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State Observers/Kalman Filters are Generally Unsuitable for Feedback Control While a Far More Rational and Superior Solution is Available

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***Abstract*--An observer that estimates system state vector x(t) is a state observer. Although a state observer can generate state feedback control signal Kx(t) where the constant gain K can bese parately designed before its realizing observer, the actual observer feedback system cannot have its loop transfer function equal $K(sI – A)^{-1}B$, the loop transfer function of the direct state feedback control, for a great majority of plant systems. Because loop transfer function determines the sensitivity function and robust properties of the corresponding feedback system, this implies that the robust properties of the Kx(t)-control are failed to be realized by state observers for a great majority of plant systems.Because robustness against system model uncertainty and terminal disturbance is foremost of feedback control even above performance, state observers are unsuitable for feedback control.**

Although this problem has initiated a vibrant robust control research in the past 40+ years, the result of that research has been unsatisfactory if parameter K is designed separately and prior to observer design. As a result, control theory remains essentially stagnant and a large amount of works still applied Kalman filters and state observers to feedback control applications.

Fortunately, this vital problem has found a fundamentally novel yet decisively satisfactory solution, that is to design parameter K based on the key observer parameters! These new observers are not restricted to be state observers, and can have freely designed and reduced observer order for the first time, and thus can guarantee the full realization of loop transfer function and robust properties of their corresponding Kx(t)-control. Such an observer exits for a great majority of plant systems!In addition, this new design principle is very simple to be learned, and adjusted very easily. Thus, a design methodology that can achieve high performance and robustness for general L-T-I systems is finally developed!

***Keywords*-- Fundamentally new design principle, general robustness and performance, simple design techniques.**

I. State Observers Are Unsuitable For Feedback Control For A Great Majority Of Plant Systems

For linear time invariant (L-T-I) state space model of an irreducible plant system (A, B, C) with n states, p inputs and m outputs. Let the general state space model of the corresponding observer be

$$d/dt\, \mathbf{z}(t) = F\mathbf{z}(t) + L\mathbf{y}(t) + TB\mathbf{u}(t) \qquad (1.1)$$

$$K\mathbf{x}(t) = K_z\mathbf{z}(t) + K_y\mathbf{y}(t) \qquad (1.2)$$

Where $\mathbf{y}(t)$ (= $C\mathbf{x}(t)$) and $\mathbf{u}(t)$ are the plant system outputs and inputs, respectively, and F, L, T, K_z, and K_y are the observer parameters to be designed.

To generate a $K\mathbf{x}(t)$ signalwhereKisconstant, we require that [1, 5]

$$TA – FT = LC \text{ and } F \text{ is stable}, \qquad (2.1)$$

which guarantees $\mathbf{z}(t)$ converge to $T\mathbf{x}(t)$,and this convergence further implies that in (1.2),

$$K = K_zT + K_yC = [K_z\!: K_y]\,[T^T\!:C^T]^T \equiv \underline{K}\underline{C},$$

And where Rank($\underline{C}$)≡ q = r + m, where r = the number of rows of T and the order of the observer/controller.

A state observer that estimates $\mathbf{x}(t) = I\mathbf{x}(t) = \underline{C}^{-1}\underline{C}\mathbf{x}(t)$, requires

$$\text{Rank}(\underline{C}) = n, \qquad (2.2)$$

And guarantees the satisfaction of the above equation K = $\underline{K}\underline{C}$ for any separately designed and arbitrarily given K, with solution $\underline{K} = K\,\underline{C}^{-1}$.

A Kalman filter with state space model

$$d/dt\, \mathbf{z}(t) = A\mathbf{z}(t) + B\mathbf{u}(t) + L[\mathbf{y}(t) – C\mathbf{z}(t)] = [A – LC]\mathbf{z}(t) + L\mathbf{y}(t) + B\mathbf{u}(t),$$

is a special state observer with F = A-LC and T = I.

