamic (MHD) is the academic discipline which studies the dynamic of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals and salt water. The MHD was originally applied to astrophysical and geophysical problems, where it is still very important. Engineers employ MHD principles in the design of heat exchanger, pumps and flow meters, in space vehicle propulsion, control and re-entry in creating novel power generating systems and developing confinement schemes for controlled fusion.

Juma *et al,* (2023), studied the effects of inclined center red conductor on MHD natural and forced convection within a heated a wavy chamber. Using finite difference method, the results indicated that an increase in Hartmann number lowers the flow rate of the fluid.

Sigey *et al,* (2013), did a study on MHD free convective flow past an infinite porous pot with joule heating. Using Finite difference approximation, the result showed that an increase in joule heating parameter causes an increase in the velocity and temperature profiles uniformly near the plate but remains constantly distributed away from the plate.

Kariuki and Otieno (2025), Investigated MHD mixed convection in a lid-driven square cavity with partial heating using the finite element method. Their research focused on the effects of Hartmann number, Reynolds number, and Richardson number on flow and heat transfer characteristics. The results indicated that magnetic fields suppress circulation strength, leading to reduced Nusselt numbers along heated walls. However, partial heating was shown to create strong localized vortices, enhancing heat transfer in specific regions of the cavity. They concluded that applying tailored heating strategies in combination with magnetic fields can achieve efficient thermal regulation in electronic cooling applications.

Achieng *et al,* (2025), Conducted a numerical study of entropy generation in MHD double diffusive convection within a porous inclined cavity. Using a Galerkin weighted residual approach, they analyzed how solutal buoyancy and magnetic damping interact. The findings showed that entropy generation increases with Rayleigh number but decreases under stronger magnetic fields. Inclination angle was also shown to significantly modify solutal and thermal stratification, resulting in diverse convective patterns. The study highlighted the practical significance of entropy minimization in energy-efficient heat exchanger and geothermal system designs.

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Cherono *et al,* (2025), Studied transient MHD convection in a rectangular chamber with oscillating thermal boundary conditions. Using an implicit finite difference scheme, they analyzed how periodic heating influences velocity and temperature oscillations under varying magnetic field strengths. The results demonstrated that oscillatory thermal forcing induces secondary flow structures, while magnetic fields damp these oscillations to restore stability. They concluded that electromagnetic control can be effectively used to stabilize or destabilize thermally driven flows depending on desired outcomes. Their study contributes to the design of smart thermal management systems in aerospace and energy applications.

Mutua and Njoroge (2025), Analyzed MHD natural convection in a trapezoidal cavity with internal heat generation. The finite volume method was employed to solve coupled momentum and energy equations under different Hartmann numbers and aspect ratios. Findings showed that magnetic fields significantly suppress plume strength, leading to weaker circulation zones. However, cavity geometry strongly influenced the position of thermal stratification layers, with trapezoidal enclosures producing asymmetric convection rolls. Their study concluded that combining cavity geometry optimization with magnetic field control can enhance heat removal in compact enclosures such as microelectronic devices.

Muli *et al,* (2023), studied on the effects of induced magnetic field, hall current and radiative heat on a 3D unsteady hydro magnetic stagnation flow of the casson fluid. Applying the collocation method, the analysis investigated that an increase in magnetic parameter and cassons parameter increases the induced magnetic field where as prandtl and Reynold numbers decreases the induce magnetic field.

Mahmuda *et al* (2023), studied on the influence of magnetic field on MHD convection in lid driven cavity with heated wavy bottom surface. Using the Galerkin weighted residual technique of finite element to solve the equations, the results showed that increasing the heat source size decreases the average nusselt number along the heated wall.

Sakthivel *et al* (2022),studied on the effects of MHD mixed convection flow over a heated plate. They used shooting technique of converting higher order ordinary differential equation to first order, they found out that due to the particles of the fluid, the heat transfer speeds up while the temperature decreases.

Soomro *et al* (2020), investigated the thermal performance due to MHD mixed convection flow in a rectangular cavity with circular obstacle. Finite element method is adopted to find the numerical solutions in the study. They found out that heat transfer rate augments due to increasing Richardson number.

Onyinkwa and Chepkwony (2023), investigated the heat and mass transfer in MHD flow about an inclined porous plate. They obtained the numerical solutions using implicit finite Centre difference method, they noticed that increasing the strength of a magnet field reduces primary and secondary velocity and increases the flow temperature.

Patil and Kulkarni (2021), did an analysis of MHD mixed convection in a Ag-TiO2 hybrid nanofluid flow past a slender cylinder. They combined the quasilinearization technique and implicit finite difference approximation to obtained the numerical solutions. The results unveiled that the inclusion of the hybrid nanoparticles in the base fluid result in high heat transfer than that of a nanofluid and base fluid.

