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Adaptive Signal Processing in Time Domain

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***Abstract*— The biggest challenges faced in processing the digital signal is overcome by the adaptive signal processing .In adaptive signal processing the signal is processed through large number of adaptive filter which overcomes the slope overload error and granular noise witnessed in delta modulation approach. The adaptive signal processing is a advance method which find it's application to the stationary signals or the parameters to be established are time varying . Few examples**

- **application in echo cancellation**
- **equalization of data communication channel in mobile communication.**
- **Time varying systems identification**

This paper discusses the various methods to implement the adaptive signal processing in time domain . The factors taken into consideration for discussion are design,analysis and implementation of system whose structure changes in response to the incoming data .

***Keywords*— LMS Least Mean Method, FIR Finite Impulse Response, IIR Infinite Impulse Responce**

I. Introduction

Adaptive signal processing is concerned with design analysis and implementation of system whose structure changes in response to incoming data. An adaptive filter is a time variant filter whose coefficients are adjusted in away to optimise a cost function or to satisfy predetermined optimization criteria .the characteristics of adaptive filters are 1] they can automatically adapt in face of changing environments and changing systems requirement.
2] they can be trained to perform specific filtering and decision making tasks according to some updating equations.

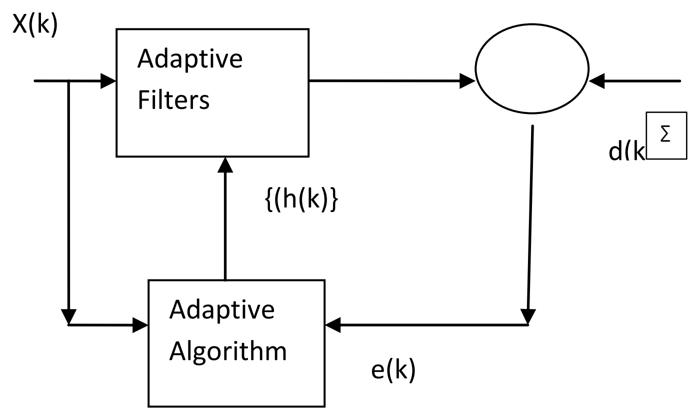

Figure1. Block diagram of adaptive filter .

In figure1 x(k) is the input signal,y(k) is the output signal ,d(k) is the desired response, h(k) is the impulse response of the adaptive filter. The relation between x(k) and d(k) can vary with time . In such situation the adaptive filter attempts to alter its value to follow the changes in this relationship as encoded by the two sequences x(k) and d(k).This behaviour is commonly referred to as tracking.

The adaptive filter can be classified in two types –FIR Filter and IIR filters .

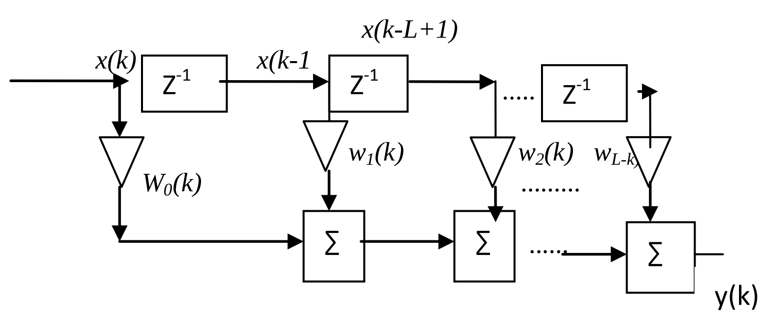

Figure2. structure of an FIR filter.

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The Figure 2 represents the direct form of Finite impulse response filter where z^{-1} represents the unit delay elements and $w_i(k)$ is a multiplicative gain within the system and in this case it corresponds to the impulse response values of the filter at time k. Output signal y(k) is given as

$$y(k) = \sum_{i=0}^{L-1} w_i\,(k)x(k-1) \qquad 1.1$$

$$= w^T(k)X(k)$$

where $X(k) = [x(k)x(k-1)................x(k-L+1)]^T$

denotes the input signal vector and t denotes vector transpose . The system requires L Multiplies and L-1 adders to implement , and these computations are easily performed by a processor or circuit so long as L is large and the sampling period for the signals is not too short . It also requires a total of 2L memory locations to store the l input signals samples and the coefficients values , respectively.

