International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347-6435(Online) Volume 12, Issue 01, January 2023)

Fuzzy Set Mathematics Theory and Applications

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***Abstract*—As a brand-new conceptual system to support human-centric framework, fuzzy set has proved quite promising and effective in modeling human involvement in human-based intelligence to attain modernity in many departments like data analyzing, data mining, image coding and explaining, as well as in intelligence systems. Fuzzy set has also become an acknowledged research subject in both pure as well as in applied mathematics and statistics, showing how this theory is highly applicable and productive in many applications. Despite being a core subject for many years, fuzzy set still attracts researchers for putting forth solutions for prime issues with certain features questioned by these notions. Fuzzy set can effectively deal with a wide spectrum of problems of the physical world via cooperation, which may be beyond the capability of classical techniques. This paper review of previous research works, which is based on the trending applications using fuzzy set.**

***Keywords*— IOT, Cyber, NIDS, HIDS, Security.**

I. INTRODUCTION

In mathematics, fuzzy sets (a.k.a. uncertain sets) are sets whose elements have degrees of membership. Fuzzy sets were introduced independently by Lotfi A. Zadeh in 1965 as an extension of the classical notion of set.[1][2] At the same time, Salii (1965) defined a more general kind of structure called an L-relation, which he studied in an abstract algebraic context. Fuzzy relations, which are now used throughout fuzzy mathematics and have applications in areas such as linguistics (De Cock, Bodenhofer & Kerre 2000), decision-making (Kuzmin 1982), and clustering (Bezdek 1978), are special cases of L-relations when L is the unit interval [0, 1].

In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition—an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set; this is described with the aid of a membership function valued in the real unit interval [0, 1]. Fuzzy sets generalize classical sets, since the indicator functions (aka characteristic functions) of classical sets are special cases of the membership functions of fuzzy sets, if the latter only takes values 0 or 1.[3] In fuzzy set theory, classical bivalent sets are usually called crisp sets.

The fuzzy set theory can be used in a wide range of domains in which information is incomplete or imprecise, such as bioinformatics.

The evolution of the fuzzification of mathematical concepts can be broken down into three stages:

1. straightforward fuzzification during the sixties and seventies,
2. the explosion of the possible choices in the generalization process during the eighties,
3. the standardization, axiomatization, and *L*-fuzzification in the nineties.

Usually, a fuzzification of mathematical concepts is based on a generalization of these concepts from characteristic functions to membership functions.

II. LITERATURE SURVEY

Z. J. Al-Araji et al.,[1] Geographically dispersed Fog Computing architecture ubiquitously connected to a range of heterogeneous nodes at the edge of the network can provide cooperative flexible, and variable computations, communications, and storage services. Several fog computing methods, models, and techniques have been used to solve cloud issues. The fuzzy theory has also been used in many aspects of fog computing. Objectives: This work presents a systematic literature review of the use of fuzzy theory in Fog Computing, highlighting the main practical motivations, classification types in research approaches, fuzzy methods used, popular evaluation tools, open issues, and future trends. Methods: The investigations were systematically performed using fuzzy theory in fog computing, and four databases which are ScienceDirect, Web of Science (WoS), Scopus, and IEEE Xplore Digital Library from 2015 to 2022, were used to analyse their performance evaluation, architecture, and applications. Results: 94 articles were selected based on fuzzy theory in fog computing using different methods, models, and techniques, based on the proposed exclusion and inclusion criteria. The results of the taxonomy were divided into five major classes: task and resource management, intrusion detection systems, trust management, and healthcare services. Discussion: Applications requiring real-time, low latency, and quick responses are well suited for fog computing.

International Journal of Recent Development in Engineering and Technology
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These studies show that resource sharing improves the fog computing architecture by delivering reduced latency, distributed processing, improved scalability, better security, fault tolerance, and privacy. Conclusion: The majority of the time, research areas on fuzzy theory in fog computing are crucially significant. We conclude that this review will enhance research capacity, thereby expanding and creating new research domains.