Require hospitalization, and one in a thousand of these cases among children results in death from measles complications ,as per WHO (2001) and WHO (2005).

III. FORMULATION OF GOVERNING EQUATIONS

The following assumptions hold in the study.

i. The fluid is Newtonian in nature.
ii. The flow is laminar.
iii. Every flow variable is a continuous function of space.
iv. Except for the buoyancy component, density is constant in the continuity equation and all inertial terms in the momentum equation
v. Heat transmitted by conduction and radiation is negligible.
vi. The consequences of viscous dissipation in the energy equation are negligible.
vii. Gravity and magnetic field are the body forces acting on the fluid.
viii. Density gradients are caused solely by temperature differences.
ix. The fluid is electrically conducting in this case its ionized air.

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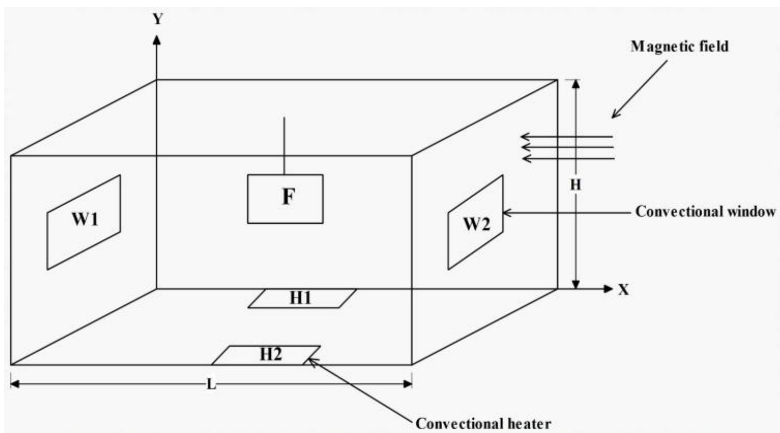
3.1 Geometry of the problem

Fig 3.1 below is a rectangular room of height H and width L is considered. Two floor heaters H_1 and H_2 are placed on the floor, while a fan F is positioned at the middle of the room.

Two convectional windows are located on the vertical walls, which are kept adiabatic. The heat transfer and fluid temperature are studied for ionized air. The fluid is considered incompressible, Newtonian, and laminar. A magnetic field B_0 is applied along the x-direction.



The viscous incompressible fluid flow and the temperature distribution inside the enclosure are described by the momentum and energy equations. Systems of Navier-stokes and energy partial differential equations with appropriate boundary conditions governing our problem are solved using a Finite Difference. The continuity (3.0), momentum (3.1 & 3.2) and energy (3.3) equations are shown below

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3.0}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.1}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta\left(T-T_o\right)-\frac{\sigma B_0^2}{\rho}v \tag{3.2}$$

$$\rho c_p u\frac{\partial T}{\partial x}+\rho c_p v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi \tag{3.3}$$

3.2 Similariry Transforms

Dimensional analysis makes the equations more concise from the physical relationship.

This process reduces the number of independent variables that defines the problem. Using the following dimensionless variables;

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{u}{u_o},V=\frac{v}{v_o},P=\frac{p}{\rho u_o^2},\theta=\frac{T-T_o}{T_1-T_o}$$

$$Ri=\frac{Gr}{\mathrm{Re}^2}\ ;\ Gr=\frac{g\beta(T_1-T_o)L^3}{\mu^2},\mathrm{Re}=\frac{u_oL}{\mu},\mathrm{Ha}=B_0L\sqrt{\frac{\sigma}{\mu}}$$

Consider equations 3.0, 3.1, 3.2 and 3.3 then by substituting the above non-dimensional variables them, we get the nondimensionalized form of the equations as shown below

The momentum equation become;

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial p}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta-\frac{\mathrm{Ha}^2}{\mathrm{Re}}V \tag{3.4}$$

And the energy equation become

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+E_c\Phi \tag{3.5}$$

3.3 Discretization of governing equations

We discretize the momentum and energy equations and form three point central difference schemes for each equation which we eventually solve using the finite difference method.

Equation (3.4) is discretized to study the effects of Re , Ri ,Ha vertical velocity profiles. Using a Hybrid difference numerical scheme, V_x, V_y, V_{xx} and V_{yy} are replaced by three point central difference approximation .When these approximations are substituted into equation (3.7), we get

$$\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta X}+\frac{V_{i,j+1}-V_{i,j-1}}{2\Delta Y}=-\frac{p_{i+1,j}-p_{i-1,j}}{2\Delta X}+\frac{1}{Re}\left[\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{\Delta X^2}+\frac{V_{i,j+1}-2V_{i,j}+V_{i,j-1}}{\Delta Y^2}\right]+Ri\cdot\theta_{i,j}-\frac{Ha^2}{Re}V_{i,j} \tag{3.17}$$