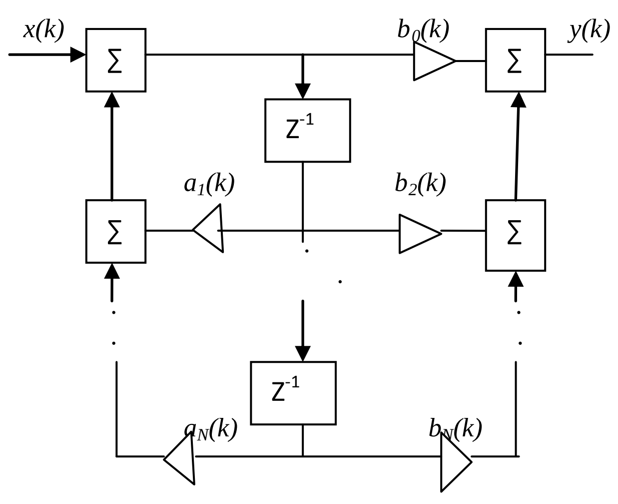

Figure3. Structure of IIR filter.

Firure3 represents the Infinite Impulse response structure . the system output is given by

$$y(k) = \sum_{i=1}^{N} a_i\,(k)y(k-1) + \sum_{j=0}^{N} b_j\,(k)x(k-j) \qquad 1.2$$

For the purpose of computing the output signals y(k), the IIR structures involves a fixed number of multiplers, adders and memory location.

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II. Method

2.1Design considerations

2.1.1 cost consideration

Choice of cost function depends on the approach used and the application of interest. The commonly used cost function- mean square error criterion : minimizes $E\{e^2(K)\}$ where E denotes expectation operation *e(k)=d(k) – y(k)* is the estimation error d(k0 is the desired response and y(k)is the actual filter output.

Exponentially weighted least square criterion minimizes factor

$$\sum_{k=0}^{k=N-1} \lambda^{N-1-k} e^2\,(k)$$

where N is the total number of samples λ denotes the exponentially weighting factor whose value is positively close to 1.

2.2Algorithm

Depends on cost function used ,convergence of algorithm, rate of convergence, misadjustments(the performance measure for algorithm that uses minimum MSE criterion),tracking capability-refers to the ability of the algorithm to track statistical variation in a non stationary environment ,computational requirement which considers the factors like number of operation ,memory size, investment required to program the algorithm on computers.

2.3 Structures

Structures and algorithm are interrelated .Choice of structure is based on quantization error ,ease of implementation and computational complexity.The commonly used structures are direct form, cascade form, parallel form and lattice structure include simple test for filter stability modular structure and low sensitivity to quantization effect.

Methods for minimizing MSE

The adaptive filter assumed here is causal FIR type for simplicity and implemented in direct form. The system block diagram is given in figure 4.

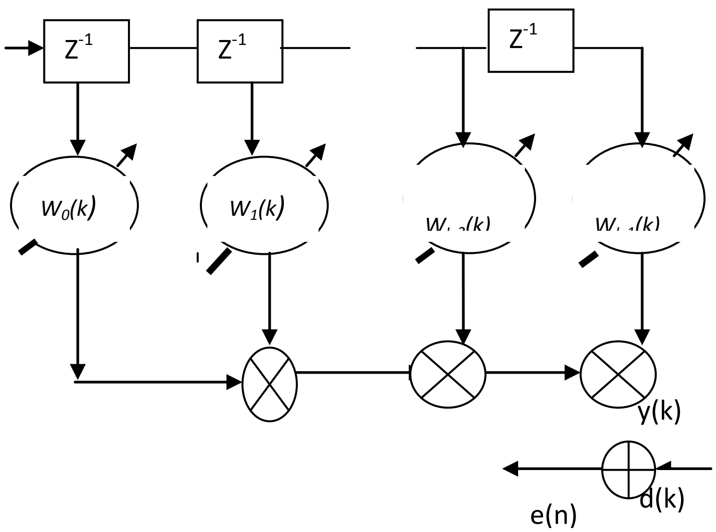

Figure 4. Causal adaptive FIR filter.

The error signal at time n is given by

$$e(k) = d(k) - y(k) \quad 2.1$$

$$y(k) = \sum_{i=0}^{L-1} w_i\,(k)x(k-1) \quad 2.2$$

$W(k)=[w_0\ w_1\ \ w_{L-2}\ \ w_{L-1}]^T$ 2.3

$X(k)=[x(k)\ x(k-1)\ \ x(k-L+2)\ \ x(k-L+1)]^T$ 2.4

Minimizing $E\{e^2(k)\}$ will give the Wiener solution in optimal filtering

$$\lim_{n\to\infty} \underline{W}(n) = \underline{W}_{MMSE} = (\underline{R}_{xx})^{-1} \cdot \underline{R}_{dx} \quad 2.5$$

In adaptive filtering, the Wiener solution is found through an iterative procedure

$$\underline{W}(n+1) = \underline{W}(n) + \Delta\underline{W}(n) \quad 2.6$$

Where $\Delta\underline{W}(n)$ is an incrementing vector.

