International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347-6435(Online) Volume 11, Issue 06, June 2022)

"A New Characterization of Thermal Stability of Couple – Stress Fluids"

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Abstract-- **This paper presents the study of Thermal stability on Couple – stress Fluids. Here, we consider, the stability analysis of couple – stress fluid heated from below with magnetic field and rotation is considered. By the Linear stability theory and normal mode technique, the dispersion relation is obtained. In the case of stationary convection, dust particles are found to have a destabilizing effect on the system, where as Couple – stress and Magnetic field have dual character to its stabilizing effect in the absence of Magnetic field and rotation. The Oscillatory modes are introduced due to the presence of Magnetic field and rotation in the system. The results are presented through graphs in each case. Graphs have been plotted by giving Numerical values to the parameters to depict the stability characteristics.**

Keywords--**Magnetic field, Rotation, Couple – stress and Dust Particles.**

I. INTRODUCTION

Stability of a dynamical system is closest to real life, in the sense that realization of a dynamical system depends upon its stability. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. A detailed account of the theoretical and experimental study of the onset of thermal instability (B'enard convection) in a fluid layer under varying assumption of hydrodynamics, has been discussed in detail by Chandrasekhar (1981).

As growing importance of non-Newtonian fluids in modern technology, the investigation of such fluids are desirable. The theory of couple – stress fluids is proposed by Stokes(1966). Couple –stress appear in noticeable magnitude in fluids with very large molecules.

Applications of couple- stress fluid occur in the attention of the study of the mechanism of lubrication of synovial joints, at which currently attract the attention of researchers.

A human joint is a dynamically loaded bearing that has an articular cartilage as the bearing and synovial fluid as the lubricant. Normal synovial fluid is clear or yellowish and is a non- Newtonian, viscous fluid. Walicki and Walicka (1999) Modeled synovial fluid as couple-stress fluid in human joints because of the long chain of lauronic acid molecules found as additives in synovial fluid. The problem of a couple-stress fluid heated from below in a porous medium is considered by Sharma and Sharma (2001) and Sharma and Thankur(2000).

Stokes (1987) has formulated the theory of a Couple - stress fluid. The presence of small amounts of additives in a lubricant can improve the bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. Joseph (1976) has given the formation and derivation of the basic equations of a layer of fluid, heated from below in porous medium, using Boussinesq approximation. The study of a layer of fluid, heated from below in porous media, is motivated both theoretically as also by its practical applications in engineering.

By spacecraft observations the dust particles play a significantrole in the dynamics of the atmosphere as well as in the diural and surface variations in the temperature.

Further, environmental pollution is the main cause of the dust to enter the human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromosomes and genetic mutation so observed are likely to breed censer as malformations in the coming progeny.

It is essential, therefore to study the presence of dust particles in astrophysical situations and fluid flow. Sunil et al.(2004) have studied the effect of suspended particles on couple- stress fluid heated and soluted from below in a porous medium and found that suspended particles have destabilizing effect on the system.

International Journal of Recent Development in Engineering and Technology
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A.K. Aggarwal and Suman Makhilja(2009) have studied the effect of thermal stability on couple-stress fluid in the presence of rotation and magnetic field and found that rotation has a stabilizing effect while Couple- Stress has stabilizing effect in the absence of rotation on the system. Kumar et al.(2004) have studied the thermal stability of Walters' B' visco-elastic fluid permeated with suspended particles in hydromagnetics in a porous medium and found that magnetic fields stabilize the system. The problem on a Rivlin-Ericksen fluid in a porous medium in the presence of uniform vertical magnetic field and rotation is also considered by Sharma et al(2001). They have found that rotation has a stabilizing effecton the system.

Sharma and Rana(2002) have studied thermosolutal instability of Walter's(Model B') visco-elastic rotation fluid permeated with suspended particles and variables gravity field in porous medium. Kumar et al.(2006) have studied the effect of magnetic field on thermal instability of a rotating Rivlin- Ericksen visco-elastic fluid Kumar et al.(2009) have studied the problem of thermalsolutal instability of couple-stress rotating fluid in the presence of magnetic field and found that magnetic field has both stabilizing and destabilizing effects on the system under certain conditions whereas rotation has a stabilizing effect on the system.