L. Qingming et al.,[2] Standard SQL statements can only perform exact information queries and do not meet the needs of the real world. Constructing query statement with fuzzy function based on fuzzy mathematics theory has certain application value. Define the fuzzy query statement format first, and then build the fuzzy knowledge table and data dictionary. Finally, a general fuzzy query parser is constructed based on fuzzy theory. Fuzzy Query Parser converts a fuzzy query statement into a standard SQL query statement to achieve a true fuzzy query and adapt to the needs of flexible queries. The validity of the Fuzzy Query Parser is verified by a simple case and the technology is applied to the data query of Mongolian medicine prescriptions. Universal fuzzy query technology can achieve user's fuzzy query requirements and can be converted to standard SQL statements. It provides a new way for data query and data analysis and has certain application value.

A. Theerens et al.,[3] In this paper we apply vague quantification to fuzzy rough sets to introduce fuzzy quantifier-based fuzzy rough sets (FQFRS), an intuitive generalization of fuzzy rough sets. We show how several existing models fit in this generalization as well as how it inspires novel models that may improve these existing models. In addition, we introduce several new binary quantification models. Finally, we introduce an adaptation of FQFRS that allows seamless integration of outlier detection algorithms to enhance the robustness of the applications based on FQFRS.

J. Zhang et al.,[4] In view of the current serious interference in the evaluation of the damage status of offshore structures, the evaluation accuracy is not good. A method for evaluating the damage status of offshore structures based on fuzzy mathematics is proposed. The damage characteristics of offshore structures are collected and identified, combined with fuzziness. The mathematical algorithm evaluates the damage state and builds an evaluation model. Finally, it is confirmed through experiments that the method of evaluating the damage state of offshore structures based on fuzzy mathematics has high practicability in practical applications.

A. I. Volkov et al.,[5] The article discusses the issues of building a base of fuzzy inference rules for solving problems of assessing the quality of mobile applications. The solution to this problem is based on the use of the main provisions of the PNST277-2018 standard "Comparative tests of mobile applications for smartphones". It is shown that the hierarchical organization of software quality models is a prerequisite for creating a base of rules for fuzzy inference. A general algebraic algorithm for constructing the structure of a mobile application quality model is presented. An example of describing fuzzy rules for the implementation of the problem of assessing the characteristics of the maintainability of a mobile application is given. It is proposed to use the Fussy Logic Toolbox MatLab package as a tool for creating a fuzzy expert system to support the process of comparative research of the quality of mobile applications.

W. Yujia et al.,[6] In view of the urgent and complex demand of the surface ship's air-defense, this paper presents a CGF air-defense decision-making method based on the theory of fuzzy mathematics and the technology of behavior tree of single ships. According to the characteristics of different nodes in the behavior tree, the CGF behavior tree model framework is constructed according to the logical sequence of single ship air-defense action flow in the actual battlefield. Aiming at the requirement of intelligence of conditional nodes, the judgment algorithm constituted by the theory of fuzzy mathematics is applied to fill the conditional nodes of behavior tree, and verified by simulation experiment, so as to construct the CGF decision model of single ship's air-defense with intelligence.

A. Sarkar et al.,[7] In this paper we demonstrated an application of Fuzzy set theory for Order of Preference by Similarity to Ideal Solution (FTOPSIS) methodology for the selection of best family car. Using a real-life data, we solved the problem. Imprecise linguistic qualitative preferences are converted into fuzzy numbers for representation and subsequent solution.

M. Gutiérrez-López et al.,[8] The FPGA is the robust platform to implement fast and complex circuits in the industry. Also, nowadays, Arduino microcontroller is very used because has typical preloaded functions that facilities its use. Intuitively it is known that FPGA is faster due its parallelization possibilities. The purpose of this work is to show in a quantitative way, the difference in performance between Arduino UNO and FPGA Xilinx Spartan 3E 100 CP132 when they are used as embedded Fuzzy system.

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This may bring arguments to decide whether to use one or another platform for this kind of applications. This paper presents a fuzzy system implementation of a Mamdani type to infer the tip given to a restaurant, where it was measured the time spent in implementing and executing; then the results were compared. These last, offered a minimal numeric difference between systems but highlighted the difference in programming time and processing time.