We investigate the effect of Re , Ri and Ha on the fluid velocity. Taking $\theta=10$, $\Delta x=\Delta y=0.25$,and V= U = 1 into (3.17), we get the scheme

$$(1-8\mathrm{Re})V_{i,j+1}-4V_{i,j}+(8\mathrm{Re}+1)V_{i,j-1}=8\mathrm{Re}\,P_{i,j-1}-8\mathrm{Re}\,P_{i,j+1}+(1-8\mathrm{Re})V_{i+1,j}+(8\mathrm{Re}+1)V_{i-1,j}+Ri\theta-\frac{Ha^2}{Re}V \tag{3.18}$$

Taking i = 1and j =1,2…..6 , we form the following systems of linear algebraic equations

$$(1-8\mathrm{Re})V_{1,2}-4V_{1,1}+(8\mathrm{Re}+1)V_{1,0}=8\mathrm{Re}\,P_{1,0}-8\mathrm{Re}\,P_{1,2}+(1-8\mathrm{Re})V_{2,1}+(8\mathrm{Re}+1)V_{0,1}+Ri\theta-\frac{Ha^2}{Re}$$

$$(1-8\mathrm{Re})V_{1,3}-4V_{1,2}+(8\mathrm{Re}+1)V_{1,2}=8\mathrm{Re}\,P_{1,1}-8\mathrm{Re}\,P_{1,3}+(1-8\mathrm{Re})V_{2,2}+(8\mathrm{Re}+1)V_{0,2}+Ri\theta-\frac{Ha^2}{Re}$$

$$(1-8\mathrm{Re})V_{1,4}-4V_{1,3}+(8\mathrm{Re}+1)V_{1,3}=8\mathrm{Re}\,P_{1,2}-8\mathrm{Re}\,P_{1,4}+(1-8\mathrm{Re})V_{2,3}+(8\mathrm{Re}+1)V_{0,3}+Ri\theta-\frac{Ha^2}{Re}$$

$$(1-8\mathrm{Re})V_{1,5}-4V_{1,4}+(8\mathrm{Re}+1)V_{1,4}=8\mathrm{Re}\,P_{1,3}-8\mathrm{Re}\,P_{1,5}+(1-8\mathrm{Re})V_{2,4}+(8\mathrm{Re}+1)V_{0,4}+Ri\theta-\frac{Ha^2}{Re}$$

$$(1-8\mathrm{Re})V_{1,6}-4V_{1,5}+(8\mathrm{Re}+1)V_{1,5}=8\mathrm{Re}\,P_{1,4}-8\mathrm{Re}\,P_{1,6}+(1-8\mathrm{Re})V_{2,5}+(8\mathrm{Re}+1)V_{0,5}+Ri\theta-\frac{Ha^2}{Re}$$

$$(1-8\mathrm{Re})V_{1,7}-4V_{1,6}+(8\mathrm{Re}+1)V_{1,6}=8\mathrm{Re}\,P_{1,5}-8\mathrm{Re}\,P_{1,7}+(1-8\mathrm{Re})V_{2,6}+(8\mathrm{Re}+1)V_{0,6}+Ri\theta-\frac{Ha^2}{Re}$$

(3.19)

The above algebraic equations can be written in matrix form as when we take initial conditions V(x,0) =1 and boundary conditions V(0,y) = 0, V(x,2) = 0 and P(x,y) = 0

$$\begin{bmatrix} 4 & (8\mathrm{Re}-1) & 0 & 0 & 0 & 0 \\ (8\mathrm{Re}+1) & 4 & (8\mathrm{Re}-1) & 0 & 0 & 0 \\ 0 & (8\mathrm{Re}+1) & 4 & (8\mathrm{Re}-1) & 0 & 0 \\ 0 & 0 & (8\mathrm{Re}+1) & 4 & (8\mathrm{Re}-1) & 0 \\ 0 & 0 & 0 & (8\mathrm{Re}+1) & 4 & (8\mathrm{Re}-1) \\ 0 & 0 & 0 & 0 & (8\mathrm{Re}+1) & 4 \end{bmatrix}\begin{bmatrix} V_{1,1} \\ V_{1,2} \\ V_{1,3} \\ V_{1,4} \\ V_{1,5} \\ V_{1,6} \end{bmatrix}=\begin{bmatrix} 8\mathrm{Re}+1+Ri\theta-\frac{Ha^2}{Re} \\ 8\mathrm{Re}+1+Ri\theta-\frac{Ha^2}{Re} \\ 8\mathrm{Re}+1+Ri\theta-\frac{Ha^2}{Re} \\ 8\mathrm{Re}+1+Ri\theta-\frac{Ha^2}{Re} \\ 8\mathrm{Re}+1+Ri\theta-\frac{Ha^2}{Re} \end{bmatrix} \quad (3.20)$$

Solving the above matrix equation (3.20), we get the solutions for varying values of Re and Ha with $\theta = 10^o$ we get results which will be represented in tables.