As for growing importance of Couple - Stress fluid, convection in a fluid layer heated from below, the present paper attempts to study the Stoke(1966) incompressible Couple-Stress fluid in the presence of dust particles, magnetic field and rotation .

II. Basic Equations And Mathematical Formulation Of The Problem

Consider a static in which an incompressible, Stokes couple - stress fluid layer of thickness d, is arranged, confined betweentwo infinite horizontal planes situated at z = 0 and z = d,which is acted upon by a vertical magnetic field **H**(0, 0, H), where H is a constant, uniform rotation **Ω**(0,0,Ω) , and variable gravity field **g**(0, 0, -g). The particles are assumed to be non - conducting. The fluid layer is heated from below leading to an adverse temperature gradient $\beta = \frac{T_0 - T_1}{}$, where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with lower and upper boundaries with $T_0 > T_1$. Let p, ρ, T, α, v, μ^1, k_r and $\vec{q}$(u, v, w) denote respectively pressure, density, temperature, thermal, coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity and velocity of the fluid.

$\vec{q_d}(\bar{x},t)$ and N($\bar{x}$,t) denote the velocity and number density of particles, respectively. K = 6 πμη where η is radius of the particle, is a constant and $\bar{x}$ = (x,y,z). Then equation of motion, continuity and heat conduction of couple-stress (Stokes, 1966 and Joseph, 1976) in hydromagnetics are

$$\frac{\delta q}{\delta t} + (q.\nabla)q = -\frac{1}{\rho_0}\nabla p + g\left(1+\frac{\delta p}{\rho_0}\right) - \left(v - \frac{\mu^1}{\rho_0}\nabla^2\right)\nabla^2 q + \frac{KN}{\rho_0}(q_d - q) + 2(q \times \Omega) + \frac{\mu_e}{4\pi\rho_0}[(\nabla \times H) \times H] \quad \ldots\ldots (1)$$

$$\nabla . q = 0 \quad \ldots\ldots (2)$$

$$\frac{\partial H}{\partial t}(H.\nabla)q + \eta\nabla^2 H \quad \ldots\ldots (3)$$

and

$$\nabla . H = 0 \quad \ldots\ldots (4)$$

The equation of state for the fluid is

$$P = \rho_0[1 - \alpha(T - T_0)] \quad \ldots\ldots (5)$$

Where α is coefficient of thermal expansion and the suffix zero refers to value at the reference level z = 0.

Assume uniform particle size, spherical shape and small relative velocities between the fluid and particles. The presence of particles add an extra force term, proportional to the velocity difference between particles and fluid, appears in equation of motion (1).

Since the force exerted by the fluid on the particles is equal and opposite to the exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The buoyancy force on the particles is neglected. Inter-particlereactions are not considered for we assume that the distance between particles is quite large as compared with their diameter. The equations of motion continuity for the particle, under the above approximation, are

$$mN\left[\frac{\partial q_d}{\partial t} + \frac{1}{\epsilon}(q_d.\nabla)q_d\right] = KN(q - q_d) \quad \ldots\ldots (6)$$

and

$$\frac{\partial N}{\partial t} + \nabla(N.q_d) = 0 \quad \ldots\ldots (7)$$

Here mN is represent the mass of the particles per unit volume. Let C_v, C_{pt} denote the heat capacity of the fluid at constant volume and the heat capacity of the particles. Assuming that the particles and fluids are in thermal equilibrium, then the equation of heat conduction given by

$$\frac{\partial T}{\partial t} + (q.\nabla)T + \frac{mNC_{pt}}{\rho_0 C_v}\left(\frac{\partial}{\partial t} + q_d.\nabla\right)T = K_T \nabla^2 T \qquad \ldots\ldots(8)$$

Where v is kinematic viscosity v, μ' is couple-stress viscosity, kT is thermal diffusivity and α is coefficient of thermal expansion which are assumed to be constants.

