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A Note on Covering Based Soft Rough Set

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Abstract— **Revising the theories and examining the principles of rough sets, the scholars of mathematics have advanced the model of covering based rough set and covering based soft sets to address the domain of the impreciseness of the objects. Further the combination of the theory of rough set and soft set are adduced with approximation operators to develop new model "Covering Based Soft Rough Set" for dealing the problem of impreciseness. This paper is aimed at defining covering based soft rough set without using full soft set a new model to handle the issues of the vagueness of objects.**
In addition, this new model has been defined in four different manners with the help of lower and upper approximation operators. .

Keywords— **Rough set, soft set, covering based rough set, soft rough set, covering based soft rough set.**

I. INTRODUCTION

From time immemorial the problem of imperfect knowledge has been a crucial issue for one and all especially for the philosophers, logicians and mathematicians. Their innovative approaches have addressed the problem for long time. However, in this age of knowledge explosion and information bombardment Professor Z.Pawlak, the great Polish mathematician and computer scientist has left a milestone in Mathematics introducing Rough Set Theory a new mathematical application in 1982 to tackle the serious problem of imperfect knowledge related to uncertainty, ambivalence, impreciseness and vagueness of object and information. The introduction and application of rough set theory has encouraged enormous research during the years in different areas giving new mathematical approaches to deal with thecomplicated problems of uncertainty emerge in different fields in our everyday life as well as different branches of knowledge.

The mathematical approaches are probability theory, fuzzy set theory, rough set theory, vague set theory, soft set theory, neutrosophic set theory etc. Moreover, combination of these theories is advanced by so many researchers to address the difficulties of uncertainty more effectively. Rough Fuzzy set, Fuzzy rough set, intuitionstic fuzzy set and rough intuitionstic fuzzy set, soft rough set, rough soft set, fuzzy soft set, rough fuzzy soft set, covering based rough set, covering based soft set and so many are the developed theories of the combination of theories. In this article we aim at introducing a new type of combination i.e. covering based soft rough set to handle the issues of impreciseness of objects arid imperfect knowledge convincingly.

This new type combination would hopefully add a new leaf in the field of research on soft set theory and application introduced by Molodstov in 1999 to tackle the problem of uncertainty.

After Z Pawlak [8], the researchers define a notion covering based rough set in various ways in ([1],[5],[10],[13],[15]). Soft set (1999) is introduced by D.Molodstov ([7]) and then covering soft set is defined by Saziye Yuksel et al ([12]) using the full soft set. Soft rough set ,the combination of rough set and soft set is studied by Feng Feng et al ([2]) and D.Mohanty ([6]). In this note we introduced four types of covering based soft rough set which are different from soft covering based rough set given by ([11],[12]). At the end we find covering based soft rough set in a new manner in which full soft set is not considered.

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II. Rough Set

The main advantage of rough set theory is that it does not need any preliminary or additional information about data. The rough set approach is based on knowledge with granular structure, which is caused by the situation when objects of interest can be distinguished and may appear to be identical. The indiscernibility relation (that is equivalence relation or knowledge) thus generated is the mathematical basis of Pawlak's Rough Set Theory.

A. Definition : (Approximation Space)

Let U be a universe of discourse, a finite set and R be an equivalence relation (knowledge) on U. R will provide a partition $U/R = \{Z_1, Z_2, Z_3, \dots Z_n\}$ *on U,* where $Z_1, Z_2, Z_3, \dots Z_n$ are the equivalence classes generated by R. Then the pair (U, R) is called approximation space.

B. Definition(Rough Set[8],[9])

For $X \subseteq U$, the lower and upper approximation of X are defined as $LR(X) = \cup\{Z_i \in U/R \mid Z_i \subseteq X\}$ and $HR(X) = \cup\{Z_i \in U/R | Zi \cap X \neq \phi$, respectively. The set X is termed as Rough set with respect to knowledge R if $LR(X) \neq HR(X)$, and X *is* said to be R - definable or an exact set with respect to knowledge R . The positive region , negative region , and boundary region of X with respect to knowledge R is given by

$POS_R(X) = LR(X)$

$NEG_R(X) = U- HR(X)$ and

$BND_R(X) = HR(X) - LR(X) = R$ - border line region of X , respectively.

C. Definition (Covering [12])

Let U be a universe of discourse of finite set and C is the family of non empty sub sets of U . Then C is called the covering of U if and only if $\cup C_i = U$, for $C_i \in C$.

