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Implementation of **Graph^{Ne} Network** and **fuzzy^{PN}** Optimization in Human Resource Allocation

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Abstract-- **This study combines Fuzzy optimization and Graph Neural Network (G^{Ne}N) to improve decision-making in Human Resource Management (HRM) when it comes to assigning employees to tasks, evaluating performance, and making recruitment selection, Here we use Fuzzy pentagonal numbers to offer a more easy way to express linguistic evaluations like "poor, average, good, very good, and excellent" criteria's. We use the HRM system as a heterogeneous graph, where employees, tasks, and departments are represented as nodes, while the relationships between skills and workload dependencies are the edges. Each node and edge compared with FPN-based attributes in evaluations and preferences. So we propose a hybrid framework that combines Graph Neural Networks (G^{Ne}N) with fuzzy pentagonal number (FPN)-based optimization. Finally, an FPN-based multi-objective optimization model is applied to derive optimal HRM decisions using Sigmoid Function such as employee-task assignments, maximizing skill-task fit and minimizing workload imbalance. This integrated approach enables more realistic, data-driven, and uncertainty-aware decision-making in HRM systems. Numerical example will illustrate the study taken as the HR selection processing between three different companies as a framework for optimization on finding best company in it**

Keywords-- **Pentagonal fuzzy number, Graph Neural network, Weighted Average Method, Sigmoid Function, Optimal Solution**

I. INTRODUCTION

In the organizational management approach , Human Resource Management (HRM) plays a pivotal role in employee engagement, workforce planning, and talent acquisition so organization increasingly rely on data-driven decision-making, through artificial intelligence (AI) and machine learning (ML) methodologies [1]. These approaches effectively capture the complex relational structures among employees and to handle the ambiguity inherent in human-centered data. In our study we make use of Graph Neural Networks (G^{Ne}N) [2] which is a powerful tool nowadays designed to operate on graph-structured data, making them for modeling organizational structures like communication networks, team dynamics, and collaborative relationships within a workplace by representing employees as nodes and their interactions as edges, GNNs can contribute HRM tasks such as employee performance prediction, attrition risk analysis, and optimal team formation. Despite the predictive strength of G^{Ne}N, [3] HR decision-making frequently involves uncertain and Imprecise information, So we undergo fuzzy Optimization,[4] particularly using Pentagonal Fuzzy Numbers (PFNs), provides a mathematical framework that models vagueness and ambiguity more effectively than traditional fuzzy sets[5]. This study presents a hybrid approach that integrates capabilities of G^{Ne}N with a Fuzzy PN–Optimization framework to improve decision-making processes in HRM and applying fuzzy optimization techniques to manage uncertainty and enhance both the accuracy and interpretability of HR analytics.

1.1 Objectives of study:

- Modeling HR data as a graph to capture employee relationships.
- Applying G^{Ne}N for prediction tasks.
- Designing a fuzzy PN–optimization model for decision-making
- Evaluating the hybrid model to perform against traditional HR analytics methods.

II. PRELIMINARIES

2.1 Fuzzy set

A fuzzy set F in the universe of discourse $\mathbf{X}$ defined as the set of ordered pairs $\ddot{A} = \{(x, \mu_A(x)); x \in X\}$ and $\mu_{\ddot{A}}$: $\mathbf{X} \to [0,1]$ is the membership function of fuzzy set F of $x \in X$

2.2 Fuzzy Number

A fuzzy number is a fuzzy set **F** defined on a set of real number R in which the memberships consist of Normal, Convex, and Upper Semi-Continuous in $X \in R$

2.3 Pentagonal Fuzzy Number – P^N

A Pentagonal Fuzzy number denoted by $Pf_N = \{\delta_1, \delta_2, \delta_3, \delta_4, \delta_5\}$ where $\delta_i \in$ R and its membership function $\mu_{A(X)}$ given below