The energy Equation (3.5) is discretized to study the effects of Re for temperature profiles. The air in the room is considered to have fixed Prandtl number 0.71. Using a three point central numerical scheme, we get

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}+U\frac{\theta_{i+1,j}-\theta_{i-1,j}}{2\Delta x}+V\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2\Delta y}=\frac{1}{\mathrm{Pr}\,\mathrm{Re}}\left[\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta x)^2}+\frac{\theta_{i+1,j+1}-2\theta_{i,j+1}+\theta_{i-1,j+1}}{(\Delta x)^2}\right]+E_c\Phi \quad (3.21)$$

We investigate the effect of Re, on the fluid velocity. We take Pr = 0.71 since the fluid is air. Taking i = 1, j = 1,2….6, $\Delta t = 0.1, \Delta x = \Delta y = 0.25$,Re=100 and V=1, U=1, we get the scheme

$$\left(1-67.2\,\mathrm{Re}\right)\theta_{i,j+1}^{n} + \left(11.2\,\mathrm{Re}-4\right)\theta_{i,j}^{n} + \left(8\,\mathrm{Re}+1\right)\theta_{i,j-1}^{n} = \left(56\,\mathrm{Re}-1\right)\theta_{i+1,j}^{n} - \left(56\,\mathrm{Re}+1\right)\theta_{i-1,j}^{n} - E_c\Phi \qquad (3.22)$$

Taking i=1 and j=1,2,3…6 we form the following systems of linear algebraic equations

$$\begin{aligned}
&\left(1-56\,\mathrm{Re}\right)\theta_{1,2}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,1}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,0}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,1}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,1}^{0} + 11.2\theta_{1,1}^{1} - E_c\Phi \\
&\left(1-56\,\mathrm{Re}\right)\theta_{1,3}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,2}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,1}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,2}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,2}^{0} + 11.2\theta_{1,2}^{1} - E_c\Phi \\
&\left(1-56\,\mathrm{Re}\right)\theta_{1,4}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,3}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,2}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,3}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,3}^{0} + 11.2\theta_{1,3}^{1} - E_c\Phi \\
&\left(1-56\,\mathrm{Re}\right)\theta_{1,5}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,5}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,3}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,4}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,4}^{0} + 11.2\theta_{1,4}^{1} - E_c\Phi \qquad (3.23)\\
&\left(1-56\,\mathrm{Re}\right)\theta_{1,6}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,6}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,4}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,5}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,5}^{0} + 11.2\theta_{1,5}^{1} - E_c\Phi \\
&\left(1-56\,\mathrm{Re}\right)\theta_{1,7}^{0} + \left(11.2\,\mathrm{Re}-4\right)\theta_{1,7}^{0} + \left(8\,\mathrm{Re}+1\right)\theta_{1,5}^{0} = \left(56\,\mathrm{Re}-1\right)\theta_{2,6}^{0} - \left(56\,\mathrm{Re}+1\right)\theta_{0,6}^{0} + 11.2\theta_{1,6}^{1} - E_c\Phi
\end{aligned}$$

Taking the initial $\theta(\mathrm{x,0,0}) = 10$ and boundary conditions, $\theta(\mathrm{0,y,t}) = 10$, $\theta(\mathrm{0,y,0}) = 10$ $\theta(\mathrm{x,y,1}) = 0$, the above system of algebraic equation becomes

$$\begin{bmatrix}
(11.2\mathrm{Re}-4) & (1-56\mathrm{Re}) & 0 & 0 & 0 & 0 \\
(8\mathrm{Re}-1) & (11.2\mathrm{Re}-4) & (1-56\mathrm{Re}) & 0 & 0 & 0 \\
0 & (8\mathrm{Re}-1) & (11.2\mathrm{Re}-4) & (1-56\mathrm{Re}) & 0 & 0 \\
0 & 0 & (8\mathrm{Re}-1) & (11.2\mathrm{Re}-4) & (1-56\mathrm{Re}) & 0 \\
0 & 0 & 0 & (8\mathrm{Re}-1) & (11.2\mathrm{Re}-4) & (1-56\mathrm{Re}) \\
0 & 0 & 0 & 0 & (8\mathrm{Re}-1) & (11.2\mathrm{Re}-4)
\end{bmatrix}
\begin{bmatrix} \theta_{1,1} \\ \theta_{1,2} \\ \theta_{1,3} \\ \theta_{1,4} \\ \theta_{1,5} \\ \theta_{1,6} \end{bmatrix}
=
\begin{bmatrix} -64\mathrm{Re}-2-E_C\Phi \\ -56\mathrm{Re}-1-E_C\Phi \\ -56\mathrm{Re}-1-E_C\Phi \\ -56\mathrm{Re}-1-E_C\Phi \\ -56\mathrm{Re}-1-E_C\Phi \\ -56\mathrm{Re}-1-E_C\Phi \end{bmatrix} \qquad (3.24)$$

Solving the above matrix equation, we get the solutions for varying values of Re and E_c. that will be represented in tables.