The ordered pair (U, C) is known as covering approximation space.

D. Definion (Minimum Description of x)

Let (U, C) be a covering approximation space . Then the minimal description of x ($x \in U$) is defined as

$Md(x) = \cup\{K : K \in C_x \wedge (\forall S \in C_x(S \subseteq K \Rightarrow K = S)\}$, where $C_x = \{K \in C : x \in K\}$. For any $x \in U$, $\cup C_x$ is called the indiscernible neighborhood of x. Also $P(U)$,the power set of U , be defined as the set of all subsets of U.

E. Example:

Let $U = \{a, b, c, d\}$, $C = \{\{a, b\}, \{b, d\}, \{c\}, \{c,d\},\{b, c, d\}\}$, be a covering of U .Then $Md(a) = \{a,b\}$, $Md(b) = \{a,b\} \cup \{b, d\}$, *as* $\{b, d\}$ is a subset of $\{b,c,d\}$, $Md(c) = \{c\}$, *as* $\{c\}$ is a subset of $\{c, d\}$ *and* $\{b, c, d\}$, $Md(d)=\{b, d\} \cup \{c, d\}$, as $\{b.d\}$ is asubset of $\{b,c,d\}$ and $\{c,d\}$ is also subset of $\{ b,c,d\}$.

F. Definition(Covering Based Rough Set [1],[13])

Let (U, C) be the covering approximation space. The operators $L, H : P(U)\rightarrow P(U)$ are defined as follows, for all $X \subseteq U$.

$L(X) = \cup \{K \in C:K \subseteq X\}$*and*

$H(X) = L(X) \cup \{Md(x) : x \in X - L(X)\}$, where $L(X)$ and $H(X)$ are known as the covering lower and covering upper approximation operators respectively.

The set $X \subseteq U$ is called the covering based rough set if $L(X) \neq H(X)$, otherwise X is said to be an exact set (definable set).

The boundary region or border line region of X of is defined by $BND_F(X) = H(X) - L(X)$.

G. Example:

Let $U = \{a, b, e, d\}$, $C = \{\{a, b\}, \{b, c\}, \{b,c,d\},\{d\}\}$ be a covering of U. If $X = \{a, c\}$, Then we find covering based lower and upper approximation, as

Here $L(X) = \phi$, $X - L(X) = X$.

$Md(a) = \{a, b\}$, $Md(c) = \{b, c\}$ as $\{b,c\} \subset \{b, c, d\}$ *and*

$\cup Md(X) = Md(a) \cup Md(c) = \{a, b\} \cup \{b, c\} = \{a, b, c\}$

So $H(X) = \{a,b,c\}$.Hence X is a covering based rough set as $L(X) \neq H(X)$. .

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III. COVERING OF SOFT SET AND SOFT ROUGH SET

Here we present the soft set introduce by Molodtsov ([7]) and covering of soft set introduced by ([4]), Suziye Yuksel et al ([11],[12]). Some application on soft set theory is given by P.K.Maji et al ([4],[3]).

A. Definition:(Soft Set [3],[4],[7])

Let U be the universe of discourse, a finite set. E denotes Set of parameter, $P(U)$ is the Power set of U. In defining a mapping $F : A \to P(U)$, *where* $A \subseteq E$. Thus a pair (F, A) is called a soft set over U. Here $F(e)$ be the set of e-approximate element of the soft set (F, A), for $e \in A$.

B. Definition: (Full Soft Set [11],[12])

Let $S = (F, A)$ be a soft set over U, If $\bigcup_{a \in A} F(a) = U$, then S is said to be a full soft set.

C. Definition: (Covering Soft Set [11],[12])

A full soft set $S = (F, A)$ over U is called as a covering soft set if $F(a) \neq \emptyset$, for each $a \in A$.

D. Examples:

1) Example:

Suppose U be the set of houses under consideration, E is this set of parameters. Each parameter is a word or a sentence. Let E = {*expensive, beautiful, wooden, cheap, in green surrounding, modern, repaired, dilapited* } = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ say.