III. Various Analytical Dicussion Related To The Variables

Stationary Convection

At stationary convection, when the instability sets, the marginal state will be characterized by $\sigma = 0$. Thus, putting $\sigma = 0$ in the equation, we get

$$R_1 = \frac{1+X}{XB}\left[\{F_1(1+x)+1\}(1+x)^2 + Q_1 + \frac{TA_1(1+x)}{[\{F_1(1+x)+1\}(1+x)^2+Q_1]}\right. \qquad \ldots\ldots(9)$$

Which expresses the modified Rayleigh number R1 as a function of the parameters B,F_1, T_{A_1} ,Q_1 and dimensionless wave number x.

To study the stability nature, effect of dust particles, couple-stress and magnetic fields, we examine the behavior of $\frac{dR_1}{dB}$, $\frac{dR_1}{dF_1}$, $\frac{dR_1}{dT_{A_1}}$, and $\frac{dR_1}{dQ_1}$ analytically.

From equation (09), we have

$$\frac{dR_1}{dB} = \frac{1+X}{XB^2}\left[\{F_1(1+x)+1\}(1+x)^2 + Q_1 + \frac{TA_1(1+x)}{[\{F_1(1+x)+1\}(1+x)^2+Q_1]}\right. \qquad \ldots\ldots(10)$$

This confirms that dust particles have a destabilizing effect on a couple - stress rotating dusty fluid on the thermal convection.

From equation (09), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xB^2}\left[1 - \frac{TA_1(1+x)}{[\{F_1(1+x)+1\}(1+x)^2+Q_1]^2}\right] \qquad \ldots\ldots(11)$$

Which shows that Couple - Stress has a stabilizing or destabilizing effect on the thermal convection under the restrictions

T_{A1} (1 + x) > or < [{ F1(1 + x) + 1}$(1+x)^2$ + Q1$]^2$

But, for the accepted values of various parameters, the said effect is stabilizing only if

T_{A_1} (1+x) < [{ F1(1 + x) + 1}$(1+x)^2$ + Q1$]^2$

In the absence of rotation and magnetic field, equation (11) becomes

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{xB} \qquad \ldots\ldots(12)$$

Which confirms that couple-stress has a stabilizing effect on the thermal convection in the absence of rotation and magnetic field as derived by Sharma and Sharma (2004).

Again from equation (09), we have

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)^2}{xB[\{F_1(1+x)+1\}(1+x)^2+Q_1]} \qquad \ldots\ldots(13)$$

Which shows that rotation has a stabilizing effect on the system.

In the absence of magnetic field, equation (13) becomes

$$\frac{dR_1}{dT_{A_1}} = \frac{1}{xB[\{F_1(1+x)+1\}]} \qquad \ldots\ldots(14)$$

Which clearly shows that rotation has a stabilizing effect onthe thermal convection of couple-stress rotating fluid in the absence of magnetic field as derived by Sharma and Sharma (2004).

Again from equation (09), we have

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xB}\left[1 - \frac{TA_1(1+x)}{[\{F_1(1+x)+1\}(1+x)^2+Q_1]^2}\right] \qquad \ldots\ldots(15)$$

T_{A_1} (1+ x) < or > [{ F1(1 + x) + 1}$(1 + x)^2$ + Q1$]^2$

But, for the permissible values of various parameters, theabove effect is stabilizing only if

T_{A_1} (1+x) < [{ F1(1 + x) + 1}$(1+x)^2$ + Q1$]^2$

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In the absence of rotation (T_{A_1}= 0), equation (15) becomes

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Which clearly shows that in the absence of rotation, magnetic field has a stabilizing effect on a couple-stress rotating dusty fluid on the thermal convection.