In addition to generate the $K\mathbf{x}(t)$ signal, we also require

$$TB = 0 \qquad (2.3)$$

Which, together with (2.1), is necessary and sufficient for the actual loop transfer function of observer feedback system, L(s), equal that of the $K\mathbf{x}(t)$-control $L_{Kx}(s) \equiv -K(sI – A)^{-1}B$, if K or $\underline{K}\underline{C}$is practically designed or is designed for a good feedback system state matrix A – BK [1: Theorem 3.4, 4, 6, 7].

The proof is very simple. The loop transfer function of the observer feedback system $L(s) = -K(sI – A)^{-1}B$ if and only if, in addition to (2.1), [4, 21]

$$L_u(s) \equiv K_z(sI - F)^{-1}TB = 0 \quad \text{for all } s. \qquad (2.4)$$

Because K_z of (1.2) must be designed only for a good matrix A – BK but not for $L_u(s) = 0$, and because $(sI – F)^{-1}$ must be invertible for all s, $L_u(s) = 0$ if and only if TB = 0.

The observer (1) satisfying (2.1) to (2.3) is equivalent of an "unknown input observer (UIO)", if matrix B is the gain of the unknown inputs. This is the only existing observer satisfying (2.3). According to [1, 3], a UIO exists if and only if the plant system satisfies $m \geq p$, Rank(CB) = p, and all transmission zeros are stable (minimum-phase). For example, Rank($\underline{C}$) = n implies dimension of matrix T must be n – m, while TB = 0 implies that T must be within the Null Space of B with dimension n – p only. Therefore $m \geq p$ is required [1].

A great majority of plant systems cannot satisfy these three conditions.The systems with m = p and Rank(CB) = p have n – m transmission zeros [15]. Based on the reasonable assumption that each transmission zero is equally likely to be stable/unstable, almost all random setsof n – m transmission zeros (n > 2m) have at least one unstable (or stable) transmission zeros among them [1]. We also assume about half of systems with m > p cannot satisfy Rank(CB) = p such as airborne systems.

Hence, assume among all plant systems, 20% is m < p, 50% is m = p, and 30% m > p, then20% of plants cannot satisfy condition $m \geq p$, almost all plants with m = p (50%) have at least one unstable transmission zero, and about half (15%) of the remaining plant systems with m > p cannot satisfyRank(CB) = p. That means altogether 85% of the plant systems cannot satisfy the above three conditions of [1, 3], or cannot realizethe loop transfer function of its K**x**(t)-control at all!

Because loop transfer function L(s) determines the sensitivity function $[I – L(s)]^{-1}$ and robustness of the corresponding feedback system, and robustness against system model uncertainty and terminal disturbance is foremost of feedback control even above performance [1, 4, 7], this implies that state observers/Kalman filters are generally unsuitable for feedback control.

This conclusion implies the well-known LQ optimal control which can be achieved only by the existing K**x**(t)-control with Rank($\underline{C}$) = n, cannot have its robust properties realized for 85% of plants!

This vital problem also appeared from practice ever since the start of modern control theory 60+ years ago. The theoretical research of this problem and loop transfer functions started in 1978 [4], and has initiated a vibrant robust control research in the past 40+ years.

Because all other existing designs followed the "separation principle" [16] so that parameter K is designed separately and prior to observer design, requirements (2.1) to (2.3) especially (2.2) are necessary and about 85% of plant systems cannot satisfy them.

The results of approximate realization of L(s) towards $L_{Kx}(s)$ are very ineffective and unpredictable either.

For example, the dominant result of approximate realization of L(s) towards $L_{Kx}(s)$is called "asymptotic LTR",and is to asymptotically increase the observer gain L (to **y**(t)) so that the gain TB (to **u**(t)) of (1.1) is overwhelmed [21]. This design approach is obviously indirect and thus very ineffective. It is also impractical because a large gain L is prohibited in any robust control design in the first place [1, 7, 20].

Furthermore, what makes these LTR results even more unsatisfactory is because sensitivity function $[I – L(s)]^{-1}$ is itself very sensitive to any variation between L(s) and$L_{Kx}(s)$, and because robustness/reliability guarantee is by definition against this variation and unpredictability.