To define a soft set, We have to point out *expensive houses*, *beautiful houses* and so on. The soft set (F, A) describes the "*attractiveness of the houses*" which Mr.Y (say) is going to purchage

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the houses, $A = \{e_l, e_2, e_3, e_4, e_5\}$, where $A \subseteq E$. We define a mapping $F : A \to P(U)$ by $F(e_l) = \{h_2, h_4\}$,$F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_3, h_4, h_5\}$, $F(e_4) = \{h_1, h_3, h_5\}$, $F(e_5) = \{h_1\}$. Then (F, A) is a soft set over U and (F, A) = { *Expensive house* / $\{h_2, h_4\}$, *beautiful house* /$\{h_1, h_3\}$, *wooden houses* / $\{h_3, h_4, h_5\}$. *cheap house* /$\{h_1, h_4, h_5\}$, *cheap house green surrounding* / $\{h_l\}\}$ = $\{e_l/\{h_2, h4\}, e_2/\{h_1, h_3\}, e_3/\{h_3, h_4, h_5\}, e_4/\{h_l, h_3, h_5\}, e_5/\{h_l\}\}$

Here $\bigcup_{e \in A} F(e) \neq U$. So (F, A) is not a full soft set, but it is a Soft set.

2) Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}$, $A = \{a_1, a_2, a_3, a_4, a_5\}$ and F is a mapping $F : A \to P(U)$, where $F(a_1) = \{h_1, h_3, h_5\}$, $F(a_2) = \{h_4\}$, $F(a_3) = \{h_3, h_4, h_6\}$, $F(a_4) = \{h_3, h_7, h_8\}$ $F(a_5) = \{h_2, h_4, h_6\}$.
Here $\bigcup_{a \in A} F(a) = U$ and $F(a) \neq \emptyset$ for any $a \in A$.
So (F, A) is a soft set, which is a full soft set and also $\{F(a_1), F(a_2), F(a_3), F(a_4), F(a_5)\}$ is a covering of U.

3) Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}$,
$A = \{a_1, a_2, a_3, a_4, a_5\}$ and F is a mapping $F : A \to P(U)$ where $F(a_1) = \{h_1, h_2, h_3\}$, $F(a_2) = \emptyset$,
$F(a_3) = \{h_3, h_5, h_7\}$,$F(a_4) = \{h_4, h_6, h_8\}$,
$F(a_5) = \{h_1, h_6, h_7\}$. Here $\bigcup_{a \in A} F(a) = U$
So (F, A) is a full soft set, but not a covering soft set of U as $F(a_2) = \emptyset$.

E. Soft Rough Set

Hybridization of two theories, soft set theory and rough set theory is yield rough soft set and soft rough set given by ([2],[6],[14]). Here we produce the definition of Soft Rough Set only.

1) Definition: (Soft lower and upper approximation)

Let $S = (F, A)$ be a soft set over U. The pair $P = (U, S)$ is called a soft approximation space. Based on the soft approximation space P, we define the following two operations for $X \subseteq U$,

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$L(apr_p(X)) = \{ u \in U: \exists\ a \in A | u \in F(a) \subseteq X\} = \bigcup_{a\in A}\{F(a) : F(a) \subseteq X\}$.

$H(apr_p(X)) = \{ u \in U: \exists\ a \in A | u \in F(a) \cap X \neq \emptyset \} = \bigcup_{a\in A} F(a) | F(a) \cap X \neq \emptyset \}$, are called the soft P - lower and P - upper approximation of X with respect to P.

2) Definition: (Soft Rough Set [2])

Feng Feng et al has given the definition of Soft Rough Set as:

The soft P - positive region ,the soft P - negative region and the soft P - boundary region of $X \subseteq U$, can be defined and denoted by

$POSp(X) = L(apr_p(X))$,

$NEGp(X) = U\text{-}H(apr_p(X))$,

$BNDp(X) = H(apr_p(X)) - L(apr_p(X))$, respectively. X is said to be soft P - rough set if $L(apr_p(X)) \neq H(apr_p(X))$, otherwise X is said to be soft P-definable. From the definition, we have X as a soft definable set, if $BNDp(X) = \phi$.

In [2] Feng Feng et al has established the fact that $L(arp_p(X)) \subseteq X$ and $L(apr_p(X)) \subseteq H(apr_p(X))$ for all $X \subseteq U$, but $X \subseteq H(apr_p(X))$ does not hold in general. This is illustrated by the following examples.