IV. Results And Discussions

The simulation results given focus on the effects of the Reynolds number *(Re)*, Eckert number *(E_c)*, Hartmann number *(Ha)* and heat absorption coefficient *(Φ)* on velocity profile and temperature distribution respectively.

Figure 4.1 shows the Effects of Reynolds number on vertical velocity profile, an increase in fluid Reynolds number leads to increases in the vertical velocity profile. As the length of the enclosure increased, the vertical velocity increased at the middle height of room except near ground and at the top. The viscous forces became dominant. As the value of Re increased, the inertia forces dominated over the viscous force and the velocity increased.

Table 4. 1: Values of vertical velocity profile for varying Reynolds number

Figure 4.1 shows Effects of Eckert number on temperature distribution, Increase in Eckert number leads to increases in the temperature distribution. With an increase in Ec, the fluid temperature increases in a region near the bottom of the room and then decreases in the region at the top of the room. Physically, the Eckert number is a ratio of kinetic energy to the enthalpy. This means that a large Eckert number implies more kinetic energy and reduced temperature difference. Kinetic energy increase and reduced temperature difference implies that more heat transfer and hence the rise in temperature profiles distribution. This implies that there is more heat generation at the bottom of the room. This outcome reveals an increase in temperature with an increase in Eckert number. Table 4.2 shows Effects of Eckert number on temperature distribution.

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Figure 4.2 shows the Effects of heat absorption coefficient on temperature distribution, Increase in heat absorption coefficient leads to increases in the temperature distribution. It is noticed that as the dimensionless heat absorption coefficient increases, the temperature is found to be decreasing in the boundary layer region. However, the change is not that significant. Further, slightly away from the place the dispersion in the temperature is considerable and thereafter as we move far away from the plate, the effect is found to be decreasing. Table 4.3 shows the eeffects of heat absorption coefficient on temperature distribution

Figure 4.3 shows Hartmann number on temperature distribution, the results illustrate how the Hartmann number *(Ha)* influences the temperature distribution within a rectangular enclosure subject to mixed convection and magnetic field effects. As the Hartmann number increases, the magnetic field strength intensifies, which in turn suppresses the fluid motion due to the Lorentz force. This suppression reduces convective heat transfer and enhances conduction-dominated behavior. Table 4.4 shows the effects of Hartmann number on temperature distribution

Figure 4.4 shows the Effects of velocity profile for varying Hartman Number, the plotted results illustrate the impact of the Hartmann number *(Ha)* on the vertical velocity profile of a fluid flow within a rectangular enclosure, where a magnetic field is applied from the top-right corner. As the Hartmann number increases i.e from $Ha = 0$ (no magnetic field) to $Ha = 50$ (strong magnetic field), the vertical velocity significantly decreases throughout the enclosure, with the most pronounced reduction occurring near the upper region where the magnetic field is strongest. This behavior is attributed to the Lorentz force induced by the magnetic field, which acts as a damping force opposing the fluid motion. When no magnetic field is present ($Ha = 0$), the vertical velocity is highest, reflecting unobstructed natural convection. However, as Ha increases, the electromagnetic resistance suppresses the motion of the fluid, thereby reducing vertical circulation and weakening the convective heat transfer. Table 4. 1 shows the values of velocity profile for varying Hartman Number