Oscillatory Convection

Using equations (1) to (8) with the boundary condition, we get

$$\sigma\left(1+\frac{M}{1+\sigma\tau_1}\right) I_1 + I_2 + FI_3 - \frac{g\alpha kTa^2}{v\beta}\left(\frac{1+\sigma^*\tau_1}{B+\sigma^*\tau_1}\right)[I_4 + \sigma^* Bp_1 I_5] + \frac{\mu_e\eta}{4\pi\rho_0 v}[I_6 + \sigma^* p_2 I_7] + d^2\left[\sigma^*\left(1+\frac{M}{1+\sigma^*\tau_1}\right)I_8 + F\, I_9 + I_{10}\right] + \frac{\mu_e\eta d^2}{4\pi\rho_0 v} + [\, I_{11} + \sigma p_2 I_{12}] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Where

$I_1 = \int(\,|DW|^2 + a^2\,|W|^2\,)dz$

$I_2 = \int(\,|D^2W|^2 + 2a^2\,|DW|^2 + a^4\,|W|^2)dz$

$I_3 = \int(\,|D^3W|^2 + 3a^2\,|D^2W|^2 + 3a^4\,|DW|^2 + a^6\,|W|^2)dz$

$I_4 = \int(\,|DQ|^2 + a^2\,|Q|^2\,)dz$

$I_5 = \int(\,|Q|^2\,)dz$

$I_6 = \int(\,|D^2K|^2 + 2a^2\,|DK|^2 + a^4\,|K|^2)dz$

$I_7 = \int(\,|DK|^2 + a^2\,|K|^2\,)dz$

$I_8 = \int(\,|Z|^2)dz$

$I_9 = \int(\,|D^2Z|^2 + 2a^2\,|DZ|^2 + a^4\,|Z|^2)dz$

$I_{10} = \int(\,|DZ|^2 + a^2\,|Z|^2\,)dz$

$I_{11} = \int(\,|DX|^2 + a^2\,|X|^2\,)dz$, and $I_{12} = \int(\,|X|^2\,)dz$,

Where σ* is the complex conjugate of σ. All the integrals I_1 to I_{12} are positive definite, putting σ = iσ, in equation andequating the imaginary parts, we obtain

$$\sigma_I\left[\sigma\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1 + \frac{g\propto k_T a^2}{v\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}\, I_4 + \frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}\, Bp_1\, I_5\right\} - \frac{\mu_e\eta}{4\pi\rho_0 v}\, p_2 I_7 - d^2\left\{1+\frac{M}{1+\sigma_i^2\tau_1^2}\right\}I_8 + \frac{\mu_e d^2\eta}{4\pi\rho_0 v}\, p_2 I_{12} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

In absence of magnetic field and rotation, equation (18) becomes

$$\sigma_I\left[\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1 + \frac{g\propto k_T a^2}{v\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}\, I_4 + \frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}\, Bp_1\, I_5\right\}\right] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(19)$$

It may be inferred from equation (19), it is obvious that all terms in the bracket are positive definite. Thus σi = 0, which means that oscillatory modes are not allowed in the system and Principle of Exchange of Stabilities (PES) is satisfied in the absence of magnetic field and rotation. It is evident form equation (18) that presence of magnetic field and rotation brings oscillatory modes (as, σ_i may not be zero) which were non-existent in their absence.

IV. Discussion On Numerical Computations

The critical thermal Rayleigh number for the onset of instability is determined for critical wave number obtained by using Newton - Raphson method, by means of the condition $\frac{dR_1}{dx}$=0

The numerical values of critical thermal Rayleigh number R1 and critical wave number x determined for various values of dust particles B, magnetic field Q1, couple - stress F1, and rotation T_{A_1}, Graphs have been potted between critical Rayleigh number R_1 and Parameters B, Q_1, F_1 and $T_{A_1}$by substituting some numerical values to them.

