International Journal of Recent Development in Engineering and Technology
Website: www.ijrdet.com (ISSN 2347-6435(Online) Volume 10, Issue 9, September 2021)

Analytical Study of Algebra of Operators and Projective Tensor Products.

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***Abstract*--_This Paper Presents the study of the Algebra of operators and Projective Tensor Products. Here, we consider R additive group of reals with discrete topology and by defining the Projective topology π on locally Convex Spaces E and F; U & V be the closed absolutely convex neighbor hoods of R in E and F respectively, forming the set Γ(U⊗V) = absolutely convex null of U⊗V in E⊗F, it is proved in this paper that the Projective Topology π is the finest locally convex topology on E⊗F for which the Canonical mapping Ψ : E X F → E⊗F is continuous.**

***Keywords*—Projective Topology, Algebraic Tensor Products, Locally Covex Spaces, Absolutely Convex null, Canonical Mapping. C* - Algebra, W* - Algebra.**

I. INTRODUCTION

E.G. EFFORTS (1), Halub, J.R.(2) and Kothe (3,4) are the pioneer workers of the present area. In fact, the present work is the extension of work done by Tomiyama, J(10), Kumar et al. (5), Srivastava et al. (6); Srivastava et al. (7), Srivastava et al. (8), and Srivastava et al. (9). In this Paper, we have studied Analytically about Algebra of operators and Projective Tensor Products.

Here, we use the following definitions and fundamental ideas:

Definition - I : Let E and F be locally convex spaces, and let U & V be the closed absolutely convex neighbourhoods of O in E and F respectively, forming the set Γ(U⊗V) = absolutely convex hull of U⊗V in E ⊗ F, (E ⊗ F is denoted as tensorial product of E & F).

Definition - II : If {U} and {V} are neighbourhood bases in E and F respectively with U, V closed absolutely convex, then the family {Γ(U ⊗ V)} is a neighbourhood basis of a locally convex topology on E ⊗ F.

This topology is called the projective topology on E ⊗ F and is denoted as $E \otimes_\pi F$.

II. MATHEMATICAL TREATMENT

Let, R = additive group of reals with discrete topology. There are several ways of constructing c*-algebras canonically associated with R.

Let A and B be c*-algebras, with algebraic tensor product A ⊙ B. In general there are several distinct (usually incomplete).

c*- norms on A ⊙ B. Two such norms are of particular interest: the maximal norm v of Guichander and the minimal (or spatial) norm α of Turumaru.

The norm α is defined as follows:

if $x \in A \odot B$, $\alpha(x)$ is the smallest non-negative real number $\mathcal{R}$ such that

$$< f \otimes g, a^*x^* \text{ x } a > \ \leq \ \mathcal{R}^2 < f \otimes g, a^*a >$$

For all $a \in A \odot B$ and all satisfies f and g of A and B respectively. If for all Bα = v on A ⊙ B, A is said to be nuclear (the terminology in due to Lance, which is an introduction to the theory of c*- tensor product. For a discrete group R, $c_\ell^*(R) = c^*(R)$ iff G is amenable, and this is the case iff $c_\ell^*(G)$ is nuclear.

Let R be a discrete group and let be the representation of c*(R) ⊙ C*(R) on $\ell^2(R)$ given by

$$\lambda (\Sigma a_i \otimes b_i) = \Sigma \lambda_i(a_i) \lambda_r(b_i)$$

Proposition 1: Let p(x) and q(y) be the semi-norms defined by U and V respectively. The set Γ (U ⊗ V) is absorbing and thus defines a semi-norm. The semi-norm of Γ (U ⊗ V) is given by

$$p \otimes q \,(Z\text{–}) \ = \ \inf \Sigma_{i=1}^{n} p(x_i)\, q(y_i) \quad (1)$$

where the infimum is taken over all representations

$$Z\text{–} = \Sigma x_i \otimes y_i \ \text{ in } E \otimes F .$$

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Proof : First we show $\Gamma (U \otimes V)$ is absorbing. Let