F. Examples:

1) Example:[2]

Let $U = \{u_1, u_2, u_3, u_4 u_5, u_6 \}$, $E = \{a_1, a_2, a_3, a_4 a_5, a_6 \}$, considering $A = \{a_1, a_2, a_3, a_4 \}$, where $F(a_1)=\{u_2,u_4\}$, $F(a_2) = \{u_1\}$, $F(a_3)=\emptyset$, $F(a_4) = \{u_2, u_3,u_6\}$, $F(a_5)= \{u_l,u_5\}$, $F(a_6) = \{n_3, u_4\}$.
Let $G = (F, A)$ *be a soft set over* U *and then* $P = (U, G)$ is the soft approximation space.

Considering $X = \{u_1, u_5, u_6\} \subseteq U$.
We have
$L(apr_p(X)) = \{u_l\}$, $H(apr_p(X)) = \{u_1, u_2, u_3, u_6 \}$,

Here $L(apr_p(X)) \neq H(apr_p(X))$, So X is a soft rough set. But $X \nsubseteq H(apr_p(X))$.

2) Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}$ be the Universal set, $E = \{e_l, e_2, e_3, e_4, e_5, e_6\}$ be the set of parameters. Let $F : A \rightarrow P(U)$, be a mapping given by $F(e_1) = \{h_1, h_3, h_5\}$, $F(e_4) = \{h_7, h_8\}$, $F(e_5) = \{h_8\}$, $F(e_6) = \{h_4, h_6, h_8 \}$.

Considering $A = \{e_l, e_4, e_5, e_6\} \subseteq E$ and $X = \{ h_2, h_6, h_7, h_8 \}$, Then $L(apr_p(X)) = \{h_7, h_8\}$, $H(apr_p(X)) = \{h_4, h_6, h_7, h_8\}$.

It is clear that $L(apr_p(X)) \neq H(apr_p(X))$. Hence X is a soft rough set but here $X \nsubseteq H(apr_p(X))$.

To overcome the difficulties, we find the following definition.

G. Definitions

1) Definition:(Not Set of A)

Let $E = \{e_1, e_2, e_3, e_4, \ldots\ldots\ldots\ldots e_n, \}$ be the set of parameters. The not set E denoted by $\sim E$ and defined by $\sim E = \{ \sim e_1, \sim e_2, \sim e_3, \sim e_4, \ldots\ldots\ldots\ldots e_n\}$, Where $\sim e_i$ = not e_i, for each i .

2) Definition:(Complement of soft set)

The complete of a soft set (F,A) is denoted by $(F, A)'$ and given by $(F,A)' = (F', \sim A)$, where $F': \sim A \rightarrow P(U)$ *is* a mapping defined by $F'(a) = U - F(\sim a)$, for all $a \in \sim A$.

3) Definition(Modified Soft Rough Set , [6])

Let $S = (F, A)$ be a soft set over U. Then the triplet $P = (U, F, A)$ *is* called soft approximation space. The soft lower and soft upper approximation defined as , for $X \subseteq U$,

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$L^*(apr_F(X)) = \{u \in U: \exists\ e \in A\ |u \in F(e) \subseteq X\} = \cup_{e\in A}\{F(e) : F(e) \subseteq X\}$, and

$$H^*(apr_F(X)) = \begin{cases} M, \text{ for } X \subseteq M \\ M \cup N \text{ for } X \nsubseteq M \end{cases}$$

respectively, where

$M = \{u \in U: \exists\ e \in A|u \in F(e), F(e) \cap X \neq \emptyset\} = \cup\{F(e): F(e) \cap X \neq \phi\}$, and
$N = \cap\{F'(e) : e \in (-A)\}$.

The $L^*(apr_F(X))$ and $H^*(apr_F(X))$ are referred to soft rough lower and upper approximation of X with respect to parameterized mapping F_A, where $F_A : A \rightarrow P(U)$ be the illustrated mapping. The set $X \subseteq U$ is called F-soft rough set, if $L^*(apr_F(X)) \neq H^*(apr_F(X))$, otherwise X is called F-soft definable set . Here we denote the notation F_A to indicate the parameter set A and the mapping F. That is F_A and F have the same mapping from A to $P(U)$. In this article we use the notation F instead of F_A everywhere.

The F-soft positive region, F-Soft negative region and F-soft boundary region of X may be defined as
$POS_F(X) = L^*(apr_F(X))$,
$NEG_F(X) = \cup_{e\in A}\{F(e) : F(e) \cap X \neq \emptyset = U - M$, for $X \subseteq M$,
$BND_F(X) = H^*(apr_F(X)) - L^*(apr_F(X))$, respectively.

