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A Study on Complex Wavelet Transform and Its Application to Image Denoising

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***Abstract*— This work addresses to a study on the wavelet transform and complex wavelet transform to demonstrate the capabilities of them in digital image denoising application. Standard Discrete Wavelet Transform (DWT) has some drawbacks like shift sensitivity, poor directionality and absence of phase information. To avoid these limitations, Complex Wavelet Transform (CWT) can be used. Complex Wavelet Transform is an extension of standard Discrete Wavelet Transform. In this paper, the capability of Complex Wavelet Transform is discussed in the process of noise removal from different images.**

***Keywords*— Discrete Wavelet Transform, Complex Wavelet Transform, Image Denoising, Thresholding, Peak Signal-to-Noise Ratio.**

I. INTRODUCTION

The need for efficient image restoration methods has grown with the massive production of digital images and movies of all kinds, often taken in poor conditions. No matter how good cameras are, an image improvement is always desirable to extend their range of action [1].

Many types of noises due to sensor or channel transmission errors often corrupt images and noise suppression becomes a particularly delicate and a difficult task [4], [7]. Applied noise removal techniques should take into account a trade-off between noise reduction and preservation of actual image content in a way that enhances the diagnostically relevant image content.

The two main limitations in image accuracy are categorized as blur and noise. Blur is intrinsic to image acquisition systems, as digital images have a finite number of samples and must satisfy the Shannon–Nyquist sampling conditions. The second main image perturbation is noise. There are different types of noises that can affect an image. Some of them are:

A. Salt and pepper noise

It is the type of noise where some black and white pixels occurs randomly on an image. A false saturation gives a white spot (salt) and a failed response gives a black spot in the image (pepper) [23], [24].

B. Gaussian white noise

This is the most common type of noise [14], [15], [23], [24] which can be generated artificially using the formula

$$Y = X + \text{sqrt}(\text{variance}) \times \text{random}(s) + \text{mean}; \qquad (1)$$

Where, X is the input image, Y is the output image, s is the size of X. The value of mean and variance is taken as input.

C. Poisson noise

In probability theory and statistics, Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed interval of time and/or space. If the expected number of occurrences in a particular time interval is λ, then probability that there are exactly k (k = 0, 1, 2 …) occurrences is given by

$$f(k,\lambda) = \frac{\lambda^k e^{-\lambda}}{k!} \qquad (2)$$

D. Speckle noise

Within each resolution cell, a number of elementary scatters reflect the incident wave towards the sensor. The received image is thus corrupted by a random granular pattern, called Speckle. A speckle noise can be modelled as

$$v = f\vartheta \qquad (3)$$

Where, v is the speckle noise, f is the noise-free image and ϑ is a unit mean random field. In this paper, the experimental work is done with Gaussian white noise [15].

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In the field of Image Processing, the wavelet transform has emerged with a great success [2], [3]. The complex wavelet transform is a specific area of wavelet transform which has so many advantages over discrete wavelet transform [5].

II. IMAGE DENOISING

The image and noise model is given as:

$$x = s + \sigma . g \tag{4}$$

Where, s is an original image and x is a noisy image corrupted by additive white Gaussian noise g of standard deviation σ. Both images s and x are of size N by M (mostly M = N and always power of 2) [12], [13], [18] [19].

A. Basic steps for image denoising

The following block diagram (Fig. 1) shows the basic steps involved in image denoising in this paper.

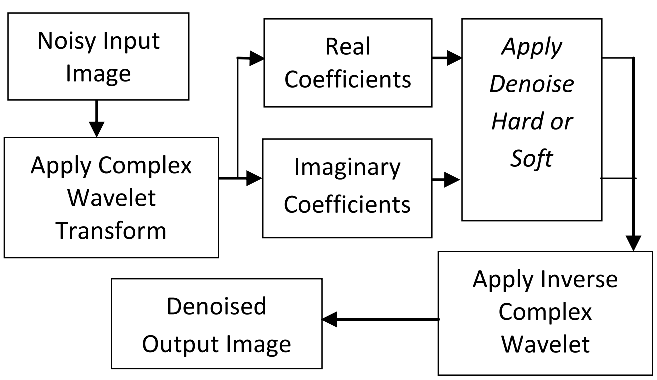


Fig. 1 Basic Steps for Image Denoising

B. Performance Measure

The performance of various denoising algorithms is quantitatively compared using MSE (mean square error) [4], [6] and PSNR [9] (Peak Signal to Noise Ratio) as

$$MSE = \frac{1}{NM}\sum_{n=1}^{N}\sum_{m=1}^{M}|s(n,m) - y(n,m)|^2 \tag{5}$$

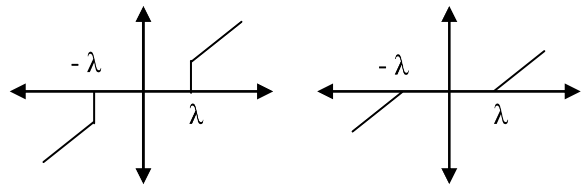
$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \tag{6}$$

Where, s is an original image and y(n,m) is a recovered image from a noisy image s(n,m).