International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347-6435(Online) Volume 11, Issue 06, June 2022)

In Figure 1. The critical Rayleigh number R_1 decreases with increase in dust particles parameter B which shows that have dust particles have destabilizing effect on the system that indicates when the critical Rayleigh number R_1 is plotted against dust particles B for fixed value of $F_1 = 10$, $T_{A_1} = 100$ and $Q_1 = 100, 300, 500$.

In Figure 2. The critical Rayleigh number R_1increses with increase in rotation parameter T_{A_1} which shows that rotation has a stabilizing effect on the system whenever the critical Rayleigh number R_1 is potted against rotation parameter T_{A_1} for fixed value of $F_1 = 10$, $B = 20$ and $Q_1 = 100, 400, 700$.

In Figure 3. The critical Rayleigh number R_1 increases with increase in rotation parameter T_{A_1} which shows that rotation has a stabilizing effect on the system when critical Rayleigh number R_1 is potted against rotation parameter T_{A_1} for fixed value of $F_1 = 10$, $Q_1 = 500$, and $B = 5, 10, 15$.

In Figure 4. The critical Rayleigh number R_1 increases with increase in magnetic field Q_1 which shows that magnetic field has a stabilizing effect on the system which indicates the critical Rayleigh number R_1 is potted against magnetic field Q_1 for fixed value of $F_1 = 10$, $B = 20$ and $T_{A_1} = 100, 500, 1000$.

V. Discussion Through Graphs

Dispersion relation governing the effects of dust particles, couple-stress, rotation and magnetic field is derived. The main results from the analysis are depicted graphically and summarized as follows:

(i) For the case of stationary convection, dust particles have a destabilizing effect on the system as can be seen from equation (10), and graphically from Figure 1.

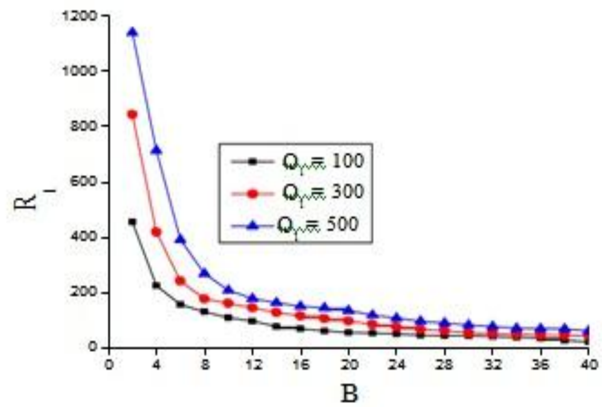

Figure 1: Variation of critical Rayleigh number R_1 with dust particles B for fixed value of $F_1 = 10$, $T_{A_1} = 100$ and $Q_1 = 100, 300, 500$.

(ii) Couple-stress has stabilizing /destabilizing effects on the system for the permissible values of various parameters which can be seen from equation (11). In the absence of rotation, couple-stress clearly has a stabilizing effect on the system as can be seen from equation (12) as derived by Sharma and Sharma (2004).

(iii) For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (13), and graphically, from Figure 2 and Figure 3.

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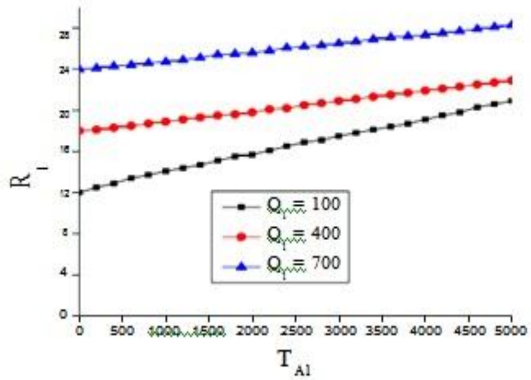

Figure 2: Variation of critical Rayleigh number R_1 with rotation T_{A_1} for fixed value of $F_1 = 10$, $B = 20$ and $Q_1 = 100, 400, 700$

