

## Trisection of an angle and its proof

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**Abstract:** This method divides an arbitrary angle into three equal angles using a compass and an unmarked straightedge. In this, any angle smaller than 180 degrees can be divided into many parts or ratios. This division is proved on the basis of the principle of correlation of arcs of equal radius. In this, any arc (or angle) and straight line can be divided into desired parts by creating semicircles on the arc and straight line.

**Keywords:** Arc at the centre point, arcs of same radius, cut point on arc, semicircles on an arc and straight line, proportional division, principle of arcs.

Research Area: Geometry

### Research Paper

#### I. CONSTRUCTION

The following steps describe how to divide an angle or arc AC with sides AB and BC into three equal angles (Fig. 1):

1. We mark a point A and draw on it an arc  $B_1B_2$  equal to the side AB.
2. Then at point  $B_2$ , draw another arc AD equal to the side AB.
3. After this, three consecutive cuts of distance slightly more than  $1/6$ th of the distance AC of the given angle are made on the arc AD, starting from point A, namely  $A_1$ ,  $A_2$ , and  $A_3$  respectively.
4. Then three semicircles are drawn on the inside of  $AA_2$ , at point  $A_1$ ,  $AA_3$ , at point  $A_2$ , and  $AA_5$  at point  $A_3$ . The distance  $A-A_5$  must be greater than the AC of the given angle.
5. A cut point C is made on semicircle  $AA_5$ , equal to the distance AC from point A.
6. From point C, a cut point B is made on arc  $B_1B_2$ .
7. At point B, draw an arc AC of length equal to side AB.
8. Now, by joining point B to the points A,  $C_1$ ,  $C_2$ , and C by straight lines, thus three equal angles ABC,  $C_1BC_2$  and  $C_2BC$  are formed.

#### II. TO PROVE THE TRISECTION METHOD OF AN ANGLE

This trisection method has been proved by the principle of correlation in the arcs of equal radius.

In this, the arcs of all the semicircles starting from a point divide the arcs of equal radius, starting from the same point, in the same proportion. As shown in Figure 2, an arc  $B_1B_2$  is drawn from point  $A_1$ , then another arc AD of the same radius is drawn from point B. From point A, some cuts  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ ...etc. of equal distance are made on the arc AD. Taking all these points on the centre, starting from point A, semicircles are drawn on all these points. Then similarly, starting from point B on arc  $B_1B_2$ , some cuts  $B_3$ ,  $B_4$ ,  $B_5$ , etc., are made at equal distance and then considering all these cuts as the centre points, many arcs are drawn starting from point A.

Here, the existence of semicircles on an arc has been described but now the existence of semicircles on a straight line is being described.

In this, starting from point A on a straight line, cuts of equal distance  $A_1$ ,  $A_2$ ,  $A_3$ ,  $A_4$ , etc. are made as shown in Figure 3.

Then, starting from point A, draw semicircles on one side at all those cut points. These points on the arc of each semicircle are placed at angles  $60^\circ$ ,  $90^\circ$ , and  $120^\circ$ . Then the points of those equal angles meet at point A to form three straight lines.

#### III. RESULT

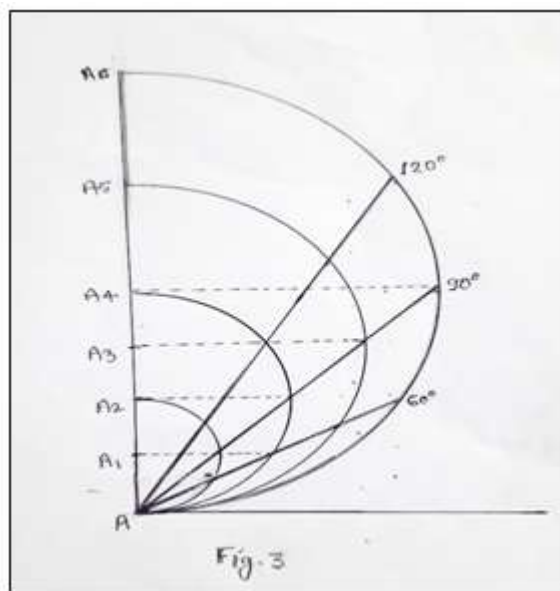
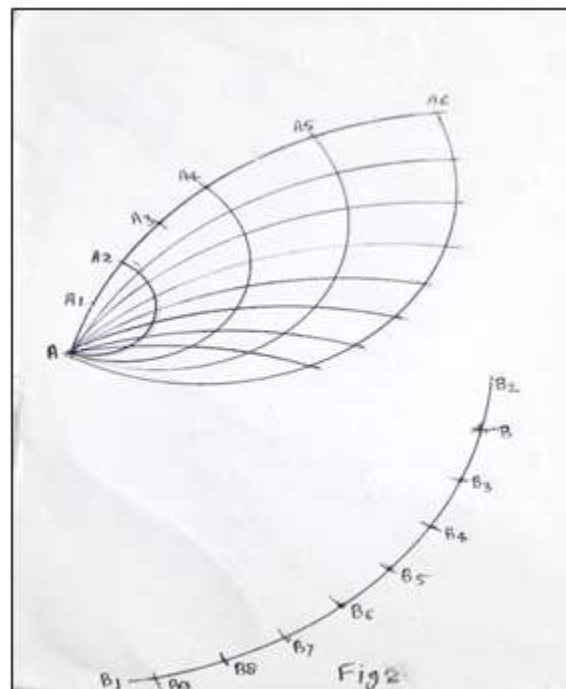
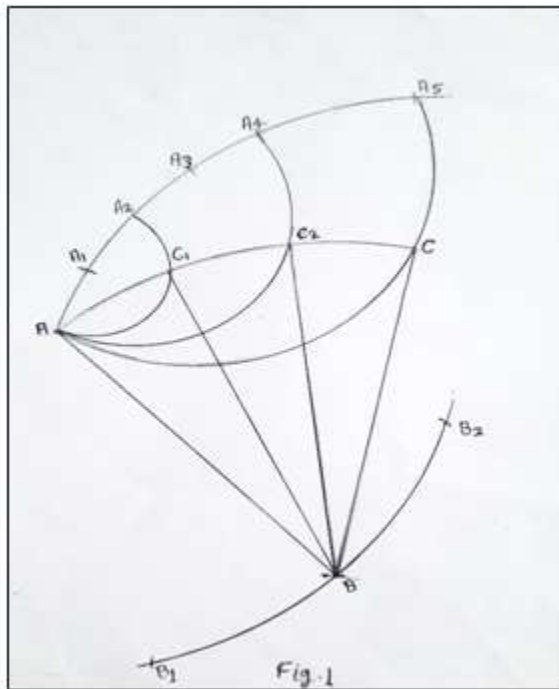
We observe that in the case of semicircles formed on an arc of the same ratio, arcs of equal radius are divided in the same ratio, but arcs of semicircles are not divided in the same ratio. But in the case of arcs of semicircles formed on a straight line, the straight lines are divided in the same ratio and the arcs are also divided in the same ratio.

#### IV. CONCLUSION

By this method, an arbitrary angle or arc can be divided into several parts or proportional parts. But the angle must be lesser than 180 degrees.

If the angle is larger, then it is halved or quartered, then after dividing by this method, it is doubled or quadrupled respectively.

This method can be used on any curved or straight line. In this method 'the principle of correlation in arcs of equal radius is applied.





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V. MESSAGE TO THE EDITOR

This research work is being sent to you for publication. I request you to publish this manuscript soon.

If there are any shortcomings in it, I will correct them promptly and send it back. I certify that this is my original and unpublished work.