This situation has forced a backward research trend to the direct design of L(s), such as minimizing the H∞ norm of $[I – L(s)]^{-1}$ starting in the 1980's. However, transfer function model is far more complicated yet reveals far less information than state space model, and classical control analysis/design techniques are far less effective than that of modern control theory.

The K**x**(t)-control aimed at improving feedback system state matrix A – BK and, indirectly, its loop transfer function $K(sI – A)^{-1}B$, of modern control theory, is based on the far more detailed and far more explicit information on system's internal structure (A, B, C) and on the "filtered" system's states **x**(t). This is why K**x**(t)-control is superior if parameter K is designed wisely, or is designed only for improving feedback system state matrix A – BK, even if K is constrained by K = $\underline{KC}$.

For example, the K**x**(t)-control has eigen-structure assignability, which is superior and far more superior than any other design results in improving feedback system performance and robustness [1, 7]. It is well-known that system poles most generally accurately determine system response and performance (as compared to bandwidth), and the sensitivity of the poles (determined by the corresponding eigenvectors) is far more focused than the raw and gross frequency response data of the sensitivity function or loop transfer function. [1, 12, 13, 14].

This should be the reason that people forwarded to modern control theory from classical control theory since the 1960's in the first place, and the reason that most applications today including those reported at 2023CCC used state observers/Kalman filters. It seems that people have been wandering between these two theories, and control textbooks especially their design part remain essentially unchangedin the past 60+ years!

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A critical question is, if high performance and robustness control cannot be realized generally for the most simple and basic L-T-I systems, then how can control theory be useful really and generally at all?

It is apparent that the *only future* of modern control theory or the whole control theory, is to find a way that can realize the loop transfer function of K**x**(t)-control generally and effectively, or that can satisfy design requirements (2.1) and (2.3) generally and effectively.

II. The Simple And Basic Reason Is In Rationality

Why is the direct K**x**(t)-control with K designed based on the assumption of ideal plant system condition with the availability of ideal information of the whole set of **x**(t), unrealizable for about 85% of all plants that are far from ideal? The answer simply lies in the words of this question itself [1, 7].

2.1 How can a K**x**(t)-control designed based on the invalid and unrealistic assumption that the plant systems are ideal, be expected realizable for 85% of actual plant systems which are far from being ideal (not satisfying the three conditions of [1, 3])? How can such an expectation be realistic and rational?

This is like expecting a true and non-disastrous realization of an ideal Communistic society, to an actual society that is deemed only at primary Socialistic economic development stage and condition.

2.2 The existing K**x**(t)-control design is aimed at improving feedback system state matrix A – BK only. It ignores parameter T which is key to the realizing observer and parameter C which is key to the output measurement information, in $K = \underline{K}\ [T^T{:}C^T]^T$. This ignorant design cannot be rational and fully realizable.

For instance, ignoring parameter C means ignoring completely whether the available number of system output measurements is 100%, or 50%, or 10%, or even 1%, of the number n of system states, when designing the state feedback control law K. How can such an ignorant design be rational and fully realizable?

2.3 The requirement of n-dimensional information insignal $\underline{C}$**x**(t) (Rank($\underline{C}$) = n) for the generation of an actual p-dimensional control signal K**x**(t) (Rank(K) = p), at the common situation that n > p, is unnecessary, excessive, and irrational.

In fact, excellent control (though not ideal control) can be designed if p + Rank($\underline{C}$)>n because it guarantees full eigenvalues and partial eigenvector assignment[1, 11]!,and powerful control (though not excellent control) orgenerical eigenvalue assignability is guaranteed if p x Rank($\underline{C}$) > n [2]!

Specifically, if q + p > n (Rank($\underline{C}$) ≡ q), then the design algorithms of [11] can assign n - q or n – p eigenvectors each with q or p basis vectors, and assign the rest of q or p eigenvectors each with q + p – n basis vectors [1, 11].

2.4 Technically, unable to satisfy generally TB = 0 means unable to avoid input feedback, which has been avoided by almost all successful and rational controllers of classical control [18]!

Input disturbance is a top concern of feedback control [1, 4, 7, 21]. This is why the failed robust realization (TB ≠ 0) is a fatal drawback of KalmanFilters/modern control theory!