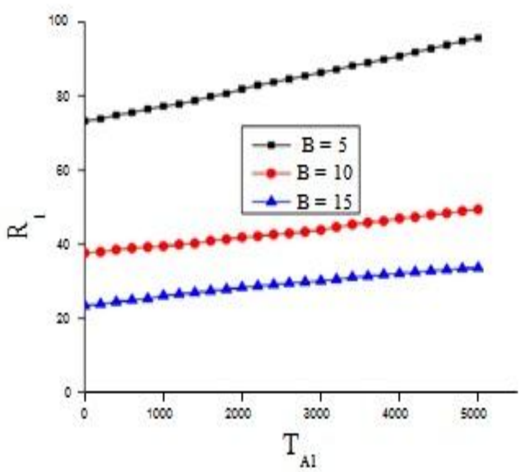

Figure 3: Variation of critical Rayleigh number R_1 with rotation T_{A1} for fixed value of $F_1 = 10$, $Q_1 = 500$, and $B = 5, 10, 15$.

(iv) Magnetic field has stabilizing/destabilizing effect on the system for the permissible values of various parameters as can be seen from equation (15), and graphically from Figure 4.

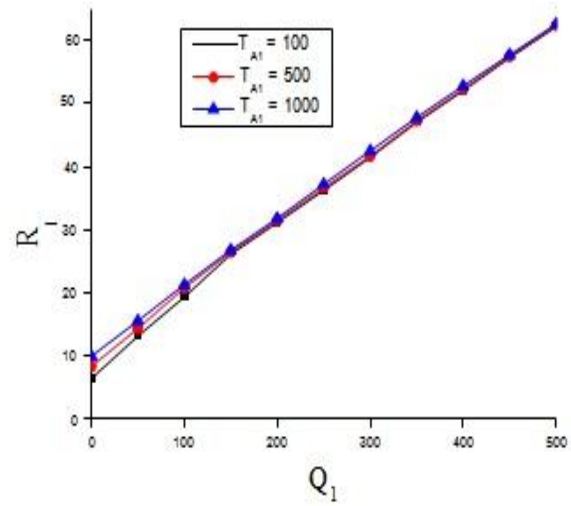

Figure 4: Variation of critical Rayleigh number R_1 with magnetic field Q_1 for fixed value of $F_1 = 10$, $B = 20$ and $T_{A_1} = 100, 500, 1000$.

(v) The Principle of Exchange of Stabilities (PES) is found to hold true in the absence of magnetic field and rotation. It is evident from equation (18) that presence of magnetic field and rotation brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

VI. CONCLUSION

In this paper, the combined effect of dust particles, on a couple - stress rotating dusty fluid heated from below in hydromagnetics is considered. In this analysis, we have investigated the effect of various parameters like dust particles, couple- stress, rotation and magnetic field on the onset of convection through numerical computations and graphs. The main results from the above analysis are listed as

i) Dust particles have destabilizing effect on a couple - stress rotating dusty fluid on the thermal convection.
ii) Couple - Stress has a stabilizing or destabilizing effect on the thermal convection under the restrictions of permissible values of various parameters.

iii) Rotation has a stabilizing effect on the thermal convection of dusty couple-stress rotating fluid.
iv) Magnetic field has a stabilizing/destabilizing effect on the system for the permissible values of various parameters and in the absence of rotation, magnetic field has a stabilizing effect on a couple-stress rotating dusty fluid on the thermal convection.

Acknowledgement

The authors and thankful to Prof. (Dr.) S.N. Jha, Ex. Head, Prof.(Dr.) P.K. Sharan Ex. Head, Prof.(Dr.) B.P. Singh, Ex. Head, Prof.(Dr.) Amita Sharma present Head of the University Deptt. Of Mathematics, Cum. C.C.D.C.B.R.A.B.U. Muzaffarpur, Bihar India and Prof. (Dr.) T.N. Singh, Ex. Head, Ex. Dean(Science), Ex. Chairman, Research Development Council, B.R.A.B.U. Muzaffarpur, Bihar, India, for extending all facilities in the completion of the present research work.

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