H. Examples:

1) Example:

Considering the example in III.*F,1* , we are now calculate lower and upper approximation of X according to definition given in III.G,
$L^*(apr_F(X)) = \{u_1\} \subseteq X$
$M = \{u_1, u_2, u_3, u_6\}$
$A = \{e_1, e_2, e_3, e_4\}$, So $\sim A = \{\sim e_1, \sim e_2, \sim e_3, \sim e_4\}$
$N = F'(\sim e_1) \cap F'(\sim e_2) \cap F'(\sim e_3) \cap F'(\sim e_4) = \{u_5\}$
$H^*(apr_F(X)) = M \cup N = \{u_1, u_2, u_3, u_5, u_6\}$, as $X \nsubseteq M$. Then $X \subseteq H^*(apr_F(X))$.

2) Example : (D. Mohanty [6])

Considering the data given in the example in III.F.2, We again illustrate lower and upper approximation of X according to definition given in III.G.
$L^*(apr_F(X)) = \{h_7, h_8\} \subseteq X$,
$M = \{h_4, h_6, h_7, h_8\}, N = \{h_2\}$
$H^*(apr_F(X)) = M \cup N = \{h_2, h_4, h_6, h_7, h_8\}$
Here $X \subseteq H^*(apr_F(X))$.

However, from these two examples we infer that the definition given for soft rough set in ([4]) is better than the previous one. (Soft rough set define in [2])

I. Proposition: [6]

Let $S = (F, A)$ be a soft set over U and $A \subseteq E$ be a set of parameters and $P = (U, F, A)$ be the corresponding soft approximation space. The F-Rough lower and F-Rough upper approximation satisfy the following properties for every $X, Y \subseteq U$.

$L^*(apr_F(\phi)) = \phi$, $\quad H^*(apr_F(\phi)) = \phi$

$L^*(apr_F(U)) = U$, $\quad H^*(apr_F(U)) = U$

$L^*(apr_F(X \cap Y)) = L^*(apr_F(X)) \cap L^*(apr_F(Y))$

$L^*(apr_F(X \cup Y)) \supseteq L^*(apr_F(X)) \cup L^*(apr_F(Y))$

$X \subseteq Y \Rightarrow L^*(apr_F(X)) \subseteq L^*(apr_F(Y))$ and $H^*(apr_F(X)) \subseteq H^*(apr_F(Y))$

$H^*(apr_F(X \cup Y) = H^*(apr_F(X)) \cup H^*(aprF(Y))$

$H^*(apr_F(X \cap Y)) \subseteq H^*(apr_F(X)) \cap H^*(apr_F(Y))$.

In the next section we define four types of soft rough set with covering based which is different from Saziye yuksel et al[12] and J Zhan and Q Wang [14].

IV. COVERING BASED SOFT ROUGH SET

In III. *C.*,it is given the definition of covering soft set written by Saziye Yuksel et al ([11],[12]). In these two articles ([11],[12]),Saziya Yuksel consider full soft set to define soft rough set and soft set is considered as covering of U.

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But, in this section we define covering of U first and then we define soft covering of U and the soft covering approximation spaces. This notion is totally different from the definition given in ([11],[12])..

A. Definition:(Covering rewriting)

Let $C = \{ C_1, C_2, C_3, \ldots\ldots C_n\}$, $C_i \subset U$, $1 \le i \le n$, then C is called covering of U if $\cup C_i = U$

B. Definition: (Soft Covering Approximation Space)

Let $K = (F, A)$ be a soft set on U and $A \subseteq E$. Then(C,K) ba a soft covering of U,for each $a \in A$, $F(a) = \cup C_i$. Then triplet $P = (U, C, K)$ be called a soft covering approximation space.

C. Definition: (Minimum Description)

Let C is a covering of U and $K = (F, A)$ be a soft set of U. And (U, C, K) be a soft covering approximation space. We denote $F(A) = \{ F(a) : a \in A\}$ and $C_x = \{T \in C: x \in T\}$ .Then minimum description of $x \in U$, can be defined as $Md_A(x) = \cup \{T \in C_x : T \in F(A) \wedge (\forall S \subseteq C_x \wedge (S \subseteq T \Rightarrow T = S))\}$.