Four more basic theoretical and rational drawbacks of the existing design are articulated in [1, 20].

Is there any actual advantage of the ideal K**x**(t)-controlif its most critical robust properties cannot be actually realized at all? Is there any rationality of separation principle that designs an ideal control but cannot be actually realized? Is there any rationality of separation principle whose state observers/Kalman filters cannot realize at all the critical robust properties of their K**x**(t)-control which they are supposed to realize?

III. A Far More Rational And Superior Remedy

The following is an excerpt from this author's previous publications:

"In 1990, and in an 85°F room one summer after noon, while stuck by the dilemma that making TB = 0 can cause Rank($\underline{C}$) <n generally, it suddenly occurred to this author that Rank($\underline{C}$) does not need to be n, and that parameter K does not need to be designed separately because parameter $\underline{K}$ can be designed instead (while satisfying (2.1) and (2.3) in priority), and that separation principle does not need to be adhered after all!".

By designing $\underline{K}$ instead of K, K = $\underline{KC}$ is satisfied automatically and design requirement (2.2) (Rank($\underline{C}$) = n) is eliminated [1, 7-9, 17].

This new design of K by designing $\underline{K}$ instead, is based on the valid and realistic assumption that information $\underline{C}$**x**(t) is available and is reliable because TB = 0 (2.3) is satisfied (see Subsection 2.1), is not ignorant of theactual parameters T and C in matrix $\underline{C}$ that are key to the realization of the $\underline{KC}$**x**(t)-control (see Subsection 2.2), is based on only q (can be less than n) signals of $\underline{C}$**x**(t) (see Subsection 2.3), and eliminates or minimizes the fatal input feedback by satisfying TB = 0 as design priority (see Subsection 2.4).

Therefore, this new design is named "synthesized design principle" [1] versus the existing "separation design principle" [16],and overcomes completely the above four basic drawbacks in rationality of the existing separation principle.

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This new design also has the following overwhelming advantage over the existing separation design principle.

Because design requirement (2.2) (Rank($\underline{C}$) = n) is eliminated, only (2.1) and (2.3) need to be satisfied. These two equations can be re-written as

$$[\boldsymbol{t}_i : \boldsymbol{l}_i]\begin{bmatrix} A - f_i I & B \\ -C & 0 \end{bmatrix} = 0 \qquad (3)$$

Where f_i is the i[th] element of a diagonal matrix F, $\boldsymbol{t}_i$ and$\boldsymbol{l}_i$are the rows of matrices T and L corresponding to f_i. It is obvious from (3) that exact non-zero solution $[\boldsymbol{t}_i:\boldsymbol{l}_i]$ exists, if the plant system (A, B, C) satisfies either one of the following two sufficient conditions: m > p, or has at least one stable transmission zerof_i and this f_i is assigned as the i[th] eigenvalue of matrix F [1, 7-9, 22]. This result can be easily generalized to generaleigenvalue cases.

Based on the same assumption of 20% m < p, 50% m = p, and 30% m > p, of plant systems of Section 1, 30% of plants satisfy m > p and almost all plants with m = p (50%) have at least one stable transmission zeros (see Section 1).Altogether, at least80% of plant systems can have the loop transfer function and robust properties of their$\underline{KC}$**x**(t)-control fully realizedusing our synthesized design principle. This is 500% more general and is decisively more general and superior than separation design principlewhich can realize robustness of K**x**(t)-control for only 15% of plant systems.!

Table 1
summarizes this comparison:

Separation Principle K**x**(t)-control Rank($\underline{C}$) = n	Synthesized Principle$\underline{KC}$**x**(t)-control Ranh($\underline{C}$) ≤ n
Three necessary conditions	**Two** sufficient conditions
1) m ≥ p -20%	1) m > p+30%
2) Minimum-phase -50%	2)Have stable zeros+50%
3) Rank(CB) = p -15%	
Ony **15%** of plants satisfy!	**80%** of plants can satisfy!