Two common gradient searching approaches for obtaining Wiener filter are

1. *Newton method*

$$\Delta w(k) = \mu R_{xx}^{-1}. - \left\{\frac{\delta E\{e^2(n)\}}{dW(n)}\right\} \quad 2.7$$

μ is the step size .It is the positive number that controls the convergence rate and stability of the algorithm. The adaptive algorithm becomes

$$W(k+1) = W(k) - \mu R_{xx}^{-1}. \left\{\frac{\delta E\{e^2(k)\}}{dW(k)}\right\} \quad 2.8$$

$$w(k+1) = (1-2\mu)w(k) + 2\mu WMSE \quad 2.9$$

$$w(k) = w_{MMSE} + (1-2\mu)^n(w(0) - w_{MMSE}) \quad 2.10$$

Where μ is $0<\mu<1$.

2. *Steepest Descent Method*

$$\Delta w(k) = -\mu \left\{\frac{\delta E\{e^2(n)\}}{dW(n)}\right\} \quad 2.11$$

$$w(k+1) = w(k) - \mu \left\{\frac{\delta E\{e^2(n)\}}{dW(n)}\right\} \quad 2.12$$

$$w(k+1) = (I - 2\mu R_{xx})(w(n) - w_{MMSE}) + w_{MMSE} \quad 2.13$$

Where *I* is the identity matrix.

2.3 Widrow's Least Mean Square(LMS) Algorithm

2.3.1 Optimization Criterion is to minimize mean square error .

Mean square error =MSE=$E\{e^2(K)\}$

2.3.2 Adaptation Procedure

It is an approximation of steepest decent methods where the expectation operator is replaced by the factor $\frac{\delta\ e^2}{\delta W}$

2.3.3 Performance analysis

Two important performance measure in LMS algorithm are rate of convergence and misadjustment (relates to steady state filter weight variance).

Convergence Analysis

$$E\{W(K+1)\} = E\{W(k) + 2\mu E\{e(k)X(k)\} \quad 2.14$$

$$E\{W(k+1)\} = E\{W(k)\} + 2\mu E\{d(k)X(k) - X(k).X^T(k)W(k)\} \quad 2.15$$

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$$E\{W(k+1)\} = E\{W(k)\} + 2\mu R_{xx}E\{W(k)\} \quad 2.16$$

$$E\{W(k+1)\} = (I - 2\mu R_{xx})E\{W(k)\} + 2\mu R_{xx}W_{MMSE}$$

The above analysis shows that convergence analysis is almost similar to steepest Decent method.

For the above discussion the W(k) will converge to Wiener filter weights if

$$\begin{bmatrix} \lim_{n\to\infty}(1-2\mu\lambda_1)^n & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lim_{n\to\infty}(1-2\mu\lambda_L)^n \end{bmatrix} \quad 2.17$$

$$|1 - 2\mu\lambda_t| < 1 \qquad i = 1,2,\dots\dots,L \quad 2.18$$

$$0 < \mu < \frac{1}{\lambda_{max}}$$

Define the geometric ratio of the p^{th} path

$$r_p = 1 - 2\mu\lambda_p \qquad p = 1,2,\dots\dots,L \quad 2.19$$

Each term in the main diagonal forms a geometric series

$$\{1, r_p^1 \dots\dots\dots r_p^{n-1}, r_p^n, r_p^{n+1}, \dots.\} \quad 2.20$$

The exponential can be fitted to each geometric series

$$r_p \approx e^{\frac{-1}{\tau p}} \quad 2.21$$

τ_p is called the p^{th} time constant for slow adaptation

$$\tau_p \approx \frac{1}{2\mu\lambda_p} \quad 2.22$$

The smaller the time constant the faster is the convergence rate. The overall convergence is limited by the slowest mode of convergence which in terms appears from the smallest eigen value R_{xx} and λmin.

In general the rate of convergence depends on step size µand the eigenvalue spread of R_{xx} and $\chi(R_{xx})$.