D. Definition :(First type of covering based Soft Rough set:)

Let $P = (U, C, K)$ be a Soft covering approximation space for a set $X \subseteq U$, the soft covering Lower and upper approximation are , respectively defined as

$FL_F(X) = \cup\{C_i : C_i \subseteq F(e) \cap X \}$

$FH_F(X) = FL_F(X) \cup \{\cup Md_A(x) : x \in X - FL_F(X)\}$.

If $FL_F(X) \neq FH_F(X)$, then X is said to be the First type of Covering Based Soft Rough Set, otherwise X is called covering based soft definable.

1) Example

We note that in ([12]) Saziye Yuksel et al define soft lower and soft upper approximations as

$\underline{S}_*(X) = \cup_{a \in A}\{ F(a) : F(a) \subseteq X \}$,

$S^*(X) = \cup \{ Md_S(X) : x \in X \}$ and Minimum description $Md_S(x) = \{ F(a) : a \in A \wedge x \in F(a) \wedge (\forall e \in A \wedge x \in F(e) \subseteq F(a) \Rightarrow F(a) = F(e)))\}$.We compare the definition giving by Saziye Yuksel et al ([11]) with our definition of First type covering soft rough set.

Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a_1, a_2, a_3, a_4, a_5\} \subseteq E$, C is covering of U, where $C_1 = \{a, b\}$, $C_2 = \{b, c\}$, $C_3 = \{d\}$, $C_4 = \{e\}$, $C_5 = \{f\}$, $C_6 = \{g\}$, $C_7 = \{g, h\}$.

F is a mapping from A to C such that $F(a_1) = C_1$, $F(a_2) = C_2 \cup C_3$, $F(a_3) = C_4 \cup C_5$, , $F(a_4) = C_6$. $F(a_5) = C_7$.. Let $X = \{ b, d, e\}$,then to find out lower and upper soft covering approximation.
$FL_F(X) = \{d\} \cup \{e\} = \{d, e\}$, $X - FL_F(X) = \{b\}$.
$FH_F(X) = \{d, e\} \cup Md_A(b) = \{d, e\} \cup \{a, b\} \cup \{b,c\} = \{a, b, c, d. e\}$. -
According to Saziye Yuksel el at, the lower approximation $\underline{S}_*(X) = \emptyset$ and upper approximation $S^*(X) = Md_A(b) \cup Md_A(d) \cup Md_A(e) = \{a, b\} \cup \{b, c, d\} \cup \{e, f\} = \{a, b, c, d, e, f\}$.

We note that our definition is better than that of Saziye et al ([12]).

Next we define other three types of covering soft rough set with an example.

E. Definition:(Second type of covering based soft rough set)

Let $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximation are respectively defined as :

$SL_F(X) = \cup \{C_i : C_i \subseteq F(e) \cap X\}$

$SH_F(X) = \cup_{e \in A} \{C_i \subseteq F(e) : F(e) \cap X \neq \phi \}$.

If $SL_F(X) \neq SH_F(X)$. then X is said to be second type of covering based soft rough set, otherwise X is called covering based soft definable.

A. Definition :(Third type of covering based soft rough set:)

Let $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximation are, respectively defined as

$TL_F(X) = \cup \{ C_i : C_i \subseteq F(e) \cap X\}$.

$TH_F(X) = \cup \{Md_A(x) : x \in X \}$,

If $TL_F(X) \neq TH_F(X)$, then X is called third type of covering based soft rough set, otherwise X is said to be covering based soft definable set.

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B. Definition :(Fourth type of covering based soft rough set:)

Let $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximation are , respectively defined as :

$FhL_F(X) = \cup \{C_i : C_i \subseteq F(e) \cap X\}$

$FhH_F(X) =$

$FhL_F(X) \cup \{Z : Z \in C \text{ and } Z \cap (X - FhL_F(X)) \neq \phi\}$.

If $FhL_F(X) \neq FhH_F(X)$, then X *is* said to be fourth type covering based soft rough set, otherwise X is said to be covering based soft definable.

C. Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Let $A = \{e_1, e_2, e_3, e_5\}$, C is covering of U, where $C_1 = \{h_1, h_7\}$, $C_2 = \{h_1, h_2\}$, $C_3 = \{h_2, h_4, h_6\}$, $C_4 = \{h_1, h_3, h_5\}$, $C_5 = \{h_2, h_5, h_7\}$, $C_6 = \{h_1, h_6, h_8\}$, $C_7 = \{h_8\}$, $C_8 = \{h_2, h_6\}$

F is a mapping from A to C such that

$F(e_1) = C_1 \cup C_4$, $F(e_2) = C_3 \cup C_5$, $F(e_3) = C_7$, $F(e_5) = C_2 \cup C_8$.