The$\underline{KC}$**x**(t)-control with eigen-structure assignability is superior and far more superior than any other design results in improving feedback system performance and robustness [1, 7].It is well-known that system poles most generally accurately determine system response and performance as compared to bandwidth, and pole sensitivity $s(\lambda_i)$ can be measured as the product of the norms of its left and right eigenvectors, and is far more focused than the raw and gross frequency response data of the sensitivity function or loop transfer function[1, 12, 13, 14].

For example, based on system poles λ_i and its sensitivity $s(\lambda_i)$ (i = 1, …, n), a new robust stability margin is proposed as $\min\{s(\lambda_i)^{-1}|Re(\lambda_i)|\}$ [13, 1]. This stability margin is proven to be far more generally accurate in indicating robust stability as well as overall system performance and robustness, than other design criteria including that of L-Q optimal control [1, 7]. This new stability margin can be easily optimized by the existing eigen-structure assignment algorithms [1, 14, 11].

IV. A Design Example

Many design examples and exercise problems were presented in [1]. The following example is presented in [20]. This is a 3[rd] order (n = 3), 2-inputs (p = 2) and 1-output (m = 1) system with state space model

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 2 & 1 \end{bmatrix}, [1 \quad 0 \quad 0]\right).$$

This system has one stable transmission zero (-1).The following design solution is presented in Figure 1:

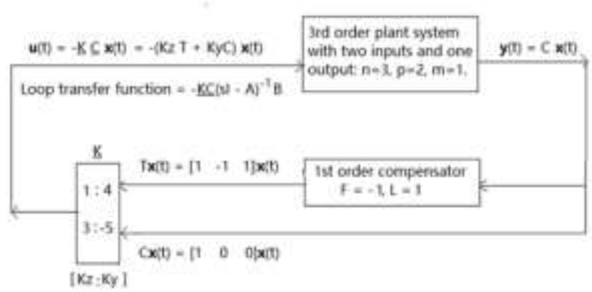

Figure 1

Because all parameters of Figure 1 satisfy (2.1) and (2.3), a $\underline{KC}$**x**(t)-control signal is generated and its loop transfer function $-\underline{KC}(sI - A)^{-1}B$ and associated robust properties are fully realized.

Although Rank($\underline{C}$) ≡ q = 2 < n in this example, the corresponding $\underline{KC}$**x**(t)-control still can assign all poles({-2, $-1 \pm j\sqrt{3}$} in this example) and one eigenvector (for eigenvalue -2 in this example) because q + p = 4 > n as predicted [1,11]. This is just a little less than the full eigenvalue/eigenvector assignability of the idealdirect K**x**(t)-control corresponding to rank($\underline{C}$) = n, and is therefore an excellent control.

In comparison, the existing direct K**x**(t)-control cannot be realized because this plant system has lessoutputs than inputs (m < p), while the existing static output feedback control cannot even assign all poles such as the poles of this example because m + p ≤ n [2, 20].

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An equally significant advantage of the new design principle is at its simplicity --- the entire design computation of this example is carried out by hand!

Therefore, synthesized design principle is a once in a life-time apperception of any control theoreticians. Its novelty and technical superiority and significance can match that of any other results in control system history.

V. Further Significance Of This New Design Principle

The comparison of overall performance and robustness levels analyzed in the previous sections, of our new synthesized design principle and the existing separation design principle, is depicted in Figure 2:

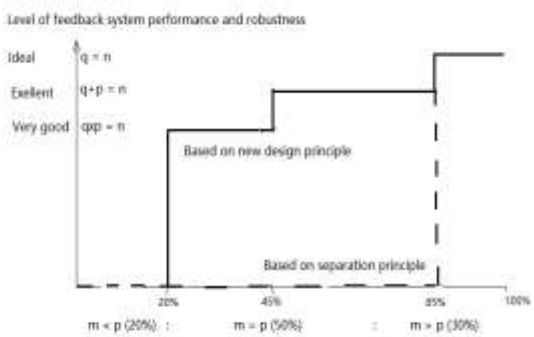

Figure 2

Figure 2 shows that the results of our new design principle (solid line) can reach at least very good level for 80%of all plant systems (or plant systems with $m \geq p$), because robustness of the KCx(t)-control can be fully realized for these plant systems.