C. Determination of Threshold

The standard thresholding of wavelet coefficients is governed mainly by either 'hard' or 'soft' thresholding function [2] as shown in figure 2.

The first function in Fig. 2(a) is a 'hard' function, and Fig. 2(b) is a 'soft' function [11].



(a) (b)

Fig. 2 Thresholding functions; (a) hard, (b) soft

The hard thresholding function is given as

$$z = hard(w) = \begin{pmatrix} w, \ for\ |w| > \lambda \\ 0, \ for\ |w| \le \lambda \end{pmatrix} \tag{7}$$

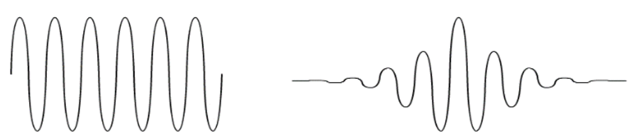
Similarly, soft thresholding function is given as [14]

$$z = soft(w) = \begin{pmatrix} signum(w) \times max(|w| - \lambda, 0), \ for\ |w| > \lambda \\ w, \ for\ |w| \le \lambda \end{pmatrix} \tag{8}$$

Where, w and z are the input and output wavelet coefficients respectively, λ is a selected threshold value for both (7) and (8).

III. WAVELET TRANSFORM

The term wavelet means a small wave. The smallness refers to the condition that this (window) function is of finite length (compactly supported). The wave refers to the condition that this function is oscillatory [21], [22].



(a) (b)

Fig. 3 Representation of a wave (a), and a wavelet (b)

The wavelet transform (WT) is a powerful tool of signal processing for its multiresolutional possibilities [22]. Unlike the Fourier transform, the WT is suitable for handling the non-stationary signals – variable frequency with respect to time.

D. Continuous Wavelet Transform (CoWT)

For a prototype function ψ (t) ∈ L2(ℜ) called the mother wavelet, the family of functions can be obtained by shifting and scaling this ψ (t) as [21], [22]

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$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right) \tag{9}$$

Where, a, b $\in \Re$, (a > 0). The CoWT of a function f (t) $\in \Re$ is then defined as

$$CoWT_f\ (a,b) = \int_{-\infty}^{\infty} \Psi_{a,b}^{*}(t) f(t) dt = \langle \Psi_{a,b}(t) f(t) \rangle \tag{10}$$

Since, the CoWT behaves like orthonormal basis decomposition, it is isometric and it preserves energy [22]. Hence the function f(t) can be recovered from its transform by the following reconstruction formula

$$f(t) = \frac{1}{C_\Psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} CoWT_f\ (a,b)\, \Psi_{a,b}(t) \frac{da\,db}{a^2} \tag{11}$$

E. Discrete Wavelet Transform (DWT)

The discrete wavelet transform (DWT) is a linear transformation that operates on a data vector whose length is an integer power of two, transforming it into a numerically different vector of the same length [10]. It separates data into different frequency components, and then matches each component with resolution to its scale. DWT is computed with a cascade of filters followed by a factor 2 subsampling (Fig. 4).

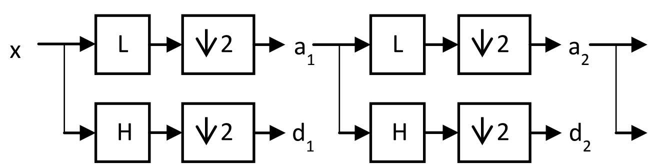


Fig. 4 DWT Tree

H and L denotes high and low-pass filters respectively, ↓2 denotes subsampling. Outputs of these filters are given by equations (12) and (13).

$$a_{j+1}[p] = \sum_{n=-\infty}^{+\infty} l[n-2p]\,a_j(n) \tag{12}$$

$$d_{j+1}[p] = \sum_{n=-\infty}^{+\infty} h[n-2p]\,a_j(n) \tag{13}$$

Elements a_j are used for next step (scale) of the transform and elements d_j, called wavelet coefficients, determine output of the transform. l[n] and h[n] are coefficients of low and high-pas filters respectively. Assume that on scale j+1 there is only half from number of a and d elements on scale j.