Misadjustment

On convergence if $\lim_{n\to\infty} W(k) = W_{MMSE}$ themn minimun MSE will be equal to

$$\varepsilon_{min} = E\left\{d^2(k)\right\} - R_{dx}^T W_{MMSE} \quad 2.23$$

This does not occur due to random noise in the weight vectors *W(k)*.the MSE of LMS is given by

$$E\{e^2(k)\} = E\{(d(k) - W(k)^T X(k))^2 \quad 2.24$$

$$E\{e^2(k)\} = e_{min} + E\{(W(k) - W_{MMSE})^T.(X(k).X(k)^T).(W(k) - W_{MMSE})\} \quad 2.25$$

$$E\{e^2(k)\} = e_{min} + E\{(W(k) - W_{MMSE})^T R_{xx}(W(k) - W_{MMSE})\} \quad 2.26$$

Excess MSE is given by

$$excess\ MSE = \lim_{n\to\infty} E\{(W(k) - W_{MMSE})^T R_{xx}(W(k) - W_{MMSE})\} \quad 2.27$$

$$excess\ MSE = \lim_{n\to\infty}\{V(k)^T.R_{xx}V(k)\} \quad 2.28$$

$$excess\ MSE = \lim_{n\to\infty}\{U(k)^T.R_{xx}\wedge U(k)\} \quad 2.29$$

$$excess\ MSE = \mu\varepsilon_{min}\sum_{i=0}^{L-1}\lambda_t = \mu\varepsilon_{min}tr[R_{xx}] \quad 2.30$$

Where $tr[R_{xx}]$ is the trace of R_{xx} which is equal to sum of all diagonal elements

$$tr[R_{xx}] = LR_{xx}(0) = LE\{x^2(k)\}$$

The Misadjustment M is given by

$$M = \frac{\lim_{k\to\infty} E\{e^2(k)\} - \varepsilon_{min}}{\varepsilon_{min}} \quad 2.31$$

$$M = \frac{\mu\varepsilon_{min}L.E\{x^2(k)\}}{\varepsilon_{min}} \quad 2.32$$

$$M = \mu\, L.E\{x^2(k)\} \quad 2.33$$

Which is proportional to step size, filter length and signal power.

3. LMS Variant

3.1Normalized LMS algorithm

Step1. The product vector *e(k)X(k)* is modified with respect to the squared Euclidian norm of the tap input vector X(k).

$$W(k+1) = W(k) + \frac{2\mu}{c + X^T.X(k)}e(k)X(k) \quad 2.34$$

Where c is a small positive constant to avoid division by 0.

Step2. Represents an LMS algorithm by varying the step size

$$\mu(n) = \frac{2\mu}{c + X^T.X(k)} \quad 2.35$$

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Substituting c=0 the normalized LMS algorithm converges if $0<\mu<0.5$,selection of step size is much easier than that of LMS algorithm.

3-2 Sign Algorithm

In high speed algorithm the time is critical ,thus the faster adaptation process is needed.

Sgn(a)={1;a>0

{0; a=0

{-1; a<1

3.2.1 Pilot LMS

The weighted coefficient are found using the following relation

$$W(k+1) = W(k) + 2\mu sgn[e(k)]X(k) \qquad 2.36$$

3.2.2 Clipped LMS

The weighted coefficient are found using the relation

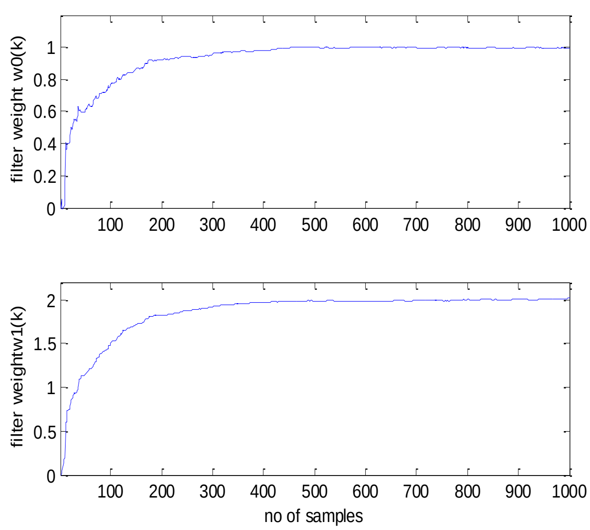
$$W(k+1) = W(k) + 2\mu e(k)sgn[X(k)] \qquad 2.37$$