D. Zhang et al.,[9] Shale gas reservoir refers to a new alternative energy. The percolation of shale gas reservoir is more complicated than that of conventional power for its unique reservoir forming characteristics and physical properties. Accordingly, to produce shale gas reservoirs in a large scale, the percolation of shale gas reservoirs should be better simulated. The conventional seepage model of shale gas reservoir assumes that reservoir permeability is a specified value. However, reservoir is usually significantly uneven and complex, suggesting that considering the permeability as a fixed value will reduce the accuracy of the model. Given this, the concept of fuzzy permeability is proposed here. On that basis, the fuzzy non-linear seepage model is established. To avoid the ergodic operation of the conventional fuzzy differential equation solving method, this study employed the fuzzy structural element method to solve the new model. Also, given the practical application problems, the fuzzy solution set is further non-fuzzy treated using the centroid method. Finally, the superiority of the nonlinear fuzzy seepage model is illustrated by the comparison between the nonlinear fuzzy seepage model and the conventional nonlinear seepage model.

Q. Lou et al.,[10] To apply intelligent model in serious practical applications like medical diagnosis, the reliability and interpretability of the model are very important to users. Among the existing intelligent models, type-2 fuzzy systems are distinctive in interpretability and modeling uncertainty. However, like most existing models, the reliability determination of fuzzy system for recognition task training is an unsolved problem. In this study, a method of constructing minimax probability interval type-2 TSK fuzzy logic system classifier (MP-IT2TSK-FLSC) based on reliability learning is proposed. The classifier can provide the lower limit of the correct classification of the model and is an important index to quantify the reliability of the model. Experimental results on medical datasets have demonstrated the advantages of this method, exhibiting remarkable interpretability and reliability of the proposed fuzzy classifier.

V. Tishkina et al.,[11] The paper presents a study about the development an enterprise analysis mobile application. An extended set of financial ratios will be used in the developed application. The user of the mobile application will be provided with detailed recommendations about the situation in the enterprise. Currently, any company is characterized by a large amount of information. This information may be contained in various documents: in financial statements, regulatory documents, laws and others. Information about the company must be converted to the type required for effective analysis of the company. Mobile business intelligence is in demand in modern enterprise management. This statement is relevant for small and large enterprises. Enterprise model based on the semantic network makes conclusions about each enterprise. Objects of the semantic network are fuzzy objects. The types of connections between objects of the semantic network are considered as fuzzy object relations. In addition, the article describes the functionality of the mobile application. The work ends with short conclusions.

III. Different Fuzzy Sets

Definition-1 [S.Dang, A.Behra and S.Nanda, 1994]

A fuzzy point x_α in X is a fuzzy set defined as follows :

$$x_\alpha(y) = \begin{cases} \alpha & if y = x \\ 0 & y \neq x \end{cases}$$

Where $0 < \alpha \leq 1$; α is called its value and x is support of x_α.

1.2 Definition [M.H.Rashid and D.M.Ali,2008]

A fuzzy point x_α is said to belong to a fuzzy set A in X (denoted by: $x_\alpha \in A$) if and only if $\alpha \leq A(x)$,for some $x \in X$.

1.3 Definition[S.Dang,A.Behra and S.Nanda, 1994]

Let A and B are fuzzy sets in X. Then $A \leq B$ if and only if $x_\alpha \in B$ for all $x_\alpha \in A$.

Definition -2 [D.H.Foster,1979]

Let A and B are fuzzy sets in X ,then :

1) $A \le B$ if and only if $A(x) \le B(x) \quad \forall x \in X$.
2) $A = B$ if and only if $A(x) = B(x) \quad \forall x \in X$.
3) $Z = A \wedge B$ if and only if $Z(x) = \min\{A(x), B(x)\} \quad \forall x \in X$.
4) $D = A \vee B$ if and only if $D(x) = \max\{A(x), B(x)\} \quad \forall x \in X$.
5) $E = A^c$ (The complement of A)if and only if $E(x) = 1 - A(x) \quad \forall x \in X$.