DWT algorithm for two-dimensional pictures is similar. The DWT is performed firstly for all image rows and then for all columns (Fig. 5).

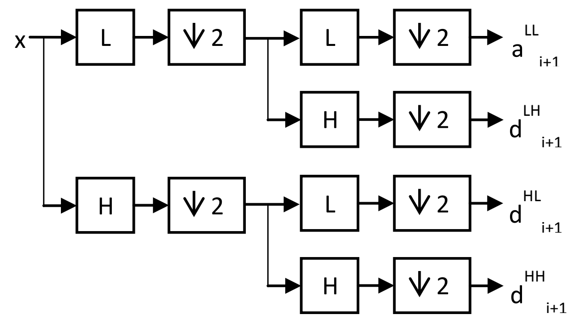


Fig. 5 Wavelet decomposition for two-dimensional pictures

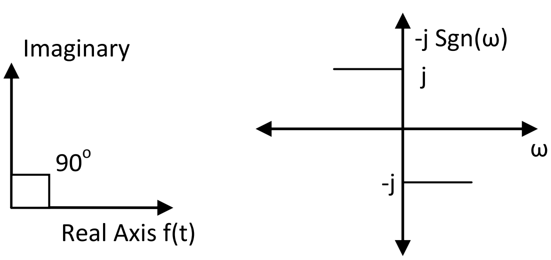
F. Complex Wavelet Transform (CWT)

Complex wavelets can provide both shift invariance and good directional selectivity [5], [8], [16]. The dual tree CWT can be used for signal and image processing applications, including motion estimation, denoising, texture analysis and synthesis, and object segmentation [17].

1) *Analytic Filter*: Gabor introduced the Hilbert transform into signal theory, by defining a complex extension of a real signal f(t) as

$$x(t) = f(t) + j\,g(t) \tag{14}$$

Where, g(t) is the Hilbert transform of f(t) and denoted as H{f(t)} and j = √(-1) [17].



(a) **(b)**

Fig. 6 Hilbert Transform in (a) polar form, (b) frequency domain

The signal g(t) is the 90° shifted version of f(t) as shown in Fig. 6(a). The real part f(t) and imaginary part g(t) of the analytic signal x(t) are also termed as the 'Hardy Space' projections of original real signal f(t) in Hilbert space. Signal g(t) is orthogonal to f(t). In the time domain, g(t) can be represented as [17]

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$$g(t) = H\{f(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(t)}{t-\tau}d\tau = f(t) * \frac{1}{\pi t} \qquad (15)$$

If F(ω) is the Fourier transform of signal f(t) and G(ω) is the Fourier transform of signal g(t), then the Hilbert transform relation between f(t) and g(t) in the frequency domain is given by

$$G(\omega) = F\{H\{f(t)\}\} = -jSgn(\omega)F(\omega) \qquad (16)$$

Where, -j Sgn(ω) is a modified 'signum' function as shown in Fig 6(b). This analytic extension provides the instantaneous frequency and amplitude of the given signal x(t) as

$$Magnitude\ of\ x(t) = \sqrt{f(t)^2 + g(t)^2} = f(t)\cos\theta + g(t)\sin\theta \qquad (17)$$

$$Angle\ of\ x(t) = \theta = \tan^{-1}\frac{g(t)}{f(t)} \qquad (18)$$

This is shown in Fig 7.

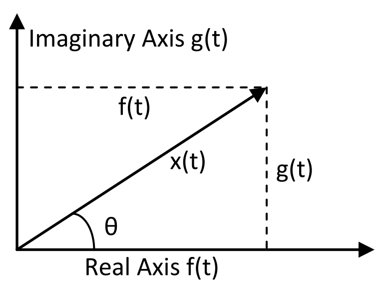

Fig. 7 Magnitude and Angle of x(t)

The formulation and interpretation of the analytic filter is shown in Fig 8.

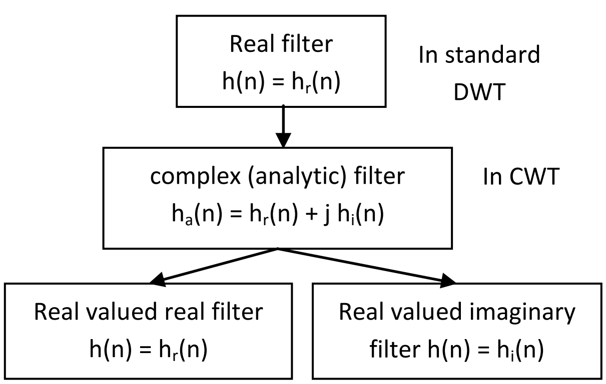

Fig. 8 The formulation and interpretation of the analytic filter

2) *Filterbank Structure of Dual-Tree DWT based CWT:* The filterbank structures for both DT-DWTs are identical. One tree is called as a real tree and other is called as an imaginary tree.