3.2.3Zero forcing LMS

The weighted elements are represented using the following relation

$$W(k+1) = w(k) + 2\mu sgn[e(k)sgn[X(n)] \qquad 2.38$$

III. RESULTS



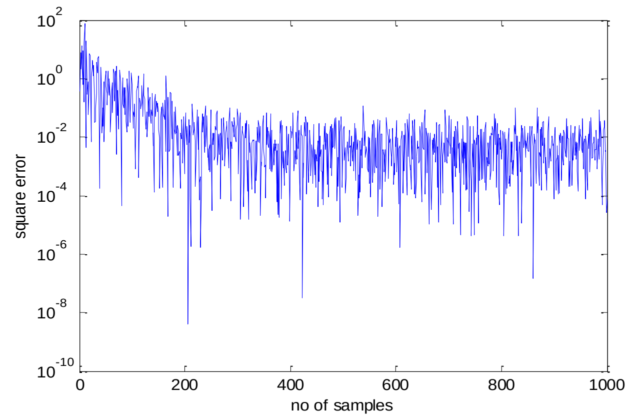


Figure 5 Convergence of filter weights

Figure 5 shows that the filter weight *w0(k)* and *w1(k)* converge at same speed and eigen values of R_{xx} are same. The square error signal e(k) is shown .

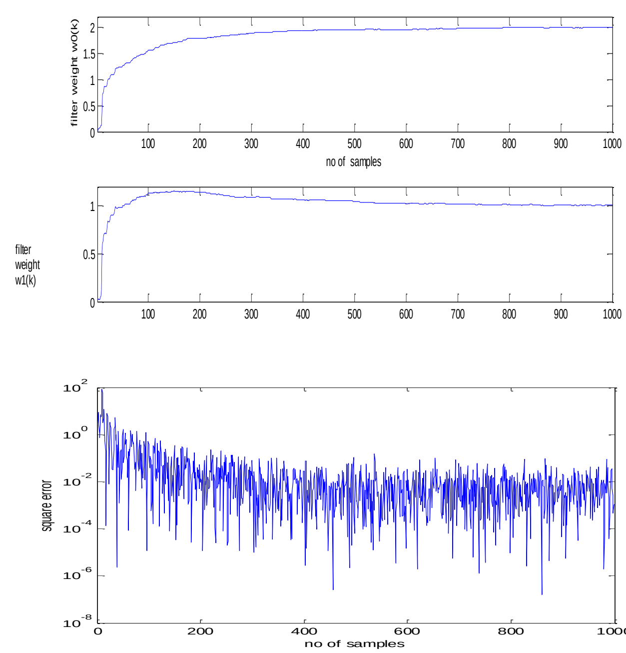


Figure 6 Convergence of filter weights .

Figure 6 shows that the filter weight w1(k) will converge at faster speed with that of filter weight w0(k). The eigen values $\lambda_{min}=o.5$ and $\lambda_{max}=1.8$

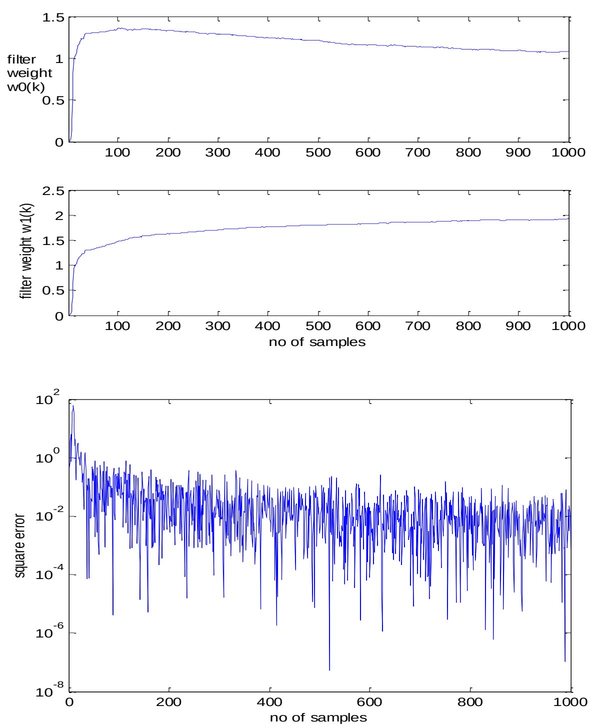


Figure 7 Convergence of Filter weights

Figure 7 shows that the filter weights *w0(k) and w1(k)* converges at the slower speed though the *w1(k)* is faster which shows that the eigen values $\lambda_{min}=o.2$ *and* $\lambda_{max}=1.8$.

IV. Conclusion

The various algorithm for Least Mean Square Error are discussed and analysed. The analysis of the signal considered is in time domain.

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