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Program Listings for Population Characteristics of Wrapped Pseudo-Lindley Distributiont

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***Abstract*—A class of models, referred to as the Wrapped Pseudo-Lindley (WPsL) distribution, is developed to model wind data. In this setting, the required population characteristics of the distribution are examined and assessed using MATLAB. The analytical expressions for circular measures such as the mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness, and kurtosis are well established and can be found in [5].**

***Keywords*—Wrapped Pseudo-Lidley distribution (WPsL), Characteristic function, population characteristics, program listings.**

I. Introduction

Constructing a circular model by wrapping a linear distribution along the circumference of unit circle is one of oldest methods [1] and is used by several researchers. To mention a few (not so recent), Wrapped exponential and Laplace in [2], [3], [4], [8] and wrapped of some of the life testing models are proposed in [7]. Wrapped Lindley distribution is introduced in [6].

II. WrappedPseudo-Lindley Distribution

In this section, the circular (wrapped) Pseudo-Lindley random variable is defined as $\theta = X(\text{mod}2\pi)$, such that for $\theta \in [0, 2\pi)$, the probability density function of θ is given by

$$g_{wpsL}(\theta) = \frac{e^{-\lambda\theta}}{\left(1-e^{-2\pi\lambda}\right)}\left(\frac{(\beta-1)\lambda}{\beta} + \frac{\theta\lambda^2}{\beta} - \frac{2\pi\lambda^2}{\beta\left(1-e^{2\pi\lambda}\right)}\right),$$

$$\theta \in [0, 2\pi) \text{ where } \lambda > 0,\ \beta \geq \frac{1+\lambda}{\lambda} \quad (2.1)$$

III. Characteristic Function And Trigonometric Moments

The characteristic function, trigonometric moments and other population characteristics like resultant length, mean, circular variance, standard deviation, coefficient of skewness and kurtosis are derived in the closed form and presented in this section.

The characteristic function of a circular random variable θ isthe doubly-infinite sequence of complex numbers $\{\phi_p : p = \pm 1, \pm 2, \pm 3, \ldots,\}$ given by $\phi_p = E\left(e^{ip\theta}\right)$

$$\phi_p = \left(\frac{\lambda\sqrt{(\lambda\beta)^2 + ((\beta-1)p)^2}}{\beta\left(\lambda^2+p^2\right)}\right) \times e^{i\left(2\tan^{-1}\left(\frac{p}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)p}{\beta\lambda}\right)\right)}$$

(3.1)

Let$\theta \sim WPsL(\lambda, \beta)$, then the order trigonometric moments of θ is given by$\phi_p = \alpha_p + i\beta_p$, where$\alpha_p = \rho_p\cos(\mu_p)$,$\beta_p = \rho_p\sin(\mu_p)$.

$$\alpha_p = \frac{\lambda\sqrt{(\lambda\beta)^2 + ((\beta-1)p)^2}}{\beta\left(\lambda^2+p^2\right)}\cos\left(2\tan^{-1}\left(\frac{p}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)p}{\beta\lambda}\right)\right)$$

(3.2)

$$\beta_p = \frac{\lambda\sqrt{(\lambda\beta)^2 + ((\beta-1)p)^2}}{\beta\left(\lambda^2+p^2\right)}\sin\left(2\tan^{-1}\left(\frac{p}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)p}{\beta\lambda}\right)\right)$$

(3.3)

Resultant length:

$$\rho = \rho_1 = \frac{\lambda\sqrt{(\lambda\beta)^2 + ((\beta-1))^2}}{\beta\left(\lambda^2+1\right)} \quad (3.6)$$

Mean direction:

$$\mu = \mu_1 = 2\tan^{-1}\left(\frac{1}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)}{\beta\lambda}\right)$$

(3.7)

Circular Variance:$V_0 = 1 - \rho = 1 - \frac{\lambda\sqrt{(\lambda\beta)^2 + ((\beta-1))^2}}{\beta(\lambda^2+1)}$

(3.8)

Circular Standard deviation:

$$\sigma_0 = \sqrt{-2\log_e(1-V_0)} = \sqrt{\log_e\left(\frac{\beta^2(\lambda^2+1)^2}{\lambda^2((\beta\lambda)^2+(\beta-1)^2)}\right)} \quad (3.9)$$

Central trigonometric moments:

$$\overline{\alpha_p} = \rho_p \cos\left(\mu_p - p\mu_1\right) = \left(\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1)p)^2}}{\beta(\lambda^2+p^2)}\right)\cos\left(\left(2\tan^{-1}\left(\frac{p}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)p}{\beta\lambda}\right)\right) - \frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\left(2\tan^{-1}\left(\frac{1}{\lambda}\right) - \tan^{-1}\left(\frac{(\beta-1)}{\beta\lambda}\right)\right)\right) \quad (3.10)$$

$$\overline{\beta_p} = \rho_p \sin\left(\mu_p - p\mu_1\right) = \left(\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1)p)^2}}{\beta(\lambda^2+p^2)}\right)\sin\left(\left(2\tan^{-1}\left(\frac{p}{\lambda}\right) - \right.\right.$$

tan−1β−1pβλ−λλβ2+β−12βλ2+12tan−11λ−tan−1β−1βλ

(3.11)

Skewness:

$$\varsigma_1^0 = \frac{\overline{\beta_2}}{V_0^{3/2}} = \frac{\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1)2)^2}}{\beta(\lambda^2+4)}\sin\left(\left(2\tan^{-1}\left(\frac{2}{\lambda}\right)-\tan^{-1}\left(\frac{(\beta-1)2}{\beta\lambda}\right)\right) - \frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\left(2\tan^{-1}\left(\frac{1}{\lambda}\right)-\tan^{-1}\left(\frac{(\beta-1)}{\beta\lambda}\right)\right)\right)}{\left(1-\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\right)^{3/2}} \quad (3.12)$$