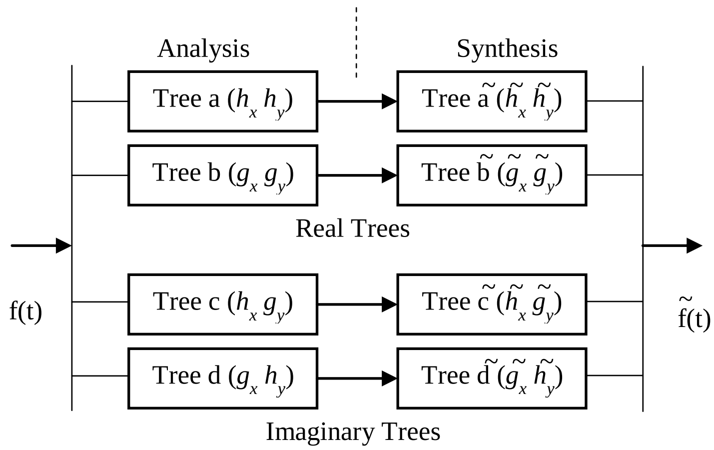

Fig. 9 Filterbank structure for 2-D DT-DWT

The filterbank structure of tree-a, similar to standard 2-D DWT(as shown in Fig. 5). All other trees- (b,c,d) have similar structures with the appropriate combinations of filters for row- and column- filtering. The tree-a and tree-b form the real pair, while the tree-c and tree-d form the imaginary pair of the analysis filterbank. Trees-(~a, ~b) and trees-(~c, ~d) are the real and imaginary pairs respectively in the synthesis filterbank similar to their corresponding analysis pairs.

IV. Experimental Results And Discussion

First of all, a clear noise free image is taken and Gaussian White Noise is added into it to get the noisy image. An example is shown in Fig. 10. This noisy image is taken as the input for image denoising using CWT.

After applying denoising technique, shown in Fig 1, the denoised images are obtained. Some of them are shown in the Fig 11 and 12.

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(a) (b)

Fig. 10 Experimental Image of lena (256×256)
(a) Original, (b) After adding Gaussian White Noise of σ=2, variance=30

(a) (b)

Fig. 11 Denoised Image using *hard* threshold (a)upto level 1, (b)upto level 8

(a) (b)

Fig. 12 Denoised Image using *soft* threshold (a)upto level 1, (b)upto level 8

The PSNR value is calculated from equation (5) and (6). The results obtained from our experiment is shown in table I.

TABLE I
PSNR VALUES FOR IMAGES OF SIZE 256×256

Image	**Th. Type**	**Wavelet Decomposition Level**			
		2	**4**	**6**	**8**
lena	Hard	33.0080	32.8678	32.8673	32.8673
	Soft	25.6550	24.3593	24.1473	24.1262
boat	Hard	33.0000	32.8879	32.8874	32.8874
	Soft	25.3327	24.1045	23.9044	23.8845
goldhill	Hard	32.7035	32.5951	32.5943	32.5942
	Soft	25.4022	24.1575	23.9550	23.9349

From the PSNR values shown in table I, it is very much clear that, as we increase the wavelet decomposition level, PSNR value gradually decreases.

(a) (b) (c)

Fig. 13 Boat Image 400% magnified (a)noisy image, (b)denoised image using hard threshold, (c) denoised image using soft threshold

V. CONCLUSION

In this paper, the advantages, applications, and limitations of popular standard DWT and its extensions are realized. Complex Wavelet Transforms (CWT), a powerful extension to real valued WT is investigated to reduce the major limitations of standard DWT and its extensions in certain signal processing applications.

The history, basic theory, recent trends, and various forms of CWTs with their applications are collectively and comprehend-sively analysed. Recent developments in CWTs are critically compared with existing forms of WTs. Potential applications are investigated and suggested that can be benefited with the use of different variants of CWTs.

Individual software codes are developed for simulation of selected applications such as Denoising both WTs and CWTs. The performance is statistically validated and compared to determine the advantages and limitations of CWTs over well-established WTs. Promising results are obtained using individual implementation of existing algorithms incorporating novel ideas into well-established frameworks.

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