1.5 Theorem[A.B.Saeid,2006]

Let X and Y be two fuzzy topological spaces and let $f: X \to Y$ be a function ,let $\{A_i\}_{i \in I}, \{B_j\}_{j \in J}$ be families of fuzzy sets in X and Y respectively ,then :

1) $f(\underset{i \in I}{\vee} A_i) = \underset{i \in I}{\vee} f(A_i)$.
2) $f(\underset{i \in I}{\wedge} A_i) \le \underset{i \in I}{\wedge} f(A_i)$.
3) $f^{-1}(\underset{j \in J}{\vee} B_j) = \underset{j \in J}{\vee} f^{-1}(B_j)$.
4) $f^{-1}(\underset{j \in J}{\wedge} B_j) = \underset{j \in J}{\wedge} f^{-1}(B_j)$.

Definition-3 [C.L.Change,1968]

Let A be a fuzzy set in X ,then :

1) The union of all fuzzy open sets contained in A is called the fuzzy interior of A and denoted by A^0.
i.e; $A^0 = \vee\{B : B \le A, B \in T\}$.

2) The intersection of all fuzzy closed sets containing A is called the fuzzy closure of A,and denoted by $\overline{A}$ .i.e; $\overline{A} = \wedge\{B : A \le B, B^c \in T\}$.

Definition -4 [S.Carlson,2005]

Let f be a function from X to Y ,and let B be a fuzzy set in Y ,then the inverse image of B under f is the fuzzy set $f^{-1}(B)$ in X with membership function defined by the rule : $f^{-1}(B)(x) = B(f(x))$ for $x \in X$.(i.e; $f^{-1}(B) = B \circ f$)

For a fuzzy set A in X , the image of A under f is the fuzzy set $f(A)$ in Y with membership function $f(A)(y)$, $y \in Y$ defined by :

$$f(A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} A(x) & \text{if } f^{-1}(y) \text{ is not empty} \\ 0 & \text{otherwise} \end{cases}$$

where $f^{-1}(y) = \{x : f(x) = y\}$.

Definition -5 [D.H.Foster,1979]

The union (respectively intersection) of the fuzzy sets $\underset{i \in I}{A_i}$ is defined by : $(\underset{i \in I}{\vee} A_i)(x) = \sup\{A_i(x) : i \in I\}$

$$(\underset{i \in I}{\wedge} A_i)(x) = \inf\{A_i(x) : i \in I\}$$

A fuzzy topological space X is said to be connected if X can not be represented as the union of two non-empty disjoint open fuzzy sets on X ,otherwise X is called disconnected space .

Definition -6 [A.M.Zahran,2000]

A family Ω of fuzzy sets is called a cover of a fuzzy set A if and only if $A \le \vee\{B_i : B_i \in \Omega\}$,and it is called a fuzzy open cover if each member B_i is a fuzzy open sets .A sub cover of Ω is a sub family of Ω which is also a cover of A .

Definition -7 [D.H.Foster,1979]

Let A be a fuzzy set in a fuzzy topological space X .

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Then A is said to be a fuzzy compact set if for every fuzzy open cover of A has a finite sub cover of A. Let $A = X$, then X is called a fuzzy compact space ,that is $A_i \in T$ for every $i \in I$ and $\underset{i \in I}{\vee} A_i = 1_X$,then there are finitely many indices $i_1, i_2, ..., i_n \in I$ such that $\underset{ij \in I}{\vee} A_{ij} = 1_X$.

A function $f : X \rightarrow Y$ is said to be fuzzy connected if and only if $f(W)$ is fuzzy connected set in Y for each W is fuzzy connected set in X ,otherwise f is called fuzzy disconnected function .

A function $f : X \rightarrow Y$ is said to be fuzzy O-connected function if and only if $f(W)$ is fuzzy connected set in Y for each W is fuzzy open and fuzzy connected set in X .

IV. Conclusion

Fuzzy sets have a great progress in every scientific research area. It found many application areas in both theoretical and practical studies. In practice, it is not always possible to fully obtain this data due to unavailability of primary observations and consequent scarcity of statistical data about the failure of components. To handle such situations, fuzzy set theory has been successfully used in novel PRA approaches for safety and reliability evaluation under conditions of uncertainty. This paper presents a review of fuzzy set theory based methodologies applied to safety and reliability engineering.