$Z– = \Sigma_{i=1}^{n} x_i \otimes y_i$ be an element of $E \otimes F$. We observe that

$$\frac{Xi}{p(Xi)} ✞ U \quad \text{if } p(x_i) \neq 0$$

and $$\frac{yi}{q(yi)} ✞ V \quad \text{if } q(y_i) \neq 0$$

also $p(x_k) = 0$ iff $\rho.x_k$ ✞ U all $\rho > 0$ and

$q(y_j) = 0$ iff $\rho.y_j$ ✞ V all $\rho > 0$. So we may write

$$Z– = \Sigma_{i=1}^{n} x_i \otimes y_i$$

$$= \sum_i p(x_i)\, q(y_i) \left[\frac{Xi}{p(Xi)} \otimes \frac{yi}{q(yi)} \right]$$

$$+ \ \delta\, \Sigma_k\, q(y_k) \left[\frac{Xk}{\delta} \otimes \frac{yk}{q(yk)} \right]$$

$$+ \ \delta\, \Sigma_j\, p(x_j) \left[\frac{Xj}{p(xj)} \otimes \frac{yj}{\delta} \right]$$

$$+ \ \delta^2\, \Sigma_m \left[\frac{Xm}{\delta} \otimes \frac{ym}{\delta} \right]$$

In each of the four terms in the sum representing Z–, the quantity in the brackets [ ] is in $\Gamma (U \otimes V)$. Given $\in > 0$, we may choose δ sufficiently small so that

(*):-$Z– \in (\Sigma_{i=1}^{n} p(x_i)\, q(y_i) + \in)\, \Gamma (U \otimes V)$.

So $\Gamma (U \otimes V)$ is absorbing.

Now $\Gamma(U \otimes V)$ is absorbing convex also, so it defines a semi-norm r(Z–) on $E \otimes F$. We now show $r(Z–) = p \otimes q(Z–)$.

(i) $r(Z–) \subseteq p \otimes q(Z–)$, r(Z–) is defined by

$r(Z–) = \inf \lambda,\ Z– \in \lambda\, \Gamma (U \otimes V)$. By (*) above

$r(Z–) \leq \inf \sum p(x_i)\, q(y_i) + \in = p \otimes q(Z–) + \in$, $\in$ arbitrary yields

$r(Z–) \leq p \otimes q(Z–)$.

(ii) $p \otimes q(Z–) \leq r(Z–)$, suppose $Z– \in \lambda\Gamma(U \otimes V)$. Then $Z– = \sum \propto_k (x'_k \otimes y'_k)$ with $p(x'_k) \leq 1$, $q(y'_k) \leq 1$, $\sum |\propto_k| \leq \lambda$ and $\propto_k \geq 0$. For this particular representation of Z–, we see

$\sum p\, (\propto_k x'_k)\, q(y'_k) = \sum |\propto_k| \leq \lambda$. So,

$p \otimes q(Z–) = \inf \sum p(x_i)\, q\, (y_i) \leq \lambda$.

This is true for every λ with $Z– \in \lambda\, \Gamma\, (U \otimes V)$.

Thus $p \otimes q(Z–) \leq \inf \lambda,\ Z– \in \lambda\, \Gamma(U \otimes V)$.

$= r(Z–)$.

This completes the proof.

Proposition 2: The projective tensor product $E \otimes_{\pi} F$ of two normed space E, p and F, q is a normed space with norm $p \otimes q$.

If E and F are metrizable locally convex spaces with semi-norms $p_1 \leq p_2 \leq$ .... and $q_1 \leq q_2 \leq$ .... respectively, then $E \otimes_{\pi} F$ is metrizable with defining semi- norms $p_1 \otimes q_1 \leq p_2 \otimes q_2 \leq$ .... .

<u>*Proof:*</u> Follows immediately from Proposition 1 and definition 2.

III. Main Result

Theorem: The projective topology π is the finest locally convex topology on $E \otimes F$ for which the canonical map Ψ : $E X F \rightarrow E \otimes F$ is continuous.

<u>*Proof:*</u> Ψ is continuous with respect to π since Ψ (U X V) $= U \otimes V \subseteq$ $(U \otimes V)$. Now let τ be any topology on $E \otimes F$ for which Ψ is continuous and let W be an absolutely convex closed τ - neighbourhood of 0. Then there exist U, V with Ψ (U X V) $= U \otimes V \subseteq W$. Since W is absolutely convex, $\Gamma (U \otimes V) \subseteq W$. So π is finer than τ.

This completes the proof of the theorem.

Acknowledgment:

The authors are thankful to Prof.(Dr.) S.N. Jha , Ex. Head of the department of Mathematics, Ex. Head, Prof. (Dr.) P.K.Sharan, Ex.Head, Prof.(Dr.) B.P. Singh, Prof. (Dr.) Amita Sharma present Head of the Deptt. Of Mathematics Cum C.C.D.C., B.R.A.B.U. Muzaffarpur, Bihar , India and Prof. (Dr.) T.N. Singh, Ex. Head ,Ex. Dean (Science) and Ex. Chairman, Research Development Council , B.R.A.B.U. Muzaffarpur, Bihar , India for extending all facilities in the completion of the present research work.

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