Kurtosis:

$$\varsigma_2^0 = \frac{\overline{\alpha_2}-(1-V_0)^4}{V_0^2} = \frac{\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1)2)^2}}{\beta(\lambda^2+4)}\cos\left(\left(2\tan^{-1}\left(\frac{2}{\lambda}\right)-\tan^{-1}\left(\frac{(\beta-1)2}{\beta\lambda}\right)\right) - \frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\left(2\tan^{-1}\left(\frac{1}{\lambda}\right)-\tan^{-1}\left(\frac{(\beta-1)}{\beta\lambda}\right)\right)\right) - \left(\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\right)^4}{\left(1-\frac{\lambda\sqrt{(\lambda\beta)^2+((\beta-1))^2}}{\beta(\lambda^2+1)}\right)^2} \quad (3.13)$$

IV. PROGRAM LISTINGS FOR POPULATION CHARACTERISTICS

Programs for graphs and computations are developed using MATLAB and code for respective work are presented here.

Program code for population characteristics

```
%Population characteristics of Wrapped Pseudo Lindley
model
a=1.0685%input('enter the value of a =');
th=linspace(0+a,2*pi,24);
h=2*pi/24;
a1=0.001
fprintf('lambda');
k=1.25%lambda
fprintf('beta');
b=(1+k+a1)/k %beta
k1=exp(-k*th)/(1-exp(-2*k*th));
k2=((b-1)*k/b)+((th.*k^2)/b)-(2*pi*k^2)./(b*(1-
exp(2*k.*th)));
f=k1.*k2;
c=[1 5 1 6 1 5 1];
C1=zeros(1,24);
S1=zeros(1,24);
for p=0:23
   yc=cos(p.*th).*f;
   ys=sin(p.*th).*f;
   s1=0;
   s2=0;
   k=p+1;
for m=1:4
for i=1:6
         s1=s1+c(i)*yc(6*(m-1)+i);
         s2=s2+c(i)*ys(6*(m-1)+i);
end
end
```

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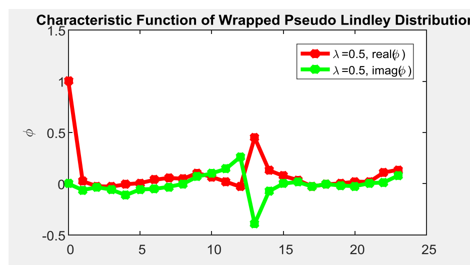
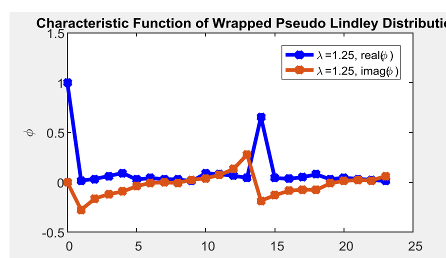
```
  s1;s2;
re=(3*h/10)*s1;
im=(3*h/10)*s2;
C1(k)=re;
S1(k)=im;
end
C1;S1;
phi=C1+j*S1;
phi(1)
p=0:23;
plot(p,real(phi),'-*r')
hold on
plot(p,imag(phi),'-*g')
ylabel('\phi')
legend('\lambda=1.25, real(\phi)', '\lambda=1.25,
imag(\phi)')
alpha=[real(phi(2)),real(phi(3))]
beta=[imag(phi(2)),imag(phi(3))]
[mu,rho1,v0,sig0,gamma1,gamma2]=circpropnew(alpha,be
ta)
title('Characteristic Function of Wrapped Pseudo Lindley
Distribution')
xlabel('p')
function[mu,rho1,v0,sig0,gamma1,gamma2]=circpropnew(
alpha,beta)
% alpha=[real(phi(2)),real(phi(3))];
% beta=[imag(phi(2)),imag(phi(3))];
rho=sqrt((beta(1)).^2+(alpha(1)).^2);
rho;
mu1=atan(beta(1)/alpha(1));%mean direction;
fprintf('mean direction');
mu1
mu=atan(beta./alpha);
for i=1:2
if beta(i)>=0 && alpha(i)>0
      mu(i)=atan(beta(i)/alpha(i));
end
if alpha(i)<0
      mu(i)=atan(beta(i)/alpha(i))+pi;
end
if beta(i)<0 && alpha(i)>=0
      mu(i)=atan(beta(i)/alpha(i))+2*pi;
end
if beta(i)>0 && alpha(i)==0
      mu(i)=pi/2;
end
end
fprintf('Mean Direction =');
 mu
fprintf('Resultant Length =');
rho1=sqrt((beta).^2+(alpha).^2)
p=1:2;
fprintf('Central Trignometric Moments alpha1 & alpha2:
\n')
  p=1:2;
alphag(p)=rho1.*cos(mu-p*mu(1))
fprintf('Central Trignometric Moments beta1 & beta2: \n')
betag(p)=rho1.*sin(mu-p*mu(1))
fprintf('circular variance =');
v0=1-rho1(1) %circular variance;
fprintf('circular standard deviation =');
sig0=sqrt(abs(log(1./((rho1).^2))))% circular standard
deviation;
fprintf('skewness =');
gamma1=(betag(2))./(v0.^(3/2))% skewness;
fprintf('kurtosis =');
gamma2=(alphag(2)-((1-v0).^4))./(v0.^2) %kurtosis;
```

Graphs of characteristic function of Wrapped Pseudo Lindley model for various values of parameter are plotted here





V. CONCLUSION

In this article, we have discussed population characteristics from the characteristic function of wrapped pseudo-Lindley distribution for modeling circular data. Using MATLAB techniques population characteristics are evaluated and program code is presented.

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