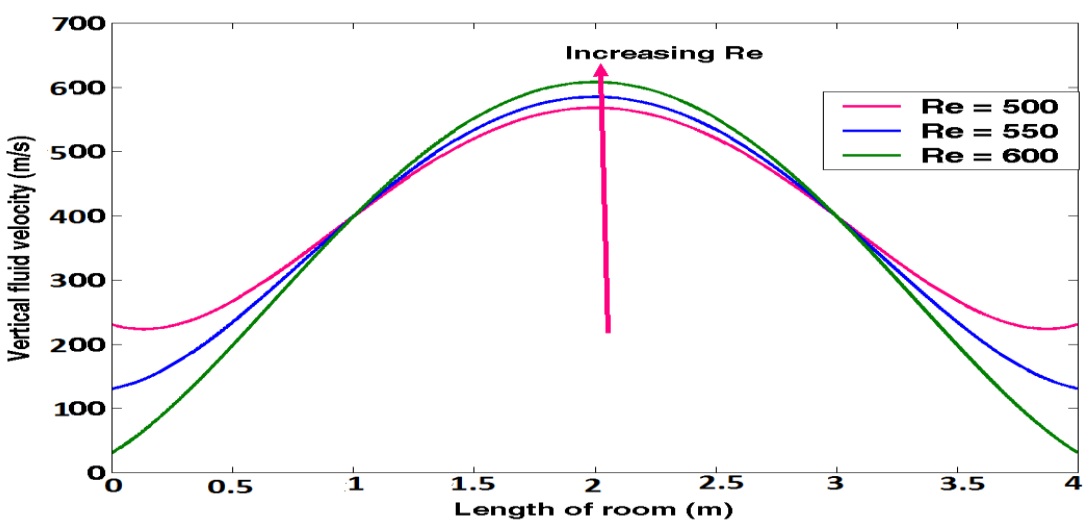

Figure 4.1 shows the Effects of Reynolds number on vertical velocity profile

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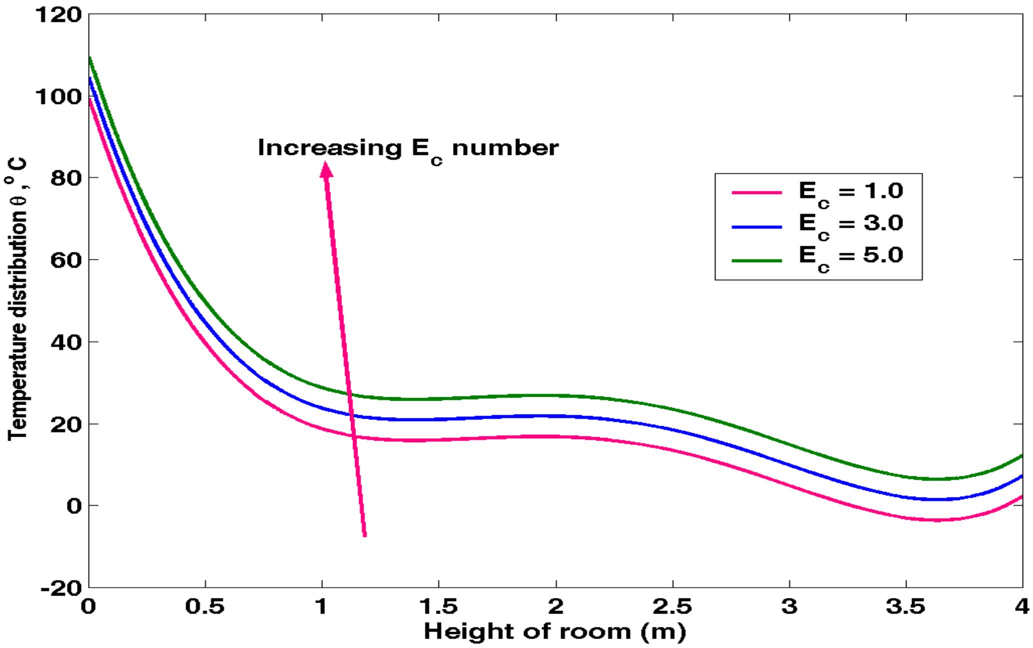