Let us consider $X = \{h_2, h_6, h_7, h_8\}$, then to find out first , second , third and fourth type of soft covering lower and upper approximation.

$FL_F(X) = C_7 \cup C_8 = \{h_2, h_6, h_8\} = SL_F(X) = TL_F(X) = FhL_F(X)$, $Md_A(h_2) = C_2 \cup C_8 \cup C_5$ *as* $C_8 \subseteq C_3$, $Md_A(h_6) = C_8$, *as* $C_8 \subseteq C_3$, $Md_A(h_7) = C_1 \cup C_5$, $Md_A(h_8) = C_7$,

$\cup Md_A(x) = C_1 \cup C_2 \cup C_5 \cup C_7 \cup C_8 = \{h_1, h_2, h_3, h_5, h_6, h_7, h_8\}$ for all $x \in X$.

$FH_F(X) = (C_7 \cup C_8) \cup (C_1 \cup C_5) = \{h_1, h_2, h_5, h_5, h_6, h_7, h_8\}$,

$SH_F(X) = C_1 \cup C_2 \cup C_3 \cup C_5 \cup C_7 \cup C_8 = \{h_1, h_2, h_4, h_5, h_6, h_7, h_8\}$

$TH_F(X) = C_1 \cup C_2 \cup C_5 \cup C_7 \cup C_8 = \{h_1, h_2, h_3, h_5, h_6, h_7, h_8\}$,

$X - FhL_F(X) = X - (C_7 \cup C_8) = \{h_7\}$,

$FhH_F(X) = (C_7 \cup C_8) \cup (C_1 \cup C_5) = \{h_1, h_2, h_5, h_6, h_7, h_8\}$.

D. Properties

Let P = (U, C, K) be a soft covering approximation space. The soft covering lower and upper approximations of First type , Second type , Third type and Fourth type is satisfied the following properties, for X ⊆ U

1) Lower approximation $(X) \subseteq X \subseteq$ upper approximation (X)
2) Lower approximation $(\phi) = \phi$,
   Upper approximation $(\phi) = \phi$
3) Lower approximation $(U) = U$,
   Upper approximation $(U) = U$
4) $X \subseteq Y \Rightarrow$ Lower approximation $(X) \subseteq$ Lower approximation (Y) and Upper approximation $(X) \subseteq$ Upper approximation (Y).

We note here that, we consider full soft set to define all the four types of covering based soft rough set.

V. NEW TYPE COVERING BASED SOFT ROUGH SET

We name it as new type of covering based soft rough set since it is different from all four types of covering based soft rough set. In the new type we define Covering Based Soft Rough Set without using full soft set, that is, $F(a) = \cup C_i$ for some i, but $\cup F(a) \neq U$ for a $\in A$.

A. Definition:(Not Set of A, rewriting)

Let $E = \{e_1, e_2, e_3, e_4, \ldots\ldots\ldots\ldots e_n\}$ be the set of parameters. The Not set E denoted by $\sim E$ and defined by $\sim E = \{\sim e_1, \sim e_2, \sim e_3, \sim e_4, \ldots\ldots\ldots\ldots e_n\}$, Where $\sim e_i$ = not e_i , for each i .

B. Definition:(Complement of soft set , rewriting)

The complete of a soft set (F,A) is denoted by $(F, A)'$ and given by $(F,A)' = (F', \sim A)$, where $F': \sim A \rightarrow P(U)$ *is* a mapping defined by $F'(a) = U - F(\sim a)$, for all $a \in \sim A$. Thus for $b \in A$, $F'(\sim b) = U - F(b.)$ It is seen that complement of a soft set is not necessarily a soft set as the parameter $\sim a \notin A$.