REFERENCES

[1] Z. J. Al-Araji, S. S. S. Ahmad, N. Kausar, A. Farhani, E. Ozbilgekahveci and T. Cagin, "Fuzzy Theory in Fog Computing: Review, Taxonomy, and Open Issues," in IEEE Access, vol. 10, pp. 126931-126956, 2022, doi: 10.1109/ACCESS.2022.3225462.

[2] L. Qingming, L. Zhanjiang and Z. Chunsheng, "Application of Fuzzy Query Technology in Data Analysis of Mongolian Medicine Prescriptions," 2022 IEEE 13th International Conference on Software Engineering and Service Science (ICSESS), Beijing, China, 2022, pp. 182-185, doi: 10.1109/ICSESS54813.2022.9930234.

[3] A. Theerens and C. Cornelis, "Fuzzy Quantifier-Based Fuzzy Rough Sets," 2022 17th Conference on Computer Science and Intelligence Systems (FedCSIS), Sofia, Bulgaria, 2022, pp. 269-278, doi: 10.15439/2022F231.

[4] J. Zhang, "Evaluation Method of Damaged Status of Offshore Structures Based on Fuzzy Mathematics," 2021 IEEE International Conference on Industrial Application of Artificial Intelligence (IAAI), Harbin, China, 2021, pp. 131-137, doi: 10.1109/IAAI54625.2021.9699937.

[5] A. I. Volkov, V. G. Semin and V. V. Semin, "Fuzzy Sets in the Problems of Assessing the Quality of Mobile Applications," 2021 International Conference on Quality Management, Transport and Information Security, Information Technologies (IT&QM&IS), Yaroslavl, Russian Federation, 2021, pp. 355-358, doi: 10.1109/ITQMIS53292.2021.9642873.

[6] W. Yujia, F. Wei, L. Chong and X. Cunxiang, "Air-defense Decision-making Model of Single Ship CGF Based on the Theory of Fuzzy Mathematics and Behavior Tree," 2020 2nd International Conference on Information Technology and Computer Application (ITCA), Guangzhou, China, 2020, pp. 755-759, doi: 10.1109/ITCA52113.2020.00162.

[7] A. Sarkar, A. Ghosh, B. Karmakar, A. Shaikh and S. P. Mondal, "Application of Fuzzy TOPSIS Algorithm for Selecting Best Family Car," 2020 International Conference on Decision Aid Sciences and Application (DASA), Sakheer, Bahrain, 2020, pp. 59-63, doi: 10.1109/DASA51403.2020.9317175.

[8] M. Gutiérrez-López, A. Montalvo-Fernández, J. -A. Padilla-Medina, F. -J. Pérez-Pinal and A. -I. Barranco-Gutiérrez, "A Mamdani fuzzy system implementation on Arduino UNO and FPGA," 2020 11th IEEE Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON), Vancouver, BC, Canada, 2020, pp. 0260-0266, doi: 10.1109/IEMCON51383.2020.9284841.

[9] D. Zhang, Z. Yu, L. Shu, D. Qiu and Z. Mo, "Fuzzy Structural Element Method for Solving Nonlinear Fuzzy Seepage Model of Shale Gas Reservoir," 2019 IEEE 14th International Conference on Intelligent Systems and Knowledge Engineering (ISKE), Dalian, China, 2019, pp. 110-117, doi: 10.1109/ISKE47853.2019.9170326.

[10] Q. Lou, Z. Deng, G. Wang and K. -S. Choi, "Reliability Learning for Interval Type-2 TSK Fuzzy Logic System with its Application to Medical Diagnosis," 2019 IEEE 14th International Conference on Intelligent Systems and Knowledge Engineering (ISKE), Dalian, China, 2019, pp. 43-50, doi: 10.1109/ISKE47853.2019.9170445.

[11] V. Tishkina, A. Pylkin, A. Kroshilin, S. Kroshilina and A. Evseev, "Enterprise Management Mobile Assistant based on Using the Theory of Fuzzy Logic and Fuzzy Sets," 2019 1st International Conference on Control Systems, Mathematical Modelling, Automation and Energy Efficiency (SUMMA), Lipetsk, Russia, 2019, pp. 247-249, doi: 10.1109/SUMMA48161.2019.8947514.