Figure 4.5 shows Effects of Eckert number on temperature distribution

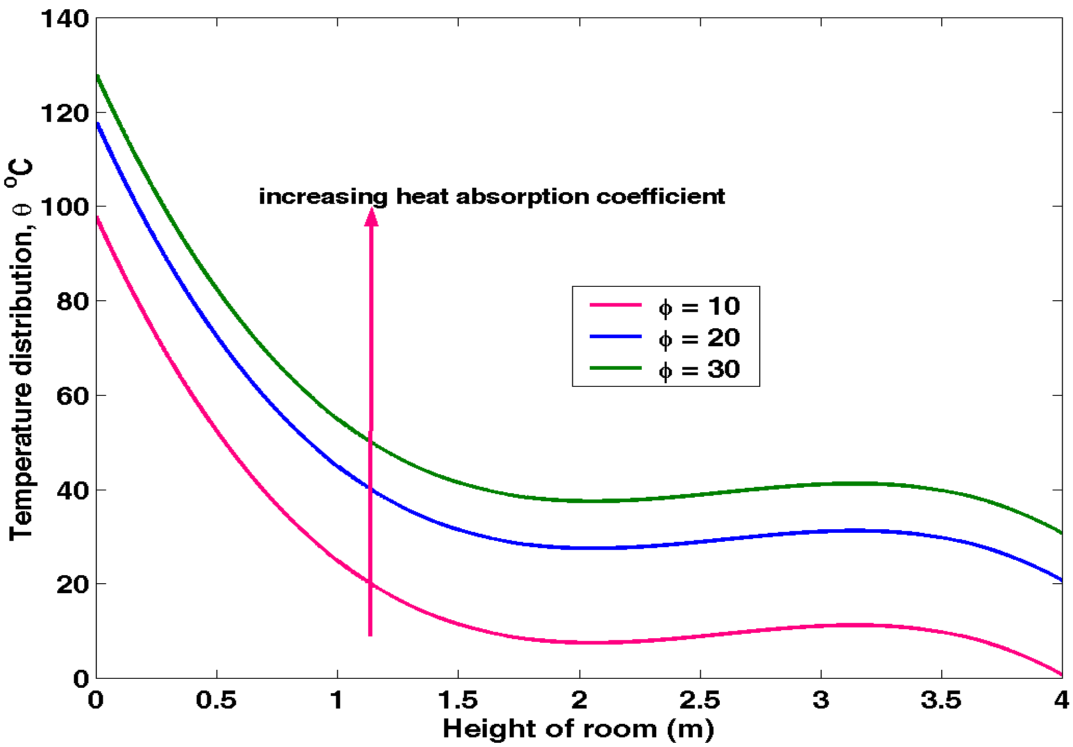

Figure 4.6 shows the Effects of heat absorption coefficient on temperature distribution