C. Definition:(New Type of Covering Based Soft Rough Set)

Let $K = (F, A)$ is a soft set of U and may not be full soft set, then (C, K) be soft covering of U. $P = (U, C, K)$ be a soft covering approximation space. For a set $X \subseteq U$, the soft covering lower and upper approximations are respectively, defined as

$NL_F(X) = \cup \{C_i : C_i \subseteq F(e) \cap X\}$, *and*

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$NH_F(X) = \begin{cases} M, \text{ for } X\ M \\ M \cup N \text{ for } X \not\subseteq M \end{cases}$

Where
$M = \cup_{e\in A} \{C_i \subseteq F(e) : F(e) \cap X \neq \emptyset\}$, and
$N = \cap\{F'(e) : e \in (\sim A)\}$. If $NL_F(X) \neq NH_F(X)$, then X is said to be new type covering based soft rough set, otherwise X is said to be covering based soft definable.

A. Example:

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, Let $A = \{e_1, e_2, e_3, e_5\}$, C_K is covering of U, where $C_1 = \{h_1, h_7\}$, $C_2 = \{h_2, h_8\}$, $C_3 = \{h_3, h_6, h_9\}$, $C_4 = \{h_4, h_7\}$, $C_5 = \{h_1, h_7, h_{10}\}$, $C_6 = \{h_2, h_9\}$, $C_7 = \{h_2, h_4, h_6\}$, $C_8 = \{h_2, h_3, h_5\}$, and F is a mapping from A to C such that $F(e_1) = C_1 \cup C_2$, $F(e_2) = C_3$, $F(e_3) = C_4$, $F(e_4) = C_7 \cup C_8$, $F(e_5) = C_2 \cup C_6$, $F(e_6) = C_5 \cup C_1$, $F(e_7) = C_3 \cup C_8$. Here $\cup_{e\in A} F(e) \neq U$ that is, (F,A) is not a full soft set.

Let us consider $X = \{h_1, h_2, h_5, h_7, h_{10}\}$,then to find out first , second and new type of soft covering lower and upper approximations.
$FL_F(X) = C_1 = \{h_1, h_7\} = SL_F(X) = NLF(X)$,
$Md_A(h_1) = C_1$, $Md_A(h_2) = C_2 \cup C_6$, $Md_A(h5) = \emptyset$,
$Md_A(h_7) = C_4$, $Md_A(h_{10}) = \emptyset$,
$X - FL_F(X) = \{h_2, h_5, h_{10}\}$,
$\cup\, Md_A(x) = C_2 \cup C_6 \cup \emptyset = \{h_2, h_8, h_9\}$,
for all $x \in (X - FL_F(X))$.
$FH_F(X) = C_1 \cup (C_2 \cup C_6) = \{h_1, h_2, h_7, h_8, h_9\}$,
$SH_F(X) = C_1 \cup C_2 \cup C_4 \cup C_6 = \{h_1, h_2, h_4, h_7, h_8, h_9\}$,
$M = C_1 \cup C_2 \cup C_4 \cup C_6 = \{h_1, h_2, h_4, h_7, h_8, h_9\}$,
Here $X \not\subseteq M$. Then we have to find out N.
$N = F'(e_1) \cap F'(e_2) \cap F'(e_3) \cap F'(e_5) = \{h_3, h_4, h_5, h_6, h_9, h_{10}\}, \cap \{h_1, h_2, h_4, h_7, h_8, h_9, h_{10}\}, \cap \{h_1, h_2, h_3, h_5, h_6, h_8, h_9, h_{10}\}, \cap \{h_1, h_3, h_4, h_5, h_6, h_8, h_9, h_{10}\} = \{h_5, h_{10}\}$

$NH_F(X) = M \cup N = \{h_1, h_2, h_4, h_7, h_8, h_9, h_{10}\}$.

Here X is a New type covering based soft rough set since $NL_F(X) \neq NH_F(X)$ and $NL_F(X) \subset X \subset NH_F(X)$. But $X \not\subseteq FH_F(X)$, $X \not\subseteq SH_F(X)$, $X \subset NH_F(X)$.This is due to $F(A)$ is not a full soft set.

So New type covering based soft rough set is better approach than other models.

VI. Conclusion

Combination of theories has not only advanced research but also helped in tackling the issues of uncertainties in real life situations. This new extension of theory i.e. Covering Based Soft Rough Set has been modeled in four different types illustrating examples. We introduced four types of covering based soft rough set which are different from soft covering based rough set given by Saziye Yuksel et al ([12]). To conclude along with introducing new extended models, this article also presents the different problem in Covering Based Soft Rough Set inviting further investigations to find the more properties related to four types of Covering Based Soft Rough Sets.

We also define the New type of covering based soft rough set without taking full soft set which is more effective.

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