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$$\mu_{\ddot{A}}(x) = \begin{cases} 0, & x < \delta_1 \\ \frac{1}{2}\frac{(x-\delta_1)}{(\delta_2-\delta_1)} & \delta_1 \le x \le \delta_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-\delta_2)}{(\delta_3-\delta_2)} & \delta_2 \le x \le \delta_3 \\ 1 & x = \delta_3 \\ \frac{1}{2} + \frac{1}{2}\frac{(\delta_4-x)}{(\delta_4-\delta_3)} & \delta_3 \le x \le \delta_4 \\ \frac{1}{2}\frac{(\delta_5-x)}{(\delta_5-\delta_4)} & \delta_4 \le x \le \delta_5 \\ 0 & x > \delta_5 \end{cases}$$

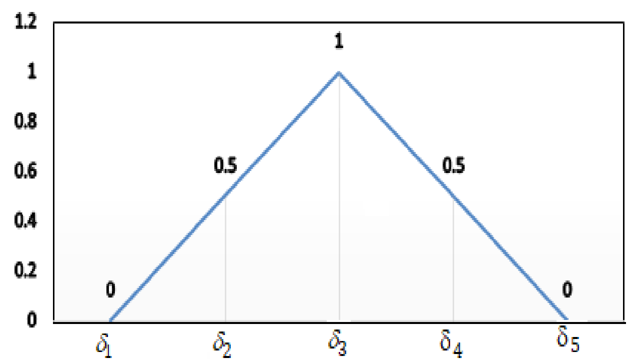


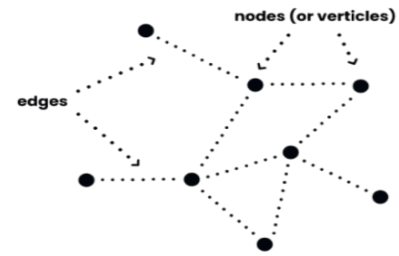
Fig 2.3 Structural Pentagonal Fuzzy Number

2.4 Neural Networks (NNs):

A Neural network is a simple processing unit that receives input, perform calculations, and pass the results. Its interconnected nodes (artificial neurons) in layers to process data, learn patterns, and make decisions or Predictions. It consists of three layered structured (input, hidden, output) and adjust connection strengths (weights and biases) to find complex relationships in data, forming the core of deep learning and powering modern AI applications. when it comes to HRM it analyze employee data to predict outcomes, automate tasks, and provide data-driven insights for better decision-making in recruitment.

2.5 Graph Neural Networks ($G^{Ne}N$)

Graph Neural Networks **($G^{Ne}N$)** is a type of machine learning software designed to analyze data presented in the form of a graph (i.e.) based on two key components of the input graph layer nodes (also known as vertices or points) and edges (the lines connecting them).



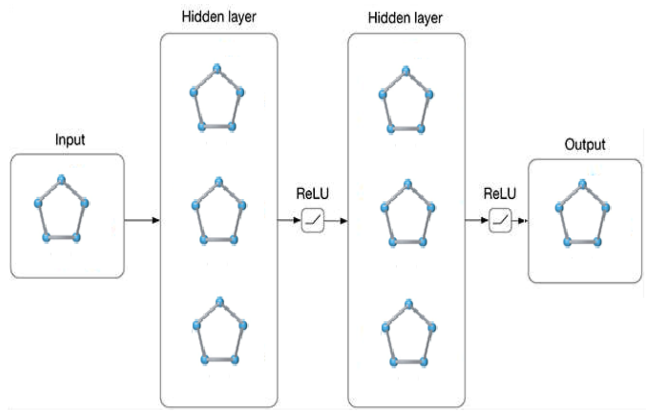


Fig 2.5 Three-pentagonal-chain-tworks

2.6 Pentagonal Graph Network PGN:

A Pentagonal graph network consists of five-sided shape that shows the fundamental building block which is connecting pentagonal structure includes various disciplines on multiple criteria with unique structure and topological characteristics in nodes. Each node represented in each pentagon gives the characteristics of HR selection process in different companies and we can find them arranged in simple linear chains (SPC_n) configurations. **V_1, V_2, V_3, V_4, V_5**