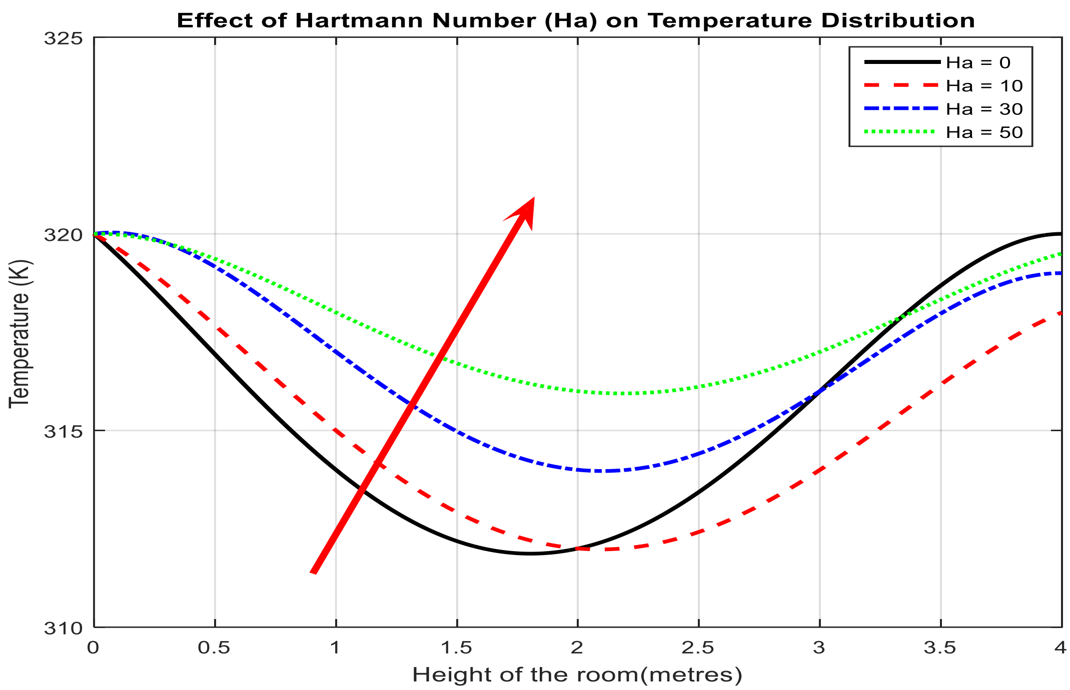

Figure 4.7 shows Hartmann number on temperature distribution

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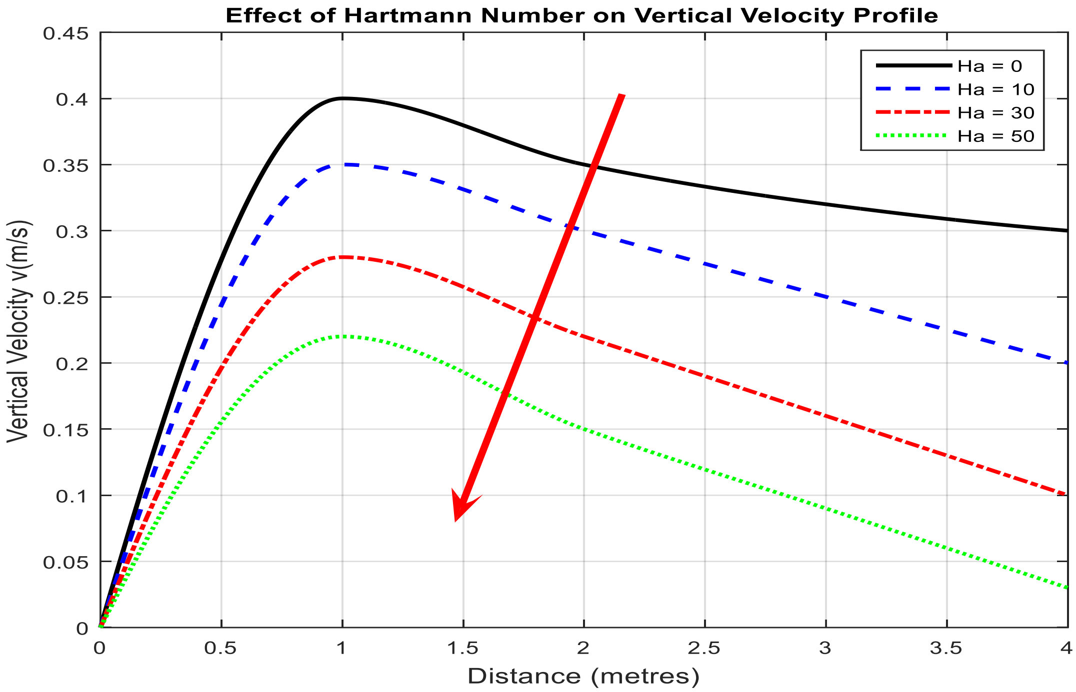

Figure 4.8 shows the Effects of velocity profile for varying Hartman Number

Table 4. 2:
Values of vertical velocity profile for varying Reynolds number

Reynolds number	**Length of room**				
	0	**1**	**2**	**3**	**4**
Re **= 500**	337.7521	0.6751667	336.9087	0.6744918	338.0895
Re **= 550**	537.0853	0.674394	370.2428	0.7567812	538.4223
Re **= 600**	737.2350	0.6763105	416.7500	0.75663456	738.2660

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Table 4. 3:
Value of temperature distribution for varying Eckert number

Eckert number	**Height of room**				
	0	**1**	**2**	**3**	**4**
E_C = 1.0	99.3448	18.71209	16.78558	4.884094	2.230255
E_C = 3.0	104.40656	23.73379	21.79607	9.887145	7.231649
E_C = 5.0	109.46862	28.73547	26.80655	14.890197	12.233042

Table 4. 4:
Value of temperature distribution for varying heat absorption coefficient

Heat absorption coefficient	**Height of room**				
	0	**1**	**2**	**3**	**4**
$\phi = 10$	99.3448	18.71209	16.78558	4.884094	2.230255
$\phi = 20$	119.4793	38.73238	36.81142	24.890711	22.246976
$\phi = 30$	129.4702	48.73928	46.81629	34.891225	32.26091

Table 4. 5:
Value of Hartmann number on temperature distribution

Hartman number	**Height of room**				
	0	**1**	**2**	**3**	**4**
$Ha = 10$	320.0	315.0	312.0	314.0	318.0
$Ha = 30$	320.0	317.0	314.0	316.0	319.0
$Ha = 50$	320.0	318.0	316.0	3317.0	319.5

Table 4. 6:
Value of velocity profile for varying Hartman Number

Hartman number	**Height of room**				
	0	**1**	**2**	**3**	**4**
$Ha = 0$	0.0	0.4	0.35	0.32	0.30
$Ha = 10$	0.0	0.35	0.30	0.25	0.20
$Ha = 30$	0.0	0.28	0.22	0.16	0.10
$Ha = 50$	0.0	0.22	0.15	0.15	0.03

V. Conclusion And Recommendation

The following conclusions were made from the analysis:

i. An increase in the magnetic parameter (Hartmann number) suppresses the velocity profile
ii. Increasing the Reynolds number results in an enhancement of the vertical velocity profile.
iii. A rise in the Eckert number causes the temperature distribution to increase.
iv. It was observed that the temperature decreases as the dimensionless heat absorption coefficient increases.
v. Increasing the Hartmann number suppresses convective heat transfer, leading to a more conduction-dominated thermal field.

5.1 Recommendations

5.1.1 consumers

i) Engineering sectors to help in predicting and controlling the flow behavior in various MHD applications, such as liquid metal pumps, fusion reactor blankets, and metallurgical processes.
ii) In pharmacy, the study is applied to understand and control fluid flow in processes like purified water distribution systems, suspension preparation, and industrial mixing. It determines if fluid flow is laminar (smooth, organized) or turbulent (chaotic, irregular), which affects factors like microbial control by influencing biofilm formation in pipes and ensuring orderly particle suspension in products. This understanding helps prevent quality issues and ensures product integrity.

5.1.2 Future researchers

i) An extension of this study to factor in a varying position of the magnetic field strength.
ii) An extension of this study to incorporate buoyancy ratio in fluids which are compressible.
iii) Use a different electrically conducting fluid.

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