For plant systems with $m = p$ (50%), they generically have $n - m$ transmission zeros and always have $n - m$ transmission zeros if Rank(CB) = p [15]. Based on a conservative yet most reasonable assumption that each transmission zero is equally likely to be stable or unstable, extensive statistical analysis in [1] indicates that almost all such plant systems (50% of all plant systems)) can satisfy $q \times p > n$, and indicates that about half of such plant systems (25% of all plant systems) can satisfy $q + p > n$. Here $q = m$ + the number of stable transmission zeros [1, 7-9, 22].

For plant systems with $m > p$ which are better than plant systems with $m = p$ (more output measurement information to use and less control signals need to generate), the cause for Rank(C)< n is either non-minimum-phase or Rank(CB) < p [1, 3]. Because systems with $m \neq p$ generically have no transmission zeros [15], Rank(C) < n is generically due to Rank(CB) < p only.

Because Rank(CB) cannot be lower than p by more than p, this lowing of Rank(C) (from n) cannot be more than p. Thus, Rank(C) + p > n generically in such systems, as demonstrated by many examples of [1].

Conclusions of the past two paragraphs are summarized in the solid line of Figure 2: 80% of plant systems can satisfy $q \times p > n$ which guarantees a very good control with generic pole assignability [2]or better, and 55% of plant systems can satisfy Rank(C) + p > n which guarantees an excellent control with full eigenvalue and partial eigenvector assignability or better [1, 11].

Figure 2 (dotted line) shows that the existing separation design principle cannot realize at all the critical loop transfer function and robustness of its Kx(t)-control, for 85% of plant systems. That means that there is actually no guaranteed robustness level at all for these systems!

The difference between the solid and dotted lines of Figure 2 is at the middle 65% of plant systems. This difference is undeniably decisive and overwhelming.

This advantage is enabled by our unique technical ability to freely design and adjust our Rank(C) from a previously fixed value of n to lower values, which is further enabled by our unique ability to freely design and adjust our number of rows T or observer/controller order r, and which is further enabled by our unique feature of solution matrix T of (2.1) of 1985 such that all rows of T are decoupled from each other [1, 7, 10].

This flexibility of our Rank(C) and controller order rfurther enables, uniquely and effectively, the tradeoff between feedback system performance and robustness: a higher order or a higher Rank(C) implies a more powerful KCx(t)-control that can better improve feedback system performance and robustness and meet the design requirements, while a lower order or a lower Rank(C) implies the robust property of our KCx(t)-control can be more easily realized (or TB = 0 can be more easily satisfied) [1, 19]. More than a dozen such design examples and problems are presented in [1].

A far more effective approximate realization to LKx(s) is to minimize TB, if a higher number of rows of T is required by the design requirement so that TB = 0 cannot be satisfied exactly [1, 19]. Thus, our design methodology can be applied to all plant system conditions and design requirements.

Overall, a design principle and methodology that can achieve high performance and robustness generally (say 80%+ of plant systems), that is basically rational and far more rational, that can adjust effectively the tradeoff between performance and robustness, and that is simple to be easily learned and implemented, has been thought after by the whole control community for 76 years since the start of control theory!

It is obvious and undeniable that, our new synthesized design principle has finally achieved this core and basic goal unachieved for so long. Therefore, this is an undeniably, a historical and monumentalachievement in control systems history!

For some reasons, this seminal and monumental result has been denied, doubted, ignored and apathetic by many and for years. This is apparently an unprecedented and continuing scandal in control theory's history. It harms the whole control theory and control community severelyas evidenced by the fact thatthe design parts of modern control textbooks remain essentially unchanged for the past 60+ years, by the fact that no other existing design technology can achieve high performance and robustness generally, and by the factthatlarge amount of recent works still applied Kalman filters and state observers to feedback control unaware of robustness.

Because of its overwhelming significance and simplicity, synthesized design principle should replace the existing separation design principle in all control textbookswithin a couple of decades, and control systems theory has finally a brilliant prospect.

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