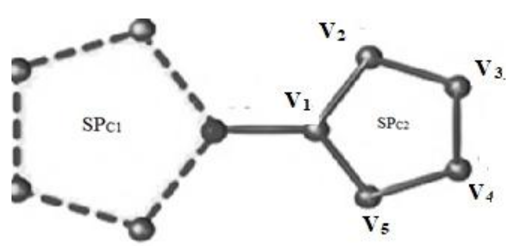


Fig 2.6 Pentagonal chain Network (SPCn)

Here, the most basic structure, where pentagons are connected edge-to-edge, creating a chain. For instance, you have the SPC_n, which is simply a chain of 'n =3' connected pentagons.

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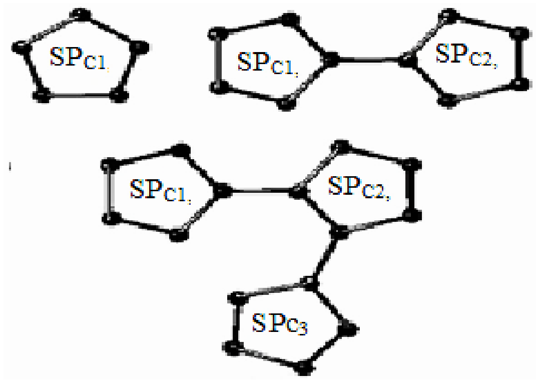

Fig 2.6.1 Three-pentagonal-chain-networks

On the other hand, a Fuzzy Neural Network (FNN) merges a traditional Multi-Layer Perceptron (MLP) with fuzzy logic, creating a unique architecture. The FNN is structured with three layers of neurons.

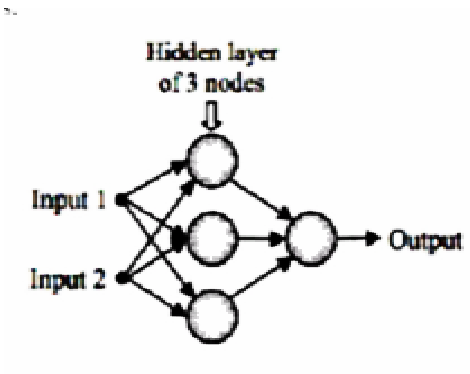

Fig 2.6.2 Layers of Neurons

The input layer of neurons takes in the input variables values, while the outputs are from the condition layer were it is analysed. This Conditional layer is structured and functions just like a hidden layer where a quick reminder that craft responses and always stick to the specified Condition related.

2.7 Pentagonal Fuzzy Graph Neural Network:

PFGs capture the fuzzy characteristics of both vertices and edges by a spectrum of possible values, each with its own degree of membership. They utilize GNNs to learn from graph-structured data that features fuzzy relationships. This approach is useful in the situations where data is naturally uncertainty or Relationships between data points are unclear.

2.8 Feed Forward Neural Network (FFNN):

Feed forward neural network (FFNN) is a simplest kind of artificial neural network where the connections between nodes has no cycles or loops. In this process, data flows in just one direction forward from the input nodes, through any hidden nodes, and finally to the output nodes. When it comes to feed forward fuzzy neural networks, it can be categorized into three types based on how inputs, outputs, and weights are fuzzified.

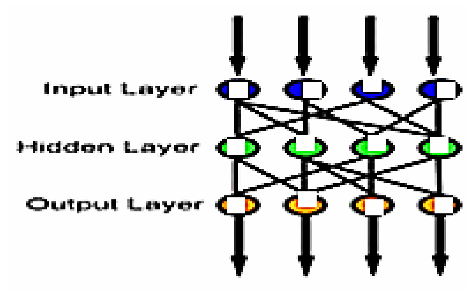

Fig 2.8 Feed Forward Neural Networks

In a feed forward network, information flows in a single direction and it never travels backward

2.9 Activation Function

An activation function is a node that you place at the output end of any neural network. Sometimes referred to as a transfer function, it can also be positioned between two neural networks. Its main job is to decide the neural network outcomes are in the form of Y or N. It takes the resulting values and maps them between 0 and 1 or -1 and 1.

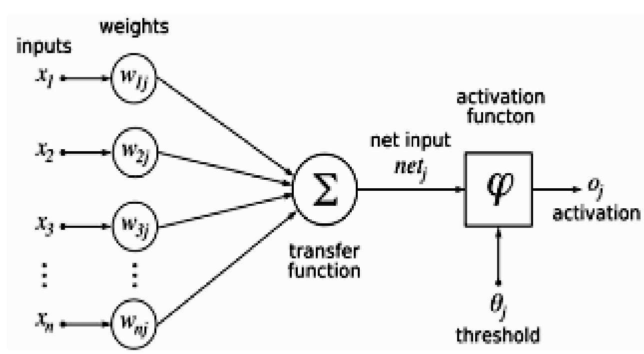

Fig 2.9 Representation of Activation function

Sigmoid Function, also known as the logistic function, is a mathematical curve that has an S-shaped or sigmoidal curve. It is defined as follows:

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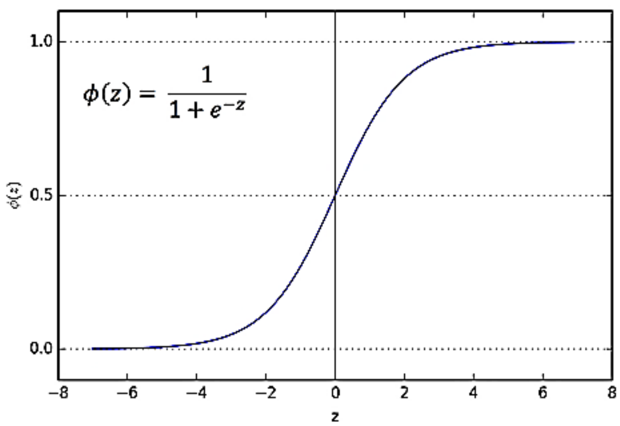

Fig 2.9.1 Sigmoid Curve Structure

The sigmoid function is differentiable, which is crucial for training neural networks using gradient-based optimization methods. The most common sigmoid function is the logistic function, defined as:

$$\text{Sigmoid Function } \delta(x) = \frac{1}{(1+e^{-x})}$$

Where 'e' is Euler's number (**approximately 2.718**)

Sigmoid converts the output of a linear model into a probability, making it suitable for predicting the probability of a binary outcome (e.g., yes/no, and true/false).

III. Working Procedure

Step 1: Begin by estimating the problem using the Pentagonal fuzzy number.

Step 2: Convert the Pentagonal fuzzy number matrix into its corresponding membership values.

Step 3: Consider the Pentagonal fuzzy number as a fuzzy Weight.

Step 4: Assume the inputs are 0 and 1.

Step 5: Calculate Weighted Average Method (WAM)

$$\text{WAM} = \sum_j^i W_{ji} x_j = W_i x_j \text{ .... (1)}$$

Step 6: The outcomes of a neuron (S f) can be represented as a function: ***S f*** **= δ(m).**

Step 7: To find the sigmoid function, apply the formula

$$\delta(x) = \frac{1}{(1+exp^{-x})} \text{ ......(2)}$$

Step 8: Lastly, identify the minimum value of δ(m) which is the optimal solution.

IV. Numerical Example

Let us assume that there are three companies that undergoes in selection process under HR namely SP_{C1}, SP_{C2}, and SP_{C3}, Let the selection process on each companies are in the form of fuzzy pentagonal number that represents Work Experience, Communication Skill, Technical Skill, Leadership , Cultural fit respectively.

Compute the Best Selection Processing Company among them

$$\begin{matrix} SP_{C1} & SP_{c2} & SP_{c3} \\ (3,2,5,3,1) & (2,3,4,2,1) & (2,1,5,1,3) \end{matrix}$$

..........(3)

Fuzzy Pentagonal Number matrix is converted into its crisp membership values

$$\text{Pentagonal Fuzzy Weight } (w_i) = \begin{pmatrix} 0.3 & 0.2 & 05 & 0.3 & 0.1 \\ 0.2 & 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.5 & 0.1 & 0.3 \end{pmatrix}$$

Let as assume the membership values of pentagonal fuzzy number as **(1, 1, 0, 1, 1)**

Weighted Average Method

$$SP_{Ci} = \sum_j^i W_{ji} x_j = W_i x_j$$

$$where\ i,j = 1,2,3,4,5$$

$$SP_{c1} = w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + w_{15}x_5$$

= 0.1(1) + 0.2(1) + 0.2(0) + 0.3(1) + 0.3(1) = 0.9

$$SP_{c2} = w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + w_{25}x_5$$

= 0.1(1) + 0.2(1) + 0.3(0) + 0.4(1) + 0.1(1) = 0.8

$$SP_{c3} = w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + w_{35}x_5$$

= 0.1(1) + 0.2 (1) + 0.2(0) + 0.3(1) + 0.1(1) = 0.7

SP_{C1} **= 0.9** SP_{C2} **= 0.8** SP_{C3} **= 0.7**

Hence, Neuron *(Sf)* is given by ***Sf (Ci)*** = δ(SP_{Ci})

Sf = δ(SP_{C1})

Sf = δ(SP_{C2})

Sf = δ(SP_{C3})

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Now we calculate the sigmoid function

$$\delta(SP_{C1}) = \frac{1}{(1 + e^{-SP_{C1}})} = 0.7109$$

$$\delta\,(SP_{C2}) = \frac{1}{(1 + e^{-SP_{C2}})} = 0.6899$$

$$\delta\,(SP_{C3}) = \frac{1}{(1 + e^{-SP_{C3}})} = 0.6882$$

Finally, determining the minimum value of the sigmoid function for optimization

$$Sf(C_1) = \delta\,(SP_{C1}) = 0.7109$$

$$Sf(C_2) = \delta\,(SP_{C2}) = 0.6899$$

$$\boldsymbol{Sf(C_3) = \delta\,(SP_{C3}) = 0.6882}$$

Minimum Value is **0.6882**.

So, Best Selection Process Company is C_3

V. CONCLUSION

This study is introduced how we can combine Graph Neural Networks (GNNs) with Fuzzy Pentagonal Numbers (FPNs) to tackle optimization challenges in Human Resource Management (HRM). Our results show that this integrated method significantly boosts the ability of HR systems to navigate ambiguity, particularly in areas like performance evaluations, team formations, and talent acquisition. GNNs excel capture relational structures, while FPNs manage the vagueness found in subjective judgments which is more comprehensive in optimization process. The model we proposed can expand field of intelligent HRM systems by enhancing decision quality, especially in situations where data may be incomplete or imprecise. Additionally, it makes a way for future research in explainable AI within HR, hybrid decision-making models, Moreover the combination of graph-based learning with fuzzy logic offers an exciting path way for optimizing HR operations, ensuring fairness, adaptability, and alignment with organizational goals.

VI. FUTURE WORK

There are still several exciting paths for future research that could boost the framework's such as Dynamic and Temporal Graph Modeling, Incorporation of Multi-Fuzzy Logic Systems, Explainable Graph Neural Networks in HR under Scalability to Large Enterprises, Integration with Real-World HR Software Systems. Our future research could involve implementing the model within existing HR software platforms (like SAP Success Factors or Workday) and evaluating